I understand that
- This is a closed books/notes exam but I can use a calculator.
- A cheat sheet including complex formulas is provided so I will NOT bring my own.
- I will choose the most appropriate answer for the questions. I will NOT get any credit by marking multiple choices for a single question. When changing my answers, I will erase the paper properly.
- I will NOT forget to define any variables I introduce.
- I will NOT use laptops, cellular phones, any cellular communication device, and I will turn off all these devices before starting my exam.
- My conduct during the exam will reside entirely within the limits of the UTD regulations governing scholastic honesty -detailed in the handbook of operating procedures Title V Chapter 49.

NAME (please print):

<table>
<thead>
<tr>
<th>Question</th>
<th>Out of</th>
<th>Points</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td></td>
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<td>Total</td>
<td>100</td>
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</table>

Q1. Put $T$ before a statement if you think that statement is true. Otherwise put $X$.

1. (       ) Economic order quantity cannot decrease with lower set up costs. X
2. (       ) Production mix at a facility can change the bottleneck. T

---

1This is a sample as such it only indicates the type of the questions but not the number of questions.
Q2: Choose the most appropriate answer and mark your answer on this paper.

1. Heuristic rules are usually applied in practice when a problem
   (a) Deals with high costs or revenues
   (b) Has a small number of alternative solutions
   (c) Is important for the upper management
   (d) Has a large number of alternative solutions
   (e) Both a and c

   D

2. A production line is to be designed for a job with three tasks. Task times are 0.5, 0.7 and 0.9 minutes. The maximum and minimum possible cycle times are:
   (a) 0.9, 0.5
   (b) 2.7, 1.5
   (c) 2.1, 0.9
   (d) 2.1, 1.5
   (e) 1.6, 0.5

   C

3. In an assembly operation at a furniture factory, seven employees assemble an average of 350 standard dining chairs per 5-day week. What is the labor productivity of this operation?
   (a) 50 chairs/(worker x week)
   (b) 70 chairs/week
   (c) 50 chairs/week
   (d) 10 chairs/day
   (e) 70 chairs/worker

   A

4. Determine which of these are true.
   I: Labor content of the final good (product/service) is higher in service industries than in manufacturing.
   II: Measuring productivity is easier in service than in manufacturing.
   III: Standardization can be achieved easily in manufacturing than in service.

   (a) I, II and III are all true
   (b) Only I and II are true
   (c) Only III is true
   (d) Only I and III are true
   (e) None are true

   D

5. Which of the following is typically dealt by operations managers?
   (a) Facility location
   (b) Set of products to be offered
   (c) Motivation of workers
   (d) Scheduling
   (e) All of the above

   E
6. Models are not built to
   (a) Improve communication
   (b) Make concepts more abstract
   (c) Allow experimentation
   (d) Standardize the situation at hand for analysis
   (e) Improve understanding

B

7. In queuing, pooling refers to
   (a) Putting different customer classes into a single class
   (b) Replacing a server with a faster server
   (c) Sending overflowing customers from one server to another
   (d) Allowing for only one queue for multiple servers
   (e) Aggregating customer service times to end up with an average service time

D

8. Erlang’s thruput loss formula gives losses due to
   (a) Server breakdowns
   (b) Impatient customers leaving the queue after waiting for a certain amount of time
   (c) Customers not entering the queue due to busy servers and unavailable buffer space
   (d) Customers not entering the queue due to busy servers and full buffers
   (e) Customers not entering the queue due to long queue

C
Essay type questions of the following form.

[Fall 15] **Refinishing Old Cabinets.** Carrollton Cabinet Conditioning Company (4C) refinishes wood kitchen and bathroom cabinet doors. Cabinet doors experience most wear/tear and can be removed easily from cabinets to be taken to 4C’s refinishing facility. Refinishing involves 4 steps: Removing the old paint with a sandpaper from the surface, Priming the surface with a colorless coat of paint to close holes and repair irregularities, Painting and Inspection. 4C employs 9 workers in total but they are assigned to each step as follows: 3 workers at Removing, 2 at Priming, 3 at Painting, 1 at Inspection. The processing times at each step are 24 minutes at Removing; 18 minutes at Priming; 21 minutes at Painting and 6 minutes at Inspection. There are plenty of storage space between these steps to keep work-in-process inventory of cabinet doors.

**a) What is maximum hourly refinished cabinet door output?**

**Answer:** Capacity is $\frac{3(60)}{24}=7.5$ cabinets per hour at Removing, $\frac{2(60)}{18}=6.67$ cabinets per hour at Priming, $\frac{3(60)}{21}=8.57$ cabinets per hour at Painting, $\frac{1(60)}{6}=10$ cabinets per hour at Inspection. Priming step is bottleneck with capacity 6.67 cabinets per hour, which is the maximum refinishing rate for all 4 steps.

**b) Today at 8 am there was no work-in-process inventory at 4C and a truck loaded with 183 cabinet doors arrived for refinishing. How long does it take to finish these doors?**

**Answer:** The first door consumes 24+18+21+10=73 minutes to complete. The subsequent doors require $\frac{60}{6.67}=9$ minutes. All doors will take $73+182*9$ minutes.

**c) In order to improve productivity, 8 workers are cross-trained. That is each worker can do all steps of removing, priming and painting. The remaining 9th worker still does inspection. When a door arrives in this configuration, a worker picks it up and does removing, priming and painting all by himself. What is maximum hourly refinished cabinet door output?**

**Answer:** Capacity is $\frac{9(60)}{24}=(24+18+21)=9(60)/63=8.57$ cabinets per hour at Removing-Priming-Painting, $\frac{1(60)}{6}=10$ cabinets per hour at Inspection. Removing-Priming-Painting step is bottleneck with capacity 8.57 cabinets per hour, which is the maximum refinishing rate for all 4 steps.

**d) Regular workers are paid hourly rate of $20 and cross-trained workers are paid at $30. How much extra does 4C pay for labor expenses after cross-training on a per door basis? In order to compensate for this extra cost, what should the minimum profit margin for refinishing a door? We assume that the refinishing system experiences enough demand to keep it running at 100% capacity.**

**Answer:** Extra cost is $3*(30-20)=30$ per hour. This extra cost is $30/6.57$ per door under the assumption of 100% utilization. By adopting cross-trained workers, 8.57-6.67=1.9 extra doors can be refinished per hour. For these doors to compensate extra labor expense, we need margin $m$ to solve $1.9m \geq 30$. The smallest value of $m$ is $15.79=30/1.9$. 
Matching of Couples. One of the fastest growing businesses in the US is online matching of individuals for dating. In this business, each individual (man or woman) is asked about his/her interests, hobbies. This information is captured in a profile vector of length 48, denoted by \( x \) for a woman and \( y \) for a man, \( 0 \leq x, y \leq 10 \).

- The first element of the profile vector can be the answer to the question: Please rate from a scale of 0 to 10, how much you would like to cook at home? If the answer is “absolutely in love” with travel for a woman \( i \), she would put 10 and her profile vector has \( x_{i,1} = 10 \).
- The forty eighth element of the profile vector can be the answer to the question: Please rate from a scale of 0 to 10, how much you would like to travel? If the answer is “somewhat” for a man \( j \), he would put 4 and his profile vector has \( y_{j,48} = 4 \).

A metric of compatibility between woman \( i \) and man \( j \) is the sum of squared distances: 
\[
(1/48) \sum_{k=1}^{48} (x_{i,k} - y_{j,k})^2
\]
A matching firm aims to match individuals that are compatible with each other once a week. If the compatibility measured by the metric is worse than the threshold level 9, woman \( i \) and man \( j \) are not matched. A week can start with \( W = 3,428 \) women and \( M = 3,012 \) men profiles to be matched. In order to show some progress, the firm requires to match at least 100 couples in a week.

\[4pts\] a) Decide what the objective of a matching firm should be and express this objective in terms of parameters described above and decision variables you may introduce. Determine if your objective is linear or not.

**Answer:** Let \( z_{i,j} = 1 \) if woman \( i \) and man \( j \) are matched; 0 otherwise. The objective is to maximize compatibility or to reduce the lack of compatibility measured by the distance between the matched couples:

\[
\min \sum_{i=1}^{3,428} \sum_{j=1}^{3,012} z_{i,j} \sum_{k=1}^{48} (x_{i,k} - y_{j,k})^2
\]

Note that profile vectors \( x \) and \( y \) are data and the objective is linear in the decision variable \( z_{i,j} \).

\[4pts\] b) For the context described above, write constraint(s) to complete your formulation.

