International Workshop on Resonance Oscillations and Stability of Nonsmooth Systems

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Minisymposium on current mathematical challenges in the theory of border collision bifurcations

The problem of singularity in impacting systems
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This talk concerns the dynamics of such mechanical systems in which the elements of the system may undergo impacts with each other. It has been observed that in such systems a periodic orbit abruptly loses stability and a large-amplitude chaotic vibration develops as the variation of a parameter drives the system from a non-impacting motion to an impacting motion. The instability is known to be induced by grazing or zero-velocity impacts.

A one-degree-of-freedom oscillator constrained by a wall serves as the archetype in the study of such systems. Nordmark [1,2] had shown that in the simple impact oscillator, the Poincare map has a square-root term, that results in a typical problem known as the square-root singularity. The square-root term causes the Jacobian assume infinite values close to the grazing condition, as a result of which there is an infinite local stretching in the state space. This causes the abrupt loss of stability and the transition to a large chaotic attractor.

Nordmark’s work resulted in a surge of interest in the dynamics of the impact oscillator and that of the square-root map. However, the approach used by researchers so far [3] treats the Jacobian as a single entity and says nothing about how the elements of the Jacobian change. A pertinent question is: Are the elements of the Jacobian subject to some constraints as the system transits from a non-impacting state to an impacting state?

Our numerical investigation revealed that the determinant of the Jacobian remains invariant and the trace undergoes an abrupt change [4,5]. Experimental investigation conducted in collaboration with the researchers at the University of Aberdeen, UK, confirmed this observation [5,6]. Subsequently we were able to show analytically why this must be so. The derivation leads to an interesting conclusion: that the singularity should vanish under some condition of the system parameters. Thus the sudden exit from the local orbit can be avoided if the parameters are carefully chosen to satisfy this relation.

In this talk I will first outline the ”zero-time discontinuity mapping” (ZDM) approach, and then will present the numerical, experimental, and analytical work of our group in the above direction. I will also present the interesting observation that the occurrence of narrow band of chaos is caused by an ”invisible grazing” and a ”dangerous bifurcation” [7].

References:
Evidence for an Unfolded Border-Collision Bifurcation in Paced Cardiac Tissue
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Coauthors: Xiaopeng Zhao, David G. Schaeffer, Wanda Krassowska Neu and Daniel J. Gauthier

Bifurcations in the electrical response of cardiac tissue can destabilize electrochemical waves in the heart. Therefore, it is important to classify these bifurcations to understand the mechanisms that cause instabilities. We have determined that the period-doubling bifurcation in paced myocardium is of the unfolded border-collision type. Furthermore, we have studied the role of calcium in inducing this bifurcation based on voltage and calcium measurements in frog ventricle.

Unstable chaos, snap-back repellers and border collision bifurcations
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A snap-back repeller is a type of homoclinic orbits which can only exist in non-invertible maps, and which imply the existence of unstable chaotic motion. I will discuss the existence and bifurcations of snap-back repellers in the normal form of border collision bifurcations. These results pave the way for a more general theory of bifurcations for snap-back repellers, with infinite cascades of subsidiary bifurcations as the critical bifurcation parameter at which a simple snap-back repeller is approached. Results about the existence of snap-back repellers are joint work with Chi Hong Wong, and have appeared in Phys. Rev. E (2009).

Shrinking Point Bifurcations of Resonance Tongues for Piecewise-Smooth, Continuous Maps
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Resonance tongues are mode-locking regions of parameter space in which stable periodic solutions occur. As was first observed for a one-dimensional sawtooth map in a 1987 paper by Yang and Hao, resonance tongues in piecewise-smooth, continuous maps commonly exhibit a distinctive lens-chain (or sausage) geometry in two-parameter bifurcation diagrams. It has since been determined that edges of these tongues usually correspond to border-collision fold bifurcations of the associated stable periodic solution.

