Homework 1

Due Wednesday, August 30, before class.

1. **Problem.** Prove the Schwartz inequality:

   \[ |\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|. \]  

   (1)

2. **Problem.** *(Problem in two-dimensions.)* Given the line:

   \[ \mathbf{L}_1 : y = 2x , \]  

   find the equation of the line \( \mathbf{L}_2 \) perpendicular to \( \mathbf{L}_1 \) passing through the point \( P = (1, 2) \).

3. **Problem.** *(Problem in two-dimensions.)* Given the ellipse:

   \[ \frac{x^2}{4} + \frac{y^2}{9} = 1 , \]  

   find the equation of the line tangent to the ellipse at the point \( P = (1, 3\sqrt{3}/2) \).

4. **Problem.** Consider the scalar field:

   \[ f(x) = a|\mathbf{r}|^2 , \]  

   (4)

   \( (a) \). Find the gradient of \( f \) at an arbitrary point \( \mathbf{r} = (x, y, z) \).

   \( (b) \). Show directly that \( \nabla \times \nabla f = 0 \).