**Answer:** 
\( z_{i,j}(1/48) \sum_{k=1}^{48} (x_{i,k} - y_{j,k})^2 \leq 9 \) for each woman \( i \) and man \( j \). This constraint can also be written as
\[
z_{i,j} \leq \frac{9(48)}{\sum_{k=1}^{48} (x_{i,k} - y_{j,k})^2} \quad \text{for } i = 1 \ldots 3,428, j = 1 \ldots 3,012.
\]

Another set of constraints is
\[
\sum_{i=1}^{3,428} \sum_{j=1}^{3,012} z_{i,j} \geq 100.
\]

\[4pts\] c) Widely different profiles render matching 100 impossible. Then the associated formulation is infeasible. In view of your constraints, identify a condition of infeasibility on the problem parameters.

**Answer:** Constraints cannot be satisfied if
\[
\sum_{i=1}^{3,428} \sum_{j=1}^{3,012} \frac{9(48)}{\sum_{k=1}^{48} (x_{i,k} - y_{j,k})^2} < 100.
\]

\[4pts\] d) The matching firm earns revenue from weekly subscription fees of the registrants. For revenue stability, it targets to have at least 2,000 women and 2,000 men subscriptions after this week’s matches who stop their subscriptions. Add a constraint to ensure these requirements.

**Answer:** If \( z_{i,j} \geq 2,000 \) and \( z_{i,j} \geq 2,000 \), which are both satisfied by
\[
\sum_{i=1}^{3,428} \sum_{j=1}^{3,012} z_{i,j} \leq 1,012.
\]

\[4pts\] e) Discuss in at most 3 sentences if the compatibility of matched couples improves with the increasing number of subscriptions.

**Answer:** Yes compatibility improves, because then there are more options to match each individual with.
Commonly demanded types of bottled water at supermarkets are spring water, purified water and filtered water. Spring water is often bottled at the water spring while filtered bottle can be bottled anywhere as the filtration process is simple. For example, see the filtration machine at WalMarts where you can fill your empty jug with filtered water. This question focuses on purified water only.

Purification of water on the contrary is more involved. It involves 8 steps in the following order from source to bottling: carbon filtration, pretreatment, demineralization, remineralization, micro-filtration, ultraviolet light disinfection, ozone disinfection and bottling. Water purification facilities generally work 24 hours per day and 7 days per week. The capacity of a small and a medium sized purification facility are respectively 4,000 liters and 5,000 liters per hour. These facilities do not have large enough tanks to store water before bottling so the water constantly flows in the pipes. Bottled water can be stored for at most 18 months in warehouses or stores.

Nestlé is considering to build a new purified water facility to produce purified water brand Pure Life. Pure Life is planned to be sold through only large supermarkets in packages of 12 half-liter bottles. The new facility is to meet the demand in Texas, where the major cities are DFW, Houston, San Antonio and Midland. The daily Texan demand for Pure Life is estimated to be 36,000 packages per summer day and 24,000 packages per winter day. Each of the parts below should be answered independently.

2 pts] a) Among the process types (job shop, batch, repetitive/assembly, continuous, project), which one is more appropriate for water purification? Explain in 1-2 sentences.

**Answer:** Continuous as the water continuously flows in the facility.

3 pts] b) To meet the Texan demand in the summer with small purification facilities only, how many facilities are necessary?

**Answer:** 36,000 packages per day = 432,000 bottles per day = 18,000 bottles per hour = 9000 liters per hour ≤ 3 (4000 liters per hour) = 3 small purification facilities.

3 pts] c) To meet the Texan demand in the winter with medium purification facilities only, how many facilities are necessary?

**Answer:** 24,000 packages per day = 288,000 bottles per day = 12,000 bottles per hour = 6000 liters per hour ≤ 2 (5000 liters per hour) = 2 medium purification facilities.

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2 Based on a UTD team visit to Nestlé facilities, WalMart DC and stores in Northwest Arkansas on March 14, 2013.
d) To meet the Texan demand in the summer and have the highest facility utilization, how many small and how many medium purification facilities are necessary?

**Answer:** 36,000 packages per day = 432,000 bottles per day = 18,000 bottles per hour = 9000 liters per hour = 4000 + 5000 liters per hour = 1 small and 1 medium purification facilities, which yield a utilization of 100%.

e) When a medium purification facility works at full capacity, it takes the tap water and purifies it in 24 minutes on average. How much water should we expect to find in the pipes of such a facility?

**Answer:** Throughput=5000 liters per hour, flow time=2/5 hour, pipeline inventory=(5000)2/5=2000 liters.

f) 2 small facilities do not have enough capacity to meet the summer demand, what is the capacity shortage in terms of liter per hour? If Nestlé prefers to rely only on inventory to meet the summer demand, how many cubicmeters of purified water should be on-hand at the beginning of the summer? Suppose that the summer lasts 6 months and each month has 30 days. Would this amount of on-hand water fit into a warehouse whose dimensions are 100 x 50 x 8 meters (length x width x height)?

**Answer:** Capacity=8000 liters per hour, demand=9000 liters per hour, capacity shortage=1,000 liters per hour = 1 cubicmeter per hour = 24 cubicmeter per day = 4,320 cubicmeter per summer. The volume of the warehouse is 4000 cubicmeter and is not sufficient. Nestlé’s practice is to rent extra seasonal warehouse space before the summer to hold the bottled water.

g) Nestlé determines to build 2 small facilities and wants to start the summer with 3,360 cubicmeter of on-hand bottled water inventory. How many weeks in advance of summer should Nestlé start to build this inventory?

**Answer:** Capacity=8000 liters per hour, demand=6000 liters per hour, excess capacity=2,000 liters per hour = 2 cubicmeter per hour = 48 cubicmeter per day. It takes 3,360/48=70 days = 10 weeks to build the inventory.

h) Carbon filtration causes 2% of the incoming water to this process to be wasted and the corresponding number for micro-filtration is 3%. Assuming that the other purification processes have no waste, how many liters of water needs to be input into (after bought from the municipality) the purification facility on a summer day?

**Answer:** For an output of 432,000 bottles, 216,000 liters of daily output are necessary. The input should be 216,000/((0.98)(0.97))=227,225 liters per day.
Bottling complex capacity and location. This builds on the purified water supply chain discussion of the previous question but it is independent of parts a)-h) therein. Nestlé needs to determine the location and the capacity of the new bottling (purification) complex, which can include several small and medium sized facilities. It has markets with the following characteristics:

<table>
<thead>
<tr>
<th>Market</th>
<th>x-coordinate in 10 miles</th>
<th>y-coordinate in 10 miles</th>
<th>Demand per summer day in 1000 packages</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFW</td>
<td>10</td>
<td>25</td>
<td>11</td>
</tr>
<tr>
<td>Houston</td>
<td>19</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>Midland</td>
<td>-20</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>San Antonio</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

Water has a high density (1 kilogram per liter = 1 ton per cubic meter) so trucks carrying water from a purification facility to markets reach their weight capacity when about half-filled by volume. Packages of bottled water are put on pallets and shrink-wrapped to reduce damage. Immediately over a pallet, packages are put on a 4x6 configuration, that allows for 24 packages on the first level. 8 packages can be put on each other to make up 8 levels with 4 x 6 packages on each level. Attempting to put more levels will bend and damage the water bottles. In summary, a pallet carries packages in 4 x 6 x 8 configuration (length x width x height). Trucks can carry only an integer number of pallets and have a weight capacity of 24 tons (=24,000 kilogram). Irrespective of what the truck carries, the trucking company charges $10 per mile (= $100 per 10 miles).

\[ \text{2pts] a) Recall that the capacity of a small and a medium sized purification facility are respectively 4,000 liters and 5,000 liters per hour. Express these capacities in terms of 1000s of packages per day. Suppose that the daily cost of operating small and medium facilities are } K_s \text{ and } K_m. \]

**Answer:** 4000 liters per hour = 96,000 liters per day = 192,000 bottles per day = 16,000 packages per day. 5000 liters per hour = 120,000 liters per day = 240,000 bottles per day = 20,000 packages per day.