In this talk I will introduce a symbolic description for a class of ”rotational” periodic solutions that display lens-chain structures for a general $N$-dimensional, piecewise-smooth, continuous map. Using symbolic dynamics I will describe an unfolding of the codimension-two, shrinking point bifurcation, where the tongues have zero width. A number of codimension-one bifurcation curves emanate from shrinking points and those that form tongue boundaries are determined.
Minisymposium on applications of topological degree to study non-smooth systems

Planar nonlinear Dirac systems: from the shooting method to global bifurcation
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We describe a global bifurcation result for a planar nonlinear Dirac problem in a half-line. To this end, we apply an abstract result in the framework of $\alpha$-contraction mappings; solutions are then distinguished by an index related to the classical concept of rotation number.

Planar Dirac systems in unbounded domains: the eigenvalue problem and a Maslov-type index
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We present a complete characterization of the eigenvalues of a Dirac operator on an unbounded domain. This is obtained using a Maslov-type index for linear problems.

First order periodic ODEs without uniqueness: complicated dynamics in simple problems
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An elementary approach, based on a systematic use of lower and upper solutions, is employed to detect the qualitative properties of solutions of first order scalar periodic ordinary differential equations. This study is carried out in the Carathéodory setting, avoiding any uniqueness assumption, in the future, or in the past, for the Cauchy problem. Various classical and recent results are recovered and generalized.

Detecting stability and instability in periodic parabolic problems via lower and upper solutions
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The method of lower and upper solutions is an elementary but powerful tool in the existence theory of initial and periodic problems for semilinear differential equations for which a maximum principle holds, even in cases where no special structure is assumed on the nonlinearity. The aim of this talk is to show that this method is also quite effective for investigating the qualitative properties of solutions, at the same extent of generality for which the existence theory is developed. Indeed, we try to work out, under a minimal set of basic assumptions, a qualitative theory for second order parabolic problem in cases where neither uniqueness for the initial value problem, nor validity of comparison principles are guaranteed, so that the analysis of these equations cannot be performed applying the standard theory of order preserving dynamical systems. We are especially interested here in studying, with the aid of lower and upper solutions, the following three basic questions: (i) existence of periodic solutions and their localization; (ii) qualitative properties of periodic solutions, with special reference to their stability or instability; (iii) asymptotic behavior of solutions of the initial value problem.

References:


Complex dynamics in piecewise linear planar systems: a topological approach
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Coauthors: Anna Pascoletti (University of Udine)

In the first part of the talk, we present a few different approaches, for the search of periodic points and chaotic-like dynamics, based on topological degree or related topological fixed point methods. Then we show some recent applications to periodically perturbed piecewise linear second order ODEs in which the geometry of the problem presents strong analogies to the theory of "linked twist maps".
Minisymposium on methods of analysis of systems with hysteresis and multi-scale systems

An Investigation of Shape Memory Alloy Dynamics
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Shape Memory Alloys (SMA) display significant hysteresis in the relationship between temperature and deformation due to a solid state phase transition. Heating the material using electrical current is a common method of inducing the phase transition. However, the highly nonlinear relationship between deformation and electrical resistance inhibit the use of current as control variable.

We present a program for the investigation of the dynamics of SMA in a dc circuit and show some preliminary results. The program is based on experimental measurements of SMA material properties and a numerical model that incorporates the Preisach nonlinearity.

Representation of Hysteresis with Return Point Memory: Expanding the Operator Basis
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Return Point Memory is a widely observed property of hysteretic systems. In fact, ability of the Preisach model of hysteresis to naturally reproduce this property is one of the reasons for the popularity of this model. However, Preisach model, which employs simple rectangular hysteresis loop operators as its basis, imposes another condition of congruency in the shape of its minor hysteresis loops. This condition substantially restricts the model applicability. In this talk I will discuss a model with an expanded set of hysteresis operator basis. The expanded basis permits to describe all hysteresis systems with return point memory whose dependence of output on history satisfies certain general conditions of smoothness. Construction of the operator basis as well as identification of the measure that permit to fit the model to reversal curves of various orders will be discussed.

