Ex. What portion of a Normal distribution lies within 5, 26, 36 away from the mean?

\[ P(|X - \mu| \leq 6) = P \left( \frac{|X - \mu|}{\sigma} \leq \frac{6}{\sigma} \right) \]

\[ = P(-1 \leq Z \leq 1) = F(1) - F(-1) \]

\[ = .8413 - .1587 = .6826 \]

68% within 5 from \( \mu \)

\[ P(|X - \mu| \leq 26) = P(-2 \leq Z \leq 2) \]

\[ = F(2) - F(-2) = .9772 - .0228 \]

\[ = .9544 \]
\[ P\left(1X - \mu_1 \leq 3 \right) = P\left(-3 \leq \frac{X}{\sigma} \leq 3 \right) = F(3) - F(-3) = 0.9987 - 0.0013 = 0.9974 \]

\[ P_{\text{3r. Pr.}}(\text{3}) \quad X \sim \text{Normal} (\mu, \sigma) \]
\[ \mu = 5 \cdot 10^6, \quad \sigma = 5 \cdot 10^5 \]
\[ P(X > 4 \cdot 10^6) \quad \text{Standardize} \]
\[ = P\left( \frac{X - \mu}{\sigma} > \frac{4 \cdot 10^6 - 5 \cdot 10^6}{5 \cdot 10^5} \right) \]
\[ = P\left( Z > -2 \right) = 0.9772 \]
Ex. (Inverse problem)
Salaries on some planet have Normal distribution with \( \mu = 200 \) coins, \( \sigma = 45 \) coins. Households with income within the lowest 8% are getting free milk. What range of salaries is that?

**Solution**

Need such \( x \):

\[
P(X \leq x) = 0.08
\]

\[
P\left( \frac{X - \mu}{\sigma} \leq \frac{x - \mu}{\sigma} \right) = 0.08
\]

\[
F\left( \frac{x - \mu}{\sigma} \right) = 0.08
\]

Using the table backwards,

\[
z = -1.405
\]
\[
\frac{x - \mu}{\sigma} = -1.405
\]

Unstandardize:
\[
x = 6(-1.405) + \mu
\]
\[
= 45(-1.405) + 200
\]
\[
= 136.8 \text{ coins}
\]

Inverse problem:
Find such \( x \) that \( P(X \leq x) = p \)

(1) Table \( \Rightarrow \) \( z \) such that
\[
P(\bar{z} \leq z) = p
\]

(2) Unstandardize:
\[
x = 6z + \mu
\]
Ex. 1d.

\[ P(1 \leq \theta \leq 0.97) = P(-0.97 \leq \theta \leq 0.97) \]

\[ = F(0.97) - F(-0.97) \]

\[ = 0.8340 - (1 - 0.8340) \]

\[ = 0.8340 - 0.1660 \]

\[ = 0.6680 \]

\[ P(1 \geq \theta \geq 0.97) = 1 - 0.6680 = 0.3320 \]

or

\[ P(\theta \leq -0.97) + P(\theta \geq 0.97) \]

\[ = 2F(-0.97) = 2(0.1660) = 0.3320 \]
Ex. Salaries of CS graduates
≈ Normal with mean = 75K, st.deviation = 20K.

What portion will get 100K + ?

\[ X \sim \text{Normal} \left( \mu = 75, \sigma = 20 \right) \]

\[
P \left( X \geq 100 \right) = P \left( \frac{X - \mu}{\sigma} \geq \frac{100 - 75}{20} \right)
\]

\[ = P \left( Z \geq \frac{100 - 75}{20} \right) = P \left( Z \geq 1.25 \right) \]

\[ = 1 - 0.8944 = 0.1056 \]
Ex. An elevator has capacity of 4,000 lbs. Your weight is 150 lbs. Human weights are approx. Normal with $\mu = 180$ lbs, $\sigma = 45$ lbs. An elevator comes with 20 people there. Would you take it?

$P(\text{Total weight} > 4000)$

$= P \left( X_1 + \ldots + X_{20} + 150 > 4000 \right)$

$= P \left( X_1 + \ldots + X_{20} > 3850 \right) \text{ Use CLT Standardize}$

$= P \left( \frac{X_1 + \ldots + X_{20} - n\mu}{\sigma \sqrt{n}} > \frac{3850 - 20 \cdot 180}{45 \sqrt{20}} \right)$

$= P \left( Z > 1.24 \right)$

$= 1 - 0.8925 = 0.1075$
Ex. Rangers play 80 games, win each game with prob. 0.6. Find the probability of winning exactly 50 games.

\[ X = \# \text{ wins} = \text{Binomial} \left( n=80, p=0.6 \right) \]

\[ \approx \text{Normal} \left( \mu = np = 48, \sigma = \sqrt{np(1-p)} = \sqrt{19.2} = 4.4 \right) \]

\[ P(X = 50) = \left( \begin{array}{c} 80 \\ 50 \end{array} \right) \cdot 0.6^{50} \cdot 0.4^{30} \text{ for Binomial} \]

but \[ 0 \text{ for Normal} \]

\[ \frac{50}{80} \]

should \[ \approx \frac{1}{1} \]
Selection: continuity correction

\[ P(X = 50) = P(49.5 \leq X \leq 50.5) \]

\[ = P \left( \frac{49.5 - 48}{4.4} \leq \frac{X - \mu}{\sigma} \leq \frac{50.5 - 48}{4.4} \right) \]

\[ = P \left( 0.34 \leq Z \leq 0.57 \right) \]

\[ = F(0.57) - F(0.34) \]

\[ = 0.7157 - 0.6331 \]

\[ = 0.0826 \]

\[ \delta X \cdot P(\text{win } 75+ \text{ games}) \]

\[ = P(X \geq 75) = P \left( \frac{X - \mu}{\sigma} \geq \frac{75 - 48}{4.4} \right) \]

\[ = P(Z \geq 6.14) \]
$p(z \geq 6.14) \approx 0$