\[ \text{[4pts] b) Find the transportation cost } c \text{ of shipping 1000 packages over 10 miles.} \]

**Answer:** With a capacity of 24 tons a truck can carry 24,000 liters. A pallet has 4x6x8=192 packages, which is 2304 bottles or 1152 liters. 24,000/1,152 is slightly over 20, so a truck can carry 20 pallets and charges $100 per 10 mile. In other words, 20x192=3840 packages are transported at $100 per 10 mile, that is paying $26 (=100/3.84) per 1000 packages over 10 miles. So } c = 26 \text{ per 1000 packages and per 10 miles.} \]

\[ \text{[10pts] c) Define the decision variables necessary to formulate this location and capacity problem. Express the objective of minimizing daily cost of operating facilities and of transportation on a summer day in terms of decision variables and } c, K_s \text{ and } K_m. \]

**Answer:** Location of new facility complex \((a, b)\), the number of small and medium sized facilities built in that complex \(y_s\) and \(y_m\), which are integer variables. Let \(d^x_D\) be the east-west distance travelled to get to the DFW from the bottling complex. Let \(d^y_D\) be the north-south distance travelled to get to the DFW from the bottling complex. Similarly define \(d^x_H\) and \(d^y_H\) for Houston, \(d^x_M\) and \(d^y_M\) for Midland, and \(d^x_S\) and \(d^y_S\) for San Antonio. Hence, \(d^x_D \geq a - x_D\), \(d^y_D \geq x_D - a\) and \(d^x_H \geq b - y_D\), \(d^y_H \geq y_D - b\). This leads to following constraints to define the distances:

\[
\begin{align*}
    d^x_D & \geq a - x_D, & d^x_H & \geq b - y_D, & d^x_M & \geq a + 20, & d^x_S & \geq a, & d^x & \geq a - x_D, & d^x & \geq b - y_D & \text{ for Houston.} \\
    d^y_D & \geq 10 - a, & d^y_H & \geq 19 - a, & d^y_M & \geq 20 - b, & d^y_S & \geq -a, & d^y & \geq b - y_D & \text{ for Midland.} \\
    d^H & \geq 25 - b, & d^M & \geq 25 - b, & d^S & \geq b, & d^H & \geq 1 - b & \text{ for San Antonio.}
\end{align*}
\]

The objective is minimize \(K_s y_s + K_m y_m + 11c(d^x_D + d^y_D) + 13c(d^x_H + d^y_H) + 4c(d^x_M + d^y_M) + 8c(d^x_S + d^y_S)\).

To meet the demand of 36,000 packages, we have the capacity constraint \(16,000 y_s + 20,000 y_m \geq 36,000\).
TexEx (Texan Express) is a credit card company that collects check payments from three major regions in the U.S.: South (S), West (W) and East (E). The average daily value of payments mailed to TexEx are $120 K from customers in the South, $80 K from customers in the West and $100 K from customers in the East. TexEx must decide where these payments should be sent to. Three potential locations are San Angelo (s), Texas for the southern customers; Walla Walla (w), Washington for the western customers; Edison (e), New York for the eastern customers. Annual cost of operating a check-receiving office is $200 K in San Angelo, $120 K in Walla Walla and $250 K in Edison. If all offices are operated, TexEx will assign customer regions to offices as follows: S-s, W-w, E-e. However, given the cost of operating the offices, operating offices in all three locations may not be profitable.

2 pts a) A check sent from anywhere in the South to San Angelo arrives in two days. What is the total value of checks at any time in the mail sent from the South to San Angelo?

Answer: The question gives throughput and time in the (postal) system and asks for inventory. The inventory is 2*120 K=240 K.

2 pts b) This is independent of a). TexEx assesses the value of checks in the mail sent from the South to San Angelo. That value is $360 K but this amount is in the mail and cannot be invested by TexEx. TexEx can earn 20% annually by investing the payments. What is the annual opportunity cost of not immediately investing the payments sent from the South to San Angelo?

Answer: TexEx has an inventory of $360 K in the mail and the annual cost of that is 0.2*360 K=72 K.

4 pts c) This is independent of a) and b). TexEx can earn 10% annually by investing the payments. The annual opportunity costs of not immediately investing the payments sent from a region (S, W, E) to offices (s, w, e) are given by the following table.

<table>
<thead>
<tr>
<th>K $</th>
<th>s</th>
<th>w</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>36</td>
<td>60</td>
<td>48</td>
</tr>
<tr>
<td>W</td>
<td>40</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td>E</td>
<td>40</td>
<td>60</td>
<td>30</td>
</tr>
</tbody>
</table>

In this table, the numbers in the first row 36, 60, 48 are obtained by assuming in turn that southern customers are assigned to s, or w, or e. Similar comments apply to the remaining two rows. Customers in a region are all assigned to an office. If the office in San Angelo is not operated and southern customers are assigned to Edison, these customers know only Edison and cannot send payments to any other office. Use the above table to deduce the number of days it takes for a mail to arrive from a region to a city, and fill in the table below:

<table>
<thead>
<tr>
<th>Days</th>
<th>s</th>
<th>w</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>3=36/(0.1*120)</td>
<td>5=60/(0.1*120)</td>
<td>4=48/(0.1*120)</td>
</tr>
<tr>
<td>W</td>
<td>5=40/(0.1*80)</td>
<td>3=24/(0.1*80)</td>
<td>6=48/(0.1*80)</td>
</tr>
<tr>
<td>E</td>
<td>4=40/(0.1*100)</td>
<td>6=60/(0.1*100)</td>
<td>3=30/(0.1*100)</td>
</tr>
</tbody>
</table>

Answer:

Each entry worths 4/9 points.
The rest of this question depends on c). To assign regions to offices \((x)\) and to determine operating offices \((y)\), TexEx needs a formulation. Define the decision variables \((x, y)\) with appropriate subscripts/superscripts for this formulation and decide if they are fractional (continuous), integer or binary.

**Answer:** To determine operating offices, we use binary variables \(y_s, y_w, y_e\). If \(y_s = 1\), open the San Angelo office; otherwise do not. Other \(y\) variables are defined similarly. To determine region to office assignment, we use binary variables \(x_{Ss}, x_{Sw}, x_{Se}, x_{Ws}, x_{Ww}, x_{We}, x_{Es}, x_{Ew}, x_{Ee}\).

The assignment of S-s, W-w, E-e is captured by \(x_{Ss} = 1, x_{Ww} = 1, x_{Ee} = 1\) and all other \(x\) variables are zero. The assignment of S-w, W-w, E-e is captured by \(x_{Sw} = 1, x_{Ww} = 1, x_{Ee} = 1\) and all other \(x\) variables are zero. The assignment of S-w, W-w, E-w is captured by \(x_{Sw} = 1, x_{Ww} = 1, x_{Ew} = 1\) and all other \(x\) variables are zero. In general, an \(x\) variable is indexed by a region and an office and it takes the value of 1 only when that region is assigned to that office.

**Answer:** To minimize opportunity cost of not investing immediately plus the cost of operating offices. 

**Answer:** Min \(200y_s + 120y_w + 250y_e + 36x_{Ss} + 48x_{Ss} + 40x_{Ww} + 24x_{Ww} + 48x_{Ww} + 40x_{Es} + 60x_{Ew} + 30x_{Ee}\).

**Answer:**

\[ x_{Ss} + x_{Sw} + x_{Se} = 1 \] for southern region;
\[ x_{Ws} + x_{Ww} + x_{We} = 1 \] for western region;
\[ x_{Es} + x_{Ew} + x_{Ee} = 1 \] for eastern region.

**Answer:**

\[ x_{Ss} + x_{Ws} + x_{Es} \leq 3y_s \] can assign to San Angelo if it is operating;
\[ x_{Sw} + x_{Ww} + x_{Ew} \leq 3y_w \] can assign to Walla Walla if it is operating;
\[ x_{Se} + x_{Ww} + x_{Ee} \leq 3y_e \] can assign to Edison if it is operating.
Spring 11] PlaIn insurance company of Plano has three claim adjusters in its main office. The office works 5 days per week. People with claims arrive at the office on an average rate of 20 per 8-hour day. An adjuster spends on average 40 minutes with a claimant. Assume that both service and interarrival times are exponentially distributed. 