Modelling of macroeconomic and financial systems with hysteresis.
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The term hysteresis has been present in the economic literature for some time now, with the rise to prominence of the concept usually identified with the economic crises of the 1970s. However, most of the behaviour described as ”hysteresis” is linear in character, and does not represent ”true” hysteretic behaviour. In recent years this has begun to change, with economic justifications proposed for the irreversible and non-smooth character associated with hysteresis. These include sunk costs involved in investment decisions and cognitive factors such as feedback and bias.

This presentation will examine these justifications for the use of hysteresis models in economic and financial systems, and present some new results from one such model - a novel operator-differential equation with vector input.
The study of propagation phenomena in reaction-diffusion systems is a central topic in non-equilibrium physics. One aspect that has received recent attention concerns the effects of a cut-off of the reaction kinetics on the propagation speed of the corresponding traveling fronts. Brunet and Derrida introduced a cut-off function in the classical Fisher-Kolmogorov-Petrovskii-Piscounov (FKPP) equation to model N-particle systems in which concentrations below a small threshold are not attainable. They conjectured that the cut-off causes a substantial shift in the front speed that is largely independent of the specific choice of cut-off function.

We give a proof of the conjecture of Brunet and Derrida based on geometric singular perturbation theory and the blow-up method, which lead naturally to the identification of the reasons for the universality of the first-order correction to the unperturbed FKPP propagation speed. Finally, we show how our results can be extended to more general families of scalar reaction-diffusion equations with cut-off.

Bistability and hysteresis effect in slow-fast systems are associated with folding the of slow manifold. As in the classical Van der Pol example of relaxation oscillations, such folding can develop through a perturbation of non-hysteretic system. Similarly, a positive feedback loop transforms a non-hysteretic ideal relay into a non-ideal relay, an elementary hysteretic transducer. We consider hysteresis loops and relaxation oscillations generated by perturbation of systems with non-smooth nonlinearities such as Devil’s staircase.

Fenichel’s Theorem guarantees that hyperbolic periodic orbits of smooth dynamical systems persist under small stable singular perturbations. We discuss what happens if we add a singular perturbation to an ordinary differential equations with piecewise smooth discontinuous right-hand-side (a Filippov system). We notice, for example, that the strong persistence statement is in general not true for periodic orbits that contain a so-called sliding segment.
Minisymposium on generic singularities of nonsmooth systems: focusing on the 3D case and its consequences for dynamics

On the existence and classification of Teixeira Singularities in Switched Control Systems
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We analyse the occurrence of the Teixeira singularity in three-dimensional discontinuous feedback control systems. This has been pointed out in the literature as a mechanism for dynamical transitions. We show that such singularities cannot occur in classical single-input single-output systems in the Lure form. However, in multiple-input multiple-output (MIMO) switched control systems, Teixeira singularities can occur and are a potentially dangerous phenomenon. The theoretical derivation is illustrated by means of a representative example.

Robustness of Teixeira singularity dynamics and bifurcations.
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We will give an overview of the various dynamical regimes associated with a Teixeira singularity and their codimension-1 bifurcations, and highlight some technical points that may or may not be relevant to practical applications.


Invariant manifolds and nondeterministic dynamics around the Teixeira Singularity
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If the simplest characteristics of dynamical systems are revealed by their ‘generic’ singularities, then the Teixeira singularity is an important stepping stone towards understanding discontinuities in general. At a Teixeira singularity, orbits are tangent to both sides of a discontinuity (or switching) manifold. These organising centres contain all the richness of Filippov dynamics: crossing of the manifold, stable and unstable sliding, nondeterministic trajectories that intersect the singularity itself. Simple local geometry reveals invariant manifolds – nonsmooth balls and saddles. Further away lie limit cycles and routes to nondeterministic chaos. We will discuss these and other aspects of this pivotal point in nonsmooth dynamics.
Symmetries and Bifurcation in Non-smooth Systems
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This talk concerns the dynamics of non-smooth systems in presence of symmetries. We introduce the concepts of equivariance and reversibility in the non-smooth universe and discuss some bifurcation problems involving the fold-fold singularity.
Methods of reduction for nonsmooth systems (averaging, Melnikov’s method, center manifold approach)

Chaos in discontinuous systems
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Following a functional analytic approach the problem is studied of chaotic behaviour in time-perturbed discontinuous systems whose unperturbed part has a piecewise $C^1$ homoclinic solution that crosses transversally the discontinuity manifold. We show that if a certain Melnikov function has a simple zero at some point, then the system has solutions that behave chaotically.