[6pts] a) What is the utilization of adjusters? How many hours per week an adjuster spends with a claimant? What is the number of available adjusters on average?

Answer: \( a = \frac{8 \times 60}{20} = 24 \text{ mins}, \ p = 40 \text{ mins}, \ m = 3, \ u = \frac{p}{am} = \frac{40}{72} = \frac{5}{9} = 0.55 \). An adjuster spends 40*0.55=22.2 hours per week with a claimant. 

3*(1-5/9)=12/9=4/3 adjusters are available on average.

[4pts] c) How much time does a claimant expect to spend in the main office?

Answer:

\[
T_q = \frac{40}{3} \frac{(5/9)^{2(3+1)-1}}{1 - 5/9} \frac{1^2 + 1^2}{2} \\
= \frac{40}{3} \frac{9}{4} (5/9)^{8-1} \\
= 30(0.34) \\
= 10.2 \text{ minutes of waiting for an available adjuster.}
\]

The service time is 40 minutes, adding this to the waiting time, a claimant spends about 50 minutes in the PlaIn office.

[2pts] e) Write an expression for the probability that a claimant will find all adjusters busy. Do not attempt to compute the probability.

Answer: \( r = \frac{p}{a} = \frac{40}{24} = \frac{5}{3} \). That probability with \( m = 3 \) is 

\[
\frac{(5/3)^3 / 3!}{\sum_{i=0}^{3} (5/3)^i / i!}
\]
UTD has a safe-walk campus escort service (www.utdallas.edu/enroll/visit/safety.php) available 24 hours a day, 365 days a year. To request an escort call ext. 2331. Give your name and the nearest exit door to your location. A safe-walk escort will meet you at that building door and walk with you to any area on campus. Requests for escorts are received on average every 5 minutes, with a coefficient of variation of 1. After receiving a request, the dispatcher answering number 2331 contacts an available escort by a mobile phone. The available escort immediately proceeds to pick up the student and walk him/her to his/her destination. This in total takes about 25 minutes, with a coefficient of variation of 1. Currently there are 7 escorts at UTD. If they are all unavailable, the dispatcher puts the requests in a queue until an escort becomes available.

**a) What is the utilization of escorts?**

\[
a = 5, \quad p = 25, \quad m = 7, \quad u = p/(am) = \frac{25}{35} = \frac{5}{7} = 0.714.
\]

**b) What is the number of available escorts on average?**

\[
7 \times (1 - \frac{5}{7}) = 2 \text{ escorts are available on average.}
\]

**c) What is the waiting time in the queue and how long does it take for a student to arrive his/her destination after placing a call to 2331?**

\[
T_q = \frac{25}{7} \left( \frac{5}{7} \right)^3 \frac{1^2 + 1^2}{2} = \frac{25}{7} \cdot \frac{25}{2} \left( \frac{5}{7} \right)^3 = 4.55 \text{ minutes of waiting for an available escort.}
\]

The service time (pick up by escort and delivery to destination) is 25 minutes. The total time it takes to arrive the destination is 29.55 minutes.

**d) A recently established UTD security committee has decided that the maximum acceptable waiting time in the queue is 2 minutes. What is the minimum number of escorts to achieve this acceptable level?**

\[
T_q = \frac{25}{8} \left( \frac{5}{8} \right)^3 \frac{1^2 + 1^2}{2} = 3.125 \frac{0.635^{3.24}}{0.375} = 1.91 \text{ minutes of waiting for an available escort.}
\]

Thus, 8 escorts suffice.

**e) If UTD operates with 6 escorts, write an expression for the probability that a request will find all escorts busy. Do not attempt to compute the probability.**

\[
r = p/a = 5. \text{ That probability with } m = 6 \text{ is}
\[
\frac{5^6/6!}{\sum_{i=1}^{6} 5^i/i!}
\]
need to write only the modifications above. For clarity, we present the complete formulation:

Maximize \(5(x_{1F} + x_{1G}) + 7(x_{2F} + x_{2G}) + 3(x_{3F} + x_{3G})\)

ST: \(3x_{1F} + 4x_{2F} + 2x_{3F} \leq 30; \quad 4x_{1G} + 6x_{2G} + 2x_{3G} \leq 40\)

\(x_{1F} + x_{1G} \leq 7; \quad x_{2F} + x_{2G} \leq 5; \quad x_{3F} + x_{3G} \leq 9\)

\(x_{1F}, x_{1G}, x_{2F}, x_{2G}, x_{3F}, x_{3G} \geq 0.\)

All parts below are independent.

\[2pts\] a) ( ) This formulation assumes that same amount of profit is made from a product no matter where it is produced. Either write True (T) or False (X) into the parentheses. No explanation is necessary. \textbf{ANSWER:} \ T

\[3pts\] b) What is profit made from the sale of 1 unit of product 1? How many hours does it take to produce 1 unit of product 2 in Grapevine plant? What is the maximum number of units of product 3 that can be sold in the market? Write 3 numbers in the order of questions, no explanation is necessary: \ldots; \ldots; \textbf{ANSWER:} \ 5; 6; 9.

\[5pts\] c) The new vice president of operations wants to reduce the product offerings to avoid undue diversification of the product line. Modify the LP formulation to ensure that \textbf{at most two products} are produced.

\textbf{ANSWER:} Let \(y_i = 1\) if product \(i\) is produced; it is 0 otherwise. Modify the sales potential constraints:

\[
x_{1F} + x_{1G} \leq 7y_1 \quad \text{Sales potential of product 1 if it is produced.}
\]

\[
x_{2F} + x_{2G} \leq 5y_2 \quad \text{Sales potential of product 2 if it is produced.}
\]

\[
x_{3F} + x_{3G} \leq 9y_3 \quad \text{Sales potential of product 3 if it is produced.}
\]

Add the new constraints: \(y_1 + y_2 + y_3 \leq 2\) and \(y_1, y_2, y_3 \in \{0, 1\}\). Students need to write only the modifications above. For clarity, we present the complete formulation:

Maximize \(5(x_{1F} + x_{1G}) + 7(x_{2F} + x_{2G}) + 3(x_{3F} + x_{3G})\)

ST: \(3x_{1F} + 4x_{2F} + 2x_{3F} \leq 30; \quad 4x_{1G} + 6x_{2G} + 2x_{3G} \leq 40\)

\(x_{1F} + x_{1G} \leq 7y_1; \quad x_{2F} + x_{2G} \leq 5y_2; \quad x_{3F} + x_{3G} \leq 9y_3\)

\(y_1 + y_2 + y_3 - 2 \leq 0\)

\(x_{1F}, x_{1G}, x_{2F}, x_{2G}, x_{3F}, x_{3G} \geq 0, \quad y_1, y_2, y_3 \in \{0, 1\}.
\]

\[5pts\] d) There is a cost of undue diversification stemming from loss of efficiencies in the production. This cost is estimated to be 10 (in the units that the profit is measured in LP formulation). It is incurred only when all three new products are produced. Modify the LP formulation to \textbf{appropriately account for the cost.}

\textbf{ANSWER:} Let \(y_i = 1\) if product \(i\) is produced; it is 0 otherwise. Let \(\bar{y} = 1\) when all three products are produced. That is \(\bar{y} = 1\) if and only if \(y_1 + y_2 + y_3 = 3\).

Insert new constraints: \(y_1 + y_2 + y_3 - 2 \leq \bar{y}\) and \(\bar{y} \in \{0, 1\}\). Deduct the cost of 10 from the objective: Maximize \(5(x_{1F} + x_{1G}) + 7(x_{2F} + x_{2G}) + 3(x_{3F} + x_{3G}) - 10\bar{y}\). Modify the sales potential constraints as in c).

Since we are maximizing, we want \(\bar{y} = 0\). However this is not possible when \(y_1 + y_2 + y_3 = 3\), which forces \(\bar{y} = 1\) by the constraint \(y_1 + y_2 + y_3 - 2 \leq \bar{y}\). Only when \(y_1 + y_2 + y_3 = 3\), we have \(\bar{y} = 1\) and we pay the cost. Students need to write only the modifications above. For clarity, we present the complete formulation:

Maximize \(5(x_{1F} + x_{1G}) + 7(x_{2F} + x_{2G}) + 3(x_{3F} + x_{3G}) - 10\bar{y}\)

ST: \(3x_{1F} + 4x_{2F} + 2x_{3F} \leq 30; \quad 4x_{1G} + 6x_{2G} + 2x_{3G} \leq 40\)

\(x_{1F} + x_{1G} \leq 7y_1; \quad x_{2F} + x_{2G} \leq 5y_2; \quad x_{3F} + x_{3G} \leq 9y_3\)

\(y_1 + y_2 + y_3 - 2 \leq \bar{y}\)

\(x_{1F}, x_{1G}, x_{2F}, x_{2G}, x_{3F}, x_{3G} \geq 0, \quad y_1, y_2, y_3, \bar{y} \in \{0, 1\}.
\]

Note that by setting \(\bar{y} = 0\), we obtain the formulation in c).

\[5pts\] e) The new VP wants use only one of the plants to produce 3 products. Apparently producing new products at these plants require some investment and PlaPharma is willing to make that investment only at one of the plants. Modify the LP formulation to ensure that \textbf{exactly one plant} is utilized to produce new products.

\textbf{ANSWER:} Let \(z_F = 1\) when Frisco plant is used. If \(Z_F = 0\), Grapevine plant is used. We add constraint \(z_F \in \{0, 1\}\) to ensure that exactly one of the plants is used. We also need to modify the plant capacity constraints:

\(3x_{1F} + 4x_{2F} + 2x_{3F} \leq 30z_F\) \quad Work hours capacity constraint in Frisco if this plant is used.

\(4x_{1G} + 6x_{2G} + 2x_{3G} \leq 40(1 - z_F)\) \quad Work hours capacity constraint in Grapevine if this plant is used. Students need to write only the modifications above. For clarity, we present the complete formulation:

Maximize \(5(x_{1F} + x_{1G}) + 7(x_{2F} + x_{2G}) + 3(x_{3F} + x_{3G})\)

ST: \(3x_{1F} + 4x_{2F} + 2x_{3F} \leq 30z_F; \quad 4x_{1G} + 6x_{2G} + 2x_{3G} \leq 40(1 - z_F); \quad z_F \in \{0, 1\}\)

\(x_{1F} + x_{1G} \leq 7; \quad x_{2F} + x_{2G} \leq 5; \quad x_{3F} + x_{3G} \leq 9; \quad x_{1F}, x_{1G}, x_{2F}, x_{2G}, x_{3F}, x_{3G} \geq 0.\)
RichChick is an east-Richardson company that produces two types of Chicken cutlets for sale to supermarkets. Cutlets are called Silverlet and Brownlet. Each cutlet consists of white meat and dark meat. Silverlet sells for $12/kg and must consist of at least 80% white meat. Brownlet sells for $10/kg and must consist of at least 40% white meat. At most 300 kg of Silverlet and 500 kg of Brownlet can be sold in the next month. Three types of chicken are used to manufacture the cutlets: Farm-grown chicken, Free Range chicken and Organic-fed chicken. Each chicken’s cost, and its dark and white meat yield are below:

<table>
<thead>
<tr>
<th>Chicken type</th>
<th>Farm-grown</th>
<th>Free-Range</th>
<th>Organic-fed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in $</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>White meat yield in kg</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Dark meat yield in kg</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

a) To formulate a LP to maximize RichChick’s profit in the next month, define decision variables.

Answer: \(F, R, O\): Number of Farm-grown, free-Range, Organic-fed chicken purchased. \(D_S, D_B\): Kilograms of Dark meat used in Silverlet, Brownlet. \(W_S, W_B\): Kilograms of White meat used in Silverlet, Brownlet.

b) Profit is revenue minus costs, express RichChick’s profit in terms of decision variables.

Answer: Max \(12(W_S + D_S) + 10(W_B + D_B) - 6F - 7R - 8O\)

c) Write constraints so that RichChick does not attempt to sell more cutlets than next month’s demand.

Answer: \(W_S + D_S \leq 300\) and \(W_B + D_B \leq 500\)

d) Write constraints so that RichChick does not attempt to use more white or dark meat than it buys in the form of Farm-grown chicken, Free Range chicken and Organic-fed chicken.

Answer: \(W_S + W_B \leq F + 2R + 2O\) and \(D_S + D_B \leq 2F + R + O\)

e) Finish your formulation by adding any constraints you find necessary.

Answer: Silverlet must have at least 80% white meat:

\[
\frac{W_S}{W_S + D_S} \geq 0.8.
\]

Brownlet must have at least 40% white meat:

\[
\frac{W_B}{W_B + D_B} \geq 0.4.
\]

Nonnegativity constraints: \(F, R, O, W_S, D_S, W_B, D_B \geq 0\)

Remark: If you define \(S\) and \(B\) as the amount of Silverlet and Brownlet produced, then you must also add the constraints

\[S = W_S + D_S \quad B = W_B + D_B.\]

Only after these adding these constraints can you replace \(W_S + D_S\) by \(S\) and \(W_B + D_B\) by \(B\) in the formulation above. If you forget \(S = W_S + D_S\) or \(B = W_B + D_B\), your output \(S\) and \(B\) becomes independent of the input \(D_S, D_B, W_S, W_B\).
UTD Drill is a drill bit producer established by some UTD MBA students. UTD Drill buys 5 cm long blank drill bits and shapes them with drill patterns (like twist, masonry, lip, spur) by using a lathe. Then UTD Drill packages 12 different drills in to a single box and sells them to the Lowe’s Home Improvement stores in Texas. UTD Drill requires 3000 blank drill bits every month and buys each bit at $0.05 and each box at $0.60. It costs $12 to UTD Drill to initiate an order from any of its suppliers. The holding costs are based on an annual interest rate of 12%.

[8pts] a) Determine the optimal number of drill bits that UTD Drill should purchase and the time between these purchases.

**Answer:**

\[ R = 3000 / \text{month}. \quad P = \infty. \quad K = 1200 \text{ cents}. \quad h = 5(0.01) = 0.05 \text{ cents}. \]

Then

\[ \text{EOQ} = \sqrt{\frac{2KR}{h}} = \sqrt{\frac{2 \cdot 1200 \cdot 3000}{0.05}} = \sqrt{2 \cdot 1200 \cdot 3000 \cdot 20} = \sqrt{2 \cdot 12 \cdot 3 \cdot 2 \cdot 1000} = 12,000. \]

Time between orders is

\[ \text{Length of an Inventory Cycle} = \frac{Q}{R} = \frac{12000}{3000} = 4 \text{ months}. \]

[4pts] b) What is the annual holding and ordering cost for blank drill bits?

**Answer:**

\[ \text{Total monthly cost} = C(Q = \text{EOQ}; P = \infty) = \frac{1}{2} \text{EOQ} \cdot h + \frac{KR}{\text{EOQ}} \]

\[ = \frac{1}{2} 12000 \cdot 0.05 + \frac{1200 \cdot 3000}{12000} = 300 + 300 = 600 \text{ cents}, \]

or since optimal order is used, Total monthly cost\(=\sqrt{2KRh} = \sqrt{2 \cdot 1200 \cdot 3000 \cdot 0.05} = \sqrt{1200 \cdot 300} = \sqrt{36 \cdot 100} = 600. \]

Then the annual cost is \$72 = 6 \cdot 12.

[4pts] c) Determine the optimal number of package boxes that UTD Drill should purchase.

**Answer:**

\[ R = 3000 / 12 \text{ per month}. \quad P = \infty. \quad K = 1200 \text{ cents}. \quad h = 60(0.01) = 12 \cdot 0.05 \text{ cents}. \]

Then

\[ \text{EOQ} = \sqrt{\frac{2KR}{h}} = \sqrt{\frac{2 \cdot 1200 \cdot 3000/12}{12 \cdot 0.05}} = \frac{1}{12} \sqrt{2 \cdot 1200 \cdot 3000 \cdot 20} = \frac{1}{12} \sqrt{2 \cdot 12 \cdot 3 \cdot 2 \cdot 1000} = 1,000. \]

[4pts] d) The forecaster at UTD Drill has made a mistake in computing the monthly blank drill bit demand which is actually 5000 per month. What is the annual holding and ordering cost for blank drill bits when the purchase quantity in a) is used with the correct demand of 5000 per month? Basically, update your computations in b).