References:

Periodic solutions of the perturbed symmetric Euler top
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We consider the three-dimensional system
\[
\dot{x} = -yz, \quad \dot{y} = xz, \quad \dot{z} = 0
\]
that is the symmetrical case of the Euler top, and its $T$-periodic perturbations
\[
\dot{x} = -yz + \varepsilon p(t,x,y,z), \quad \dot{y} = xz + \varepsilon q(t,x,y,z), \quad \dot{z} = \varepsilon s(t,x,y,z).
\]
The orbits of the unperturbed system are either singular points, or periodic orbits. Note that the unperturbed system is not isochronous and, beside the trivial ones, it has $T$-periodic orbits for any $T > 0$. We are interested in the existence of $T$-periodic solutions for the perturbed system when $\varepsilon$ is sufficiently small.

Investigation of oscillatory processes in some classes of impulsive systems via the method of averaging
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The talk will be devoted to two topics.
1. Linear systems with discontinuous periodic and almost periodic coefficients near to constants. The algorithm for studying of stability of such systems will be presented.
2. The problem of the existence and stability of periodic and almost periodic solutions in systems with discontinuous fast phase will be discussed. The results are applied to some classes of vibro-impact systems.
The averaging of differential inclusions and the maximum principle
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It is considered the multivalued Cauchy problem for differential inclusions containing the slow and the rapid variables. The slow variables often contain the main information of the properties of the main system. In this case it is natural to approximate main problem with respect to slow variables by the simpler Cauchy problem is called an average problem.

One of the most important task is connected with the construction of the right hand of the average problem. It is shown this construction is reduced to evaluation of the limits of maximal means for an real function with many variables. The exact upper bound is calculated over all solutions of the differential inclusion with unbounded right hand. It is proved if some conditions are fulfilled that the limit of maximal mean equals the maximum of function.

Keywords: differential inclusions, average problem, unbounded right hand, limits of maximal means

The reduction to invariant cones for nonsmooth systems
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I plan to discuss piecewise smooth systems an try to describe mechanisms to obtain a similar reduction to a lower dimensional system as has been achieved for smooth systems via the center manifold approach. It turns out that for nonsmooth systems there are invariant quantities as well which can be used for a bifurcation analysis but the form of the quantities is more complicated. In simple examples they are given as invariant cones (for piecewise linear systems) or deformations in the case of nonlinear perturbations of PWLS. Since the PWLS is itself nonlinear it is possible that multiple invariant cones occur so in general the situation is more complex.

Persistence of homoclinic trajectory in discontinuous systems
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Using Melnikov theory combined with some functional analysis we study a time-perturbed discontinuous system, whose unperturbed part exhibits a homoclinic trajectory asymptotic to the discontinuity manifold. We give sufficient conditions for the persistence of the homocline.
Classification and normal forms for nonsmooth systems and maps

Sufficient conditions for period increment big bang bifurcations
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The increasing interest in nonsmooth systems in the last years have led to the insight that these kind of systems possess some exclusive properties. Especially, a big number of analytical calculations and numerical simulations of 2-parameter families of one-dimensional discontinuous maps have demonstrated the existence of rare points in parameter space acting as organizing centers. Arbitrarily small open neighborhoods of such a point contain an infinite number of different possible periodic orbits, which are created and destroyed mainly by border collision bifurcations. These infinite number of bifurcation curves emerge from these particular points and therefore they have been referred in the literature as ”organizing centers” or ”big bang bifurcations”. Regarding the rules of creation that the symbolic representation of these orbits follow, mainly two possible scenarios are well known: a period adding and a period increment one. We formally present in our work general and sufficient conditions for a map to exhibit a ”big bang bifurcation” with a period increment scenario.

Codimension-2 singularities with infinitely many codimension-1 bifurcation branches
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We present a study of codimension two singularities of planar Filippov systems. For this purpose we establish two different definitions of topological equivalence which, in some cases, lead to different unfoldings of these singularities.