**Answer:**

\[ \text{Total monthly cost} = C(Q = \text{EOQ}; P = \infty) = \frac{1}{2} \text{EOQ} \cdot h + \frac{KR}{\text{EOQ}} \]

\[ = \frac{1}{2} 12000 \cdot 0.05 + \frac{1200 \cdot 5000}{12000} = 300 + 500 = 800 \text{ cents}, \]

Then the annual cost is \$96 = 8 \cdot 12. Note that here we are computing the cost of ordering suboptimally in 12000 units. The formula \(\sqrt{2KRh}\) cannot be used when the order quantity is suboptimal.
Assigning tasks to workers. Consider the following 8 tasks:

<table>
<thead>
<tr>
<th>Task</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>15</td>
<td>25</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td>20</td>
<td>50</td>
<td>15</td>
</tr>
</tbody>
</table>

They are to be assigned to 3 workers (Abraham, Ben and Candy) on a conveyor-paced assembly line.

(a) Provide a formulation to maximize the capacity of the line. Hint: Consider minimizing the cycle time denoted by $C$. Please define all the variables that appear in your formulation. Do not attempt to solve the formulation.

**Answer:**

Let $x_{iA} = 1$ if the $i$th task is assigned to Abraham; Otherwise $x_{iA} = 0$. Similarly define $x_{iB}$ and $x_{iC}$. Let the total time of the tasks assigned to Abraham, Ben and Candy be $t_A$, $t_B$ and $t_C$. Note that $C = \max\{t_A, t_B, t_C\}$, so $C \geq t_A$, $C \geq t_B$ and $C \geq t_C$. We use these inequalities in our formulation:

- Minimize $C$
- Subject to $[2/3 \text{pts}]$ $C - t_A \geq 0$, $C - t_B \geq 0$, $C - t_C \geq 0$; $t_A - 15x_{1A} - 25x_{2A} - 15x_{3A} - 20x_{4A} - 15x_{5A} - 20x_{6A} - 50x_{7A} - 15x_{8A} = 0$; $t_B - 15x_{1B} - 25x_{2B} - 15x_{3B} - 20x_{4B} - 15x_{5B} - 20x_{6B} - 50x_{7B} - 15x_{8B} = 0$; $t_C - 15x_{1C} - 25x_{2C} - 15x_{3C} - 20x_{4C} - 15x_{5C} - 20x_{6C} - 50x_{7C} - 15x_{8C} = 0$.
- $[2\text{pts}]$ $x_{iA} + x_{iB} + x_{iC} = 1$ for $i = 1 \ldots 8$; Every item must be assigned to exactly one of the workers.
- $[0\text{pts}]$ $x_{iA}, x_{iB}, x_{iC} \in \{0, 1\}$
- $[0\text{pts}]$ $C, t_A, t_B, t_C \geq 0$.

Definition of variables, the objective and each type of the constraint (there are 3 important ones) worth 2 points.

(b) In part a), there are no precedence relations. If there had been precedence relations among tasks, would the capacity have been more or less. Please provide a brief argument.

**Answer:** Adding constraints to the formulation above to represent the precedence relations can only remove the existing feasible solutions. If some solutions are eliminated, we may not be able to decrease $C$ as much as we can without the precedence constraints. Hence, the objective value, the optimal $C$, can increase with the constraints. In other words, the capacity of the system may decrease or remain constant. However, it cannot increase with the precedence relations.
CzePhone is a new service company that provides European mobile phones to American visitors to Czech Republic. The company currently has 80 phones available at the Prague airport. There are - on average - 25 customers per day requesting a phone. These requests arrive throughout the 24 hours the store is open. The corresponding coefficient of variation is 1. Customers keep their phones on average 72 hours. The standard deviation of this time is 100 hours.

Since CzePhone currently has no competitor in Prague airport providing equally good service, customers are willing to wait for the telephones. Yet, during the waiting period, customers are provided a free calling card. Based on prior experience, CzePhone found that the company incurred a cost of $1 per hour per waiting customer, independent of day or night.

(a) What is the average number of telephones CzePhone has in its store?

\[ u = \frac{p}{m \cdot a} = \frac{72}{80 \cdot 0.96} = 0.94\% \] means that 75 = 0.94 \times 80 phones are in use, and 5 phones are available in the store on average.

(b) How long does a customer, on average, have to wait for the phone?

\[ T_q = \frac{1}{u} \left( \frac{u \sqrt{2(m+1)} - 1}{1 - u} \right) \left( \frac{CV_a^2 + CV_p^2}{2} \right) \]

at \( p = 72, a = 0.96, m = 80, u = 0.94, CV_a = 1 \) and \( CV_p = 1.39 \),

\[ T_q = \frac{72}{80} \cdot 0.94 \cdot 1.39 = 10.58 \text{ hours} \]

(c) What are the total monthly (30 days) expenses for telephone cards?

First calculate \( I_q \), the average number of people in the queue. From the Little’s formula:

\[ I_q = (1/a)T_q = 10.58/0.96 = 11.02 \] So we can multiply 1 \times 24 \text{ hours/day} \times 30 \text{ days} \times 11.02 people in the queue = $7934.4.

(d) Assume CzePhone could buy additional phones at $1000 per unit. What is the payback period (the time the phone pays for $1000 investment) for one additional phone?

Now \( m = 81 \) so \( u = \frac{72}{81 \cdot 0.96} = 92.6\% \).

\[ T_q = \frac{72}{81} \cdot 0.926 \cdot 1.39 = 7.13/0.96 = 7.43 \] So we can multiply 1 \times 24 \text{ hours/day} \times 30 \text{ days} \times 7.43 people in the queue = $5349.6. With an extra phone, we save 7934.4 - 5349.6 = 2584.8 in a month. It takes about 1000/2584.8 months = 11.6 days to recover $1000 investment.

This question unfortunately involves some algebra. There will be partial credit if you have the correct steps but incorrect numerical answers.
The airport branch of a car rental company operates 24 hours per day and maintains a fleet of 75 SUVs. The interarrival time between requests for an SUV is 2.4 hours, on average, with a standard deviation of 2.4 hours. There is no indication of a systematic arrival pattern over the course of the day. Assume that, if all SUVs are rented, customers are willing to wait until there is an SUV available. An SUV is rented, on average, for 4 days, with a standard deviation of 1 day.

3pts] a. Let us treat this case as a queue problem. Let the SUVs be the servers. What are \( m, a, p, CV_a, CV_p \)?

\[ \text{ANSWER: We know that } a = 2.4 \text{ hours, } p = 96 \text{ hours, } CV_a = (2.4/2.4) = 1, CV_p = (24/96) = 0.25, \text{ and } m = 75 \text{ cars.} \]

5pts] b. What is the average number of SUVs parked in the company’s lot? Hint: What is the number of SUVs utilized?

\[ \text{ANSWER: To determine the number of cars on the lot, we can look at the utilization rate of our "servers" = } (1/a)/(m/p) = 53.3\%. \text{ Therefore, on average 53.3\% of the cars are in use or 40 cars, so on average 35 cars are in the lot.} \]

7pts] c. Through a marketing survey, the company has discovered that if it reduces its daily rental price of $80 by $25, the average demand would increase to 12 rental requests per day and the average rental duration becomes 3 days. Is this price decrease warranted? Provide an analysis by comparing the daily revenue under both pricing schemes. Assume that the standard deviations of the interarrival and activity times do not change.

\[ \text{ANSWER: If the average demand is increased to 12 rentals per day, then } a = 2 \text{ hours. If the average rental duration is to 4 days, then } p = 72 \text{ hours. So utilization rate becomes 48\%. This means that 36 cars are rented on average.} \]

\[ \text{With the initial rate average revenue per day } = 80 \times 40 = \$3,200. \text{ With the proposed rate average revenue per day } 55 \times 36 = \$1,980. \text{ Therefore, the company should not make the proposed changes.} \]
A company produces two products (1,2) using two machines (M,N). Product 1 requires processes on both machines M and N. On the contrary, product 2 can be produced on either machine M or N. Processing times (in minutes) on each machine are

<table>
<thead>
<tr>
<th>Product</th>
<th>Machine N</th>
<th>Machine M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

Each machine works for 8 hours every day. Due to marketing limitations, the number of Product 1 sold must be at least the number of Product 2 sold. When sold, each unit of Product 1 and 2 contributes to profit $16 and $20.

10pts a) Provide an LP to maximize daily contribution to profit.

**Answer:** Let $x_1$ be the number of Product 1 produced, and $x_{2M}$ and $x_{2N}$ be the number of Product 2 produced on machines M and N.

Max $16x_1 + 20(x_{2M} + x_{2N})$

ST

15$x_1 + 20x_{2N} \leq 8(60)$ Capacity constraint for machine N

18$x_1 + 25x_{2M} \leq 8(60)$ Capacity constraint for machine M

$x_1 \geq x_{2M} + x_{2N}$

$x_1, x_{2M}, x_{2N} \geq 0$

5pts b) Suppose that marketing limitation is lifted. Then compute how many more Product 2 can be produced by producing one fewer Product 1. Basically think of using capacity to produce Product 2 as opposed to Product 1. In this case, compare the reduction in profit due to Product 1 against the increase in profit due to Product 2. Finally argue that Product 1 will not be produced in the optimal solution without the marketing limitations.

**Answer:** With one less Product 1, 15 mins and 18 mins capacity are released on Machines N and M. This capacity can be used to produce 15/20 and 18/25 Product 2 on machines N and M. The net effect to profit is -16+(15/20)20+(18/25)20 and is positive. Reducing Product 1 production increases profit so no Product 1 is produced in the optimal solution.
**Additional Questions**

1. Three US Olympic teams and their trainers will fly back from Sydney to San Francisco with a plane that can carry 100 people. This will be a nonstop flight lasting 20 hours. Three teams are Swimming, Gymnastics and Cycling. These teams have the following number of members and trainers: Swimming 42 and 12; Gymnastics 22 and 14; Cycling 34 and 16. There must be at least one swimming trainer accompanying every three swimmers on the plane. Similarly, there must be at least one gymnastics trainer for every two gymnasts on the plane. Cyclists tend to be older and can travel by themselves without trainers. Swimming and cycling associations are equally paying for the trip and they first require that at least the 70% of the seats are allocated to swimmers, cyclists and their trainers. Second, the total number of swimmers and their trainers must equal to the total number of cyclist and their trainers.

a) Provide an LP formulation to minimize the number of people that cannot be put on this flight.

**ANSWER:**

Let $x_s$, $x_g$, $x_c$, $t_s$, $t_g$, $t_c$ be the number of swimmers, gymnasts, cycles, and their trainers put on the plane.

Min $\ 140 - (x_s + x_g + x_c + t_s + t_g + t_c)$

ST :

$x_s - 3t_s \leq 0$

$x_g - 2t_g \leq 0$

$x_s + t_s + x_c + t_c \geq 70$

$x_s + t_s - x_c - t_c = 0$

$0 \leq x_s \leq 42, \ 0 \leq x_g \leq 22, \ 0 \leq x_c \leq 34$

$0 \leq t_s \leq 12, \ 0 \leq t_g \leq 14, \ 0 \leq t_c \leq 16$

b) What is the optimal value of the objective in a)? Justify your answer. You can answer this without solving the formulation.

**ANSWER:**

Consider $x_s = 23$, $t_s = 12$, $x_g = 20$, $t_g = 10$, $x_c = 34$ and $t_c = 1$, this solution is feasible and yields an objective value of 40. You can pick another solution and discover that it also gives an objective value of 40 (consider $x_s = 24$, $t_s = 12$, $x_g = 18$, $t_g = 10$, $x_c = 30$ and $t_c = 6$). Indeed any feasible solution has an objective value of 40. Moreover, we can not reduce the objective value below 100, because the plain takes 100 people and we have 140 athletes.

c) Suppose that leaving out a gymnast costs three times as much as leaving out a swimmer or a cyclist. And also suppose that the cost of leaving out trainers is negligible. Modify your answer to a) to minimize the cost of people left behind (not put on the plane).

**ANSWER:**

Modify the objective function as $\text{Min } (42 - x_s) + 3(22 - x_g) + (34 - x_c)$. 
2. Farmer Billy Bauer has two farms in Dallas to grow wheat and barley. There are differences in the yields and costs of growing crops due to soil conditions at two farms:

<table>
<thead>
<tr>
<th></th>
<th>McKinney Farm</th>
<th>Addison Farm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barley yield/acre</td>
<td>400 bushels</td>
<td>700 bushels</td>
</tr>
<tr>
<td>Cost/acre of barley</td>
<td>$90</td>
<td>$80</td>
</tr>
<tr>
<td>Wheat yield/acre</td>
<td>350 bushels</td>
<td>300 bushels</td>
</tr>
<tr>
<td>Cost/acre of wheat</td>
<td>$110</td>
<td>$100</td>
</tr>
</tbody>
</table>

McKinney and Addison farms have 70 and 120 acres for cultivation. At least 20000 bushels of barley and 30000 bushels of wheat must be grown. Provide an LP to minimize the cost of meeting wheat and barley demand.

**Answer:**

Let BM: Area in acres dedicated for Barley production at McKinney. BA: Area in acres dedicated for Barley production at Addison. Similarly define WM and WA.

Min \(90BM + 80BA + 110WM + 100WA\)

ST:

\(BM + WM \leq 70\)

\(BA + WA \leq 110\)

\(400BM + 700BA \geq 20000\)

\(350WM + 300WA \geq 30000\)

\(BM, BA, WM, WA \geq 0\)
3. PlanoTurkey produces two types of turkey cutlets for sale to fast food restaurants. Each type of cutlet consists of white meat and dark meat. Cutlet 1 sells for $8/kg and must consist of at least 70% white meat. Cutlet 2 sells for $6/kg and must consist of at least 60% white meat. At most 50 kg of cutlet 1 and 30 kg of cutlet 2 can be sold for Thanksgiving. Two types of turkey used to manufacture the cutlets are purchased from an Addison farm. Each type 1 turkey costs $10 and yields 5 kg of white meat and 2 kg of dark meat. Each type 2 turkey costs $8 and yields 3 kg of white meat and 3 kg of dark meat. Formulate a LP to maximize PlanoTurkey’s profit.

a) Define decision variables.

**Answer:**

$T_1$: Number of type 1 turkey purchased. $D_1$: Kilograms of dark meat used in cutlet 1. $W_1$: Kilograms of white meat used in cutlet 1. Define $T_2, D_2, W_2$ similarly.

b) Profit is revenue minus costs, express the profit in terms of decision variables.

**Answer:**

Max $8(W_1 + D_1) + 6(W_2 + D_2) - 10T_1 - 10T_2$

c) Write constraints so that no more cutlets than demand is sold.

**Answer:**

$W_1 + D_1 \leq 50$ and $W_2 + D_2 \leq 30$

d) Write constraints so that PlanoTurkey does not use more white or dark meat than it buys from the Addison farm.

**Answer:**

$W_1 + W_2 \leq 5T_1 + 3T_2$ and $D_1 + D_2 \leq 2T_1 + 3T_2$

e) Finish your formulation by adding any contraints you find necessary.

**Answer:**

Cutlet 1 must have at least 70% white meat:

$$\frac{W_1}{W_1 + D_1} \geq 0.7$$

Cutlet 2 must have at least 60% white meat:

$$\frac{W_1}{W_1 + D_1} \geq 0.6$$

Nonnegativity constraints: $T_1, D_1, W_1, T_2, D_2, W_2 \geq 0$
4. The Apex Television company has to decide on the number of 27 and 20 inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27 inch sets and 10 of the 20 inch sets can be sold per month. The maximum number of work hours available is 800 hours per month. A 27 inch set requires 15 work hours and a 20 inch set requires 10 work hours. Each 27 inch set produces a profit of $120 and the same number is $80 for 20 inch sets.

a) Formulate an LP to maximize the profit:

**ANSWER:**

\( B \): Number of 27 inch sets produced per month. 
\( S \): Number of 20 inch sets produced per month.

\[
\text{Max } 120B + 80S \\
\text{Subject to: } \\
B \leq 40 \\
S \leq 10 \\
15B + 10S \leq 800 \\
B, S \geq 0
\]

b) Through commercials, TV set demand can be increased. For every $20 spent for commercials, 1 more 27 inch TV and 2 more 20 inch TV can be sold. Formulate an LP to maximize the profit with a budget of $400 for commercials.