In this talk we focus on singularities whose generic unfoldings present global codimension one bifurcations as birth of periodic orbits or separatrix connections which, in some cases, appear in infinitely many branches.

Border-collision bifurcations of two-dimensional discontinuous maps
Stephen Pring
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Two-dimensional (2D) discontinuous maps arise in many applications which include corner-impacting oscillators and electronic switching circuits. Unlike continuous piecewise-linear maps, there does not exist a complete classification theory for the types of dynamics which can occur for 2D piecewise-linear discontinuous maps. In this talk we shall discuss some of the dynamics of the normal-form 2D discontinuous map and show the difference in behaviour between real and complex eigenvalues. For real eigenvalues we show that the bifurcations are of period-adding type (Farey sequence) when the trace of the matrix is positive. We will also consider the basins of attraction for the map when the eigenvalues are complex.
Theory of stability of nonsmooth systems

Nonsmooth Dynamical Systems: an overview
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This talk constitutes a short introduction to the subject of the stability (in the sense of Lyapounov) of evolution variational inequalities and non-smooth dynamical systems. After recalling some background of convex analysis, we present basic existence results and an Invariance principle for a class of first order evolution variational inequalities. Using this approach, stability and asymptotic properties of important classes of second order dynamic systems with dry friction are studied. Using Brouwer's topological degree, necessary conditions for asymptotic stability of evolution variational inequalities are also studied.

Stability of Differential Equations with Piecewise Constant Argument of Generalized Type
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The concept of differential equations with piecewise constant argument [1,2] has been recently generalized in [3,4], where we also propose a new approach to the analysis of the equations, so that a wide spectrum of stability problems can now be fully investigated. Some of these results [3-6] will be the subject of discussion in this talk.

References:

A complete bifurcation analysis of planar conewise affine systems
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A procedure is presented to find all limit sets near bifurcating equilibria in continuous, piecewise affine systems defined on a conic partition of the plane. To guarantee completeness of the obtained limit sets, new conditions for the existence or absence of closed orbits are combined with the study of return maps. With these results a complete bifurcation analysis of a class of planar conewise affine systems is presented.
Marginally stable asymmetric solutions of a dry friction oscillator
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Special sub-harmonic solutions of a harmonically forced dry friction oscillator are analyzed. We prove that the typical non-sticking solutions are stable and symmetric, but a continuum of possible asymmetric, marginally stable solutions exist at discrete excitation frequencies.

We determine the explicit form of the one-parameter family of these solutions, and give the conditions under which our formulas are valid. We also show that the parameter domain of non-sticking symmetric solutions is smaller than it was published in earlier contributions. The stability of the solutions is examined in the third order approximation, and our analytical results are checked by numerical simulations.

Finally, we point out that a strange beating phenomenon may cause quite large numerical errors close to resonance.

References:

Stability analysis of nonsmooth limit cycles
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The stability analysis of nonsmooth limit cycles imposes many challenging problems that have attracted a lot of attention [1-3]. Various methods have been used to address this problem [4-7] with fruitful results. In this talk we will focus on a specific class of systems called Filippov systems that have degree of singularity 1 [2]. Using the formulation that was first proposed by Aizerman-Gantmaher & Filippov [8] we demonstrate that it is possible not only to determine the local stability properties of the orbit under study but we can also design supervising control laws that can stabilise an unstable orbit (of any periodicity) without changing its location in the state space. Analytical, numerical and experimental results of this method applied on various power electronic converters demonstrate its validity [9, 10].

References:
Determination of the Basin of Attraction in periodic non-smooth systems
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In this talk we consider the non-smooth scalar equation $x' = f(t, x)$ where $f$ is periodic with respect to $t$ and non-smooth at $x = 0$. We will give a sufficient and necessary condition for existence and uniqueness of an exponentially asymptotically stable periodic solution, which also provides a bound on the exponential rate of attraction and the basin of attraction of the periodic solution.