**ANSWER:**

\( C \): Commercial budget spent for TVs.

\[
\text{Max } 120B + 80S - C \\
\text{Subject to: } \\
B \leq 40 + C/20 \\
S \leq 10 + C/10 \\
C \leq 400 \\
15B + 10S \leq 800 \\
B, S \geq 0
\]
5. To celebrate the ending of the term, suppose that you go to a restaurant with the following menu:

<table>
<thead>
<tr>
<th></th>
<th>Salad</th>
<th>Soup</th>
<th>Steak</th>
<th>Chicken</th>
<th>Rice</th>
<th>Pasta</th>
<th>Fish</th>
<th>Fries</th>
<th>Cheesecake</th>
<th>Pie</th>
<th>Coffee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories</td>
<td>400</td>
<td>300</td>
<td>1200</td>
<td>1100</td>
<td>800</td>
<td>900</td>
<td>1000</td>
<td>400</td>
<td>500</td>
<td>400</td>
<td>100</td>
</tr>
<tr>
<td>Price</td>
<td>5</td>
<td>4</td>
<td>14</td>
<td>12</td>
<td>7</td>
<td>8</td>
<td>13</td>
<td>3</td>
<td>6</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Suppose that you have $25 and do not want to consume more than 5000 calories and less than 2500 calories.

(8 points) a) Your objective is to look rich, i.e. to order as many items as possible. Provide a formulation to achieve this objective.

**Answer:**

Let \( x_i \) = 1 if \( i \)th item is ordered, 0 otherwise.

Maximize \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \)

St

\[
\begin{align*}
400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} & \leq 5000 \\
400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} & \geq 2500 \\
5x_1 + 4x_2 + 14x_3 + 12x_4 + 7x_5 + 8x_6 + 13x_7 + 3x_8 + 6x_9 + 6x_{10} + 2x_{11} & \leq 25 \\
x_i & \in \{0, 1\}
\end{align*}
\]

(6 points) b) Write a constraint that does not allow ordering cheesecake and pie together.

**Answer:**

\[ x_9 + x_{10} \leq 1 \]

(6 points) c) Write a constraint so that coffee is ordered when cheesecake or pie is ordered.

**Answer:**

\[ 2x_{11} \geq x_9 + x_{10} \]
6. To celebrate the ending of the term, suppose that you go to a restaurant with the following menu:

<table>
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<td>800</td>
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<td>400</td>
<td>500</td>
<td>400</td>
<td>100</td>
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<td>4</td>
<td>14</td>
<td>12</td>
<td>7</td>
<td>8</td>
<td>13</td>
<td>3</td>
<td>6</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Suppose that you do not want to consume more than 4000 calories and less than 2000 calories but at the same time you will order at least 5 items.

(8 points) a) Provide a formulation that minimizes your expense.

**Answer:**
Let \( x_i = 1 \) if \( i \)th item is ordered, 0 otherwise.

Minimize \( 5x_1 + 4x_2 + 14x_3 + 12x_4 + 7x_5 + 8x_6 + 13x_7 + 3x_8 + 6x_9 + 6x_{10} + 2x_{11} \)
\( \text{St} \)
\( 400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} \leq 4000 \)
\( 400x_1 + 300x_2 + 1200x_3 + 1100x_4 + 800x_5 + 900x_6 + 1000x_7 + 400x_8 + 500x_9 + 400x_{10} + 100x_{11} \geq 2000 \)
\( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \geq 5 \)
\( x_i \in \{0, 1\} \)

(6 points) b) Write a constraint that forces you to order the steak if you are ordering the soup and allow to order the steak without the soup.

**Answer:**
\( x_3 \geq x_2 \)

(6 points) c) Write a constraint so that chicken is not ordered when steak and fish are ordered.

**Answer:**
Correct answer: \( x_4 \leq 2 - x_3 - x_7 \)
A wrong answer: \( x_3 + x_7 \geq x_4 - 1 \). This allows for \( x_3 = 1, x_7 = 1, x_4 = 1 \) so it is wrong.
7. Choco is a small chocolate manufacturer producing only Black (B), Milk (M) and Strawberry (S) chocolate. Manufacturing costs per ton for B, M and S are 800, 700 and 900 dollars. Marketing needs at least 8, 14 and 10 tons of B, M and S chocolate for the next month and guarantees a revenue of 3300, 3000 and 2800 dollars for each ton of B, M and S. Total labor capacity for the next month is 2400 hours but each ton of B, M and S consume 50, 60, 80 hours of capacity. Finally finance department enforces a minimum profit limit of 60,000 dollars per month.

(10 points) a) Let $C$ be the total production costs, and B, M, S be the total tons of Black, Milk and Strawberry chocolate produced in the next month. Write an LP that minimizes $C$ subject to limitations given above.

**ANSWER:**

Minimize $C$

Subject to

\[
\begin{align*}
C &= 800B + 700M + 900S \\
B &\geq 8, \ M &\geq 14, \ S \geq 10 \\
3300B + 3000M + 2800S - C &\geq 60,000 \\
50B + 60M + 80S &\leq 2400 \\
B, M, S &\geq 0
\end{align*}
\]

(5 points) b) Compute the contribution to margin for each ton of B, M, S chocolates and compare with labor hours. Is there a chocolate type which is dominated both in unit contribution to margin and labor hours, explain? Would your LP indicate a positive production quantity for that type, why?

**ANSWER:**

Contribution to margins are 2500, 2300 and 1900, and labor hours are 50, 60 and 80 per ton of B, M and S. Clearly M and S are dominated in profitability. LP has $M \geq 14, S \geq 10$, so it will set $M = 14, S = 10$.

(5 points) c) Use the labor hours constraint to find the smallest number $U$ tons such that $B \leq U$, $M \leq U$ and $S \leq U$ in any feasible solution to your LP. I am just asking for the smallest number $U$ such that if $B, M$ and $S$ are feasible then $B \leq U$, $M \leq U$ and $S \leq U$.

**ANSWER:**

If no M or S produced, we have $B \leq 2400/50 = 48$. Similarly, $M \leq 40$ and $S \leq 30$ so $U = \max\{48, 40, 30\} = 48$.

Another but more complicated way is to involve demand constraints:

\[
\begin{align*}
50B &\leq 2400 - 60 \cdot 14 - 80 \cdot 10 \\
60M &\leq 2400 - 50 \cdot 8 - 80 \cdot 10 \\
80S &\leq 2400 - 60 \cdot 14 - 50 \cdot 8
\end{align*}
\]

Let $U = \max\{B, M, S\}$. Both ways get the full mark.
8. Refer to the statement of the previous problem. The marketing group at Choco is convinced that forcing manufacturing to produce above some predetermined quantities is not a good idea. They have stopped requiring at least 8, 14, 10 tons of B, M and S, and have gave freedom to manufacturing to set these numbers to zero if it is profitable to do so. On the other hand manufacturing group has just realized that production costs are not entirely proportional to production quantities. Actually there are fixed costs that are paid to start up the production for each chocolate, these costs are independent of production quantities. Fixed cost for a chocolate is incurred only if that chocolate is produced (in positive quantities). Fixed costs are 10,000, 12,000 and 8000 dollars for chocolates B, M and S.

(12 points) a) Define appropriate variables and provide a formulation to minimize the total cost in light of new marketing policy and new fixed costs.

**Answer:**
Let $y_B$, $y_M$ and $y_S$ be binary variables becoming 1 if B, M and S are produced, respectively.

Minimize $C$

Subject to

\[ C = 800B + 700M + 900S + 10,000y_B + 12,000y_M + 8,000y_S \]
\[ B \leq 48y_B, \quad M \leq 48y_M, \quad S \leq 48y_S \]
\[ 3300B + 3000M + 2800S - C \geq 60,000 \]
\[ 50B + 60M + 80S \leq 2400 \]
\[ B, M, S \geq 0, \quad y_B, y_M, y_S \in \{0, 1\} \]

Note we computed 48 tons as the smallest uniform upper bound on B, M and S from the 3.c. But you do not have to use this bound. Any number larger than 48 will work as well.

(8 points) b) Write a constraint to make sure that: at least 16 tons of Milk chocolate must be produced only when no Black chocolate is produced.

**Answer:**
\[ M \geq 16(1 - y_B) \]