The condition is a generalisation of Borg’s criterion measuring the local rate of attraction between two adjacent solutions. It involves a periodic, non-smooth function $W$ which acts as a weight function for a modified distance between two adjacent solutions.

References:

Troublesome tangencies: the singularities of piecewise-smooth systems
Mike Jeffrey

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The study of the singularities and discontinuity-induced bifurcations in piecewise-smooth systems has revealed many phenomena depending essentially upon the occurrence of ‘grazing’ points, where a dynamical flow is tangent to a discontinuity set. Seemingly innocuous, grazing points represent no pathological behaviour of the constituent smooth flows, yet they lie at the heart of nonsmooth bifurcations. A study of the geometry around grazing points reveals a complete classification of orbit bifurcations associated with grazing, and shows how they govern the appearance of invariant manifolds, equilibria, and regions of sliding. Most striking of all is the two-fold singularity, a generic point in three dimensions where loci of grazing points intersect, providing mechanisms for the catastrophic destruction of limit cycles. The two-folds include the Teixeira-singularity, a little understood and deceptively simple point that may lurk at the heart of many control and other piecewise-linear systems, from which springs a diabolical invariant manifold, and routes to nondeterministic chaos. On the surface these reveal novel consequences of piecewise-smooth behaviour, but they also pose vital questions about the notion of structural stability in nonsmooth systems.
A new approach to the qualitative analysis of the dissipative systems with symmetries based
on the generalized Smale diagrams is proposed.

The qualitative analysis of the dynamics of the dissipative systems with symmetries is discussed.

Let’s suppose that dissipative forces acting on a mechanical system depend on a small parameter:
if the parameter equals zero, these forces are partially dissipative, otherwise they are total dissipative.
The full mechanical energy is a non-increasing function along the motions of the system
\( H(v, r) \leq h \) (here \( v \) are quasi-velocities, \( r \) are quasi-coordinates, \( h \) is an initial value of the energy).
Besides let’s suppose that, firstly, when the parameter vanishes the system admits a first integral
\( K(v, r) = k \), that is linear on \( v \), secondly, the absolute value of this integral doesn’t increase along the motions when
the parameter is non-zero: \( K^2 \leq k^2 \) (here \( k \) is an initial value of \( K \)).

The qualitative analysis of the original system (the parameter does not equal zero) is based on
the analysis of the unperturbed system (the parameter is zero). Let’s consider an effective potential for the
unperturbed system: \( V_k(r) \) is a minimum of the function \( H \) with respect to the variables \( v \) while the
value of the linear integral is fixed. This potential is defined on the configuration space of the system
and depends on \( k^2 \). Each critical point \( r = r_0(k^2) \) of the effective potential corresponds to steady-
state motion of the unperturbed system, that is stable (unstable) if the critical point is minimum (not
minimum). Along the steady-state motions full mechanical energy of the unperturbed system
conserves its initial value. On the plane \((k^2, h)\) the critical points correspond to the family of the
curves \( h = h(k^2) = V_k(r_0(k^2)) \), that forms the generalized Smale bifurcation diagram (in other words,
the Smale bifurcation set). Each point \((k^2, h)\) of this set is invariant under the phase flow of the
unperturbed system, other points move along the lines \( k^2 = \text{const} \) so \( h \) decreases, and each of them
asymptotically tends to some point of the bifurcation set. Thus the generalized Smale diagram provides
the global qualitative analysis of the unperturbed system: for any initial state of the system it is possible
to determine the initial coordinates of the representative point on the Smale plane \((k^2, h)\)
and thereby determine the final motions of the unperturbed system, that are steady-state motions. If
only one of them is stable, then it will be a final motion of the unpertubed system with a probability
equals 1.

If the parameter doesn’t equal zero there is no neither any linear integral nor steady-states. The
linear integral of the unperturbed system transforms into a slowly decreasing on its absolute value
function, and the steady states become quasisteady states. The latter determine the transform of the
original system with total dissipation from the arbitrary initial state to the stable equilibrium state
corresponding to the minimum of the initial potential \( V(r) = V_0(r) \).

As an example the motion of the “tippe-top” on the horizontal plane with sliding, spinning and
rolling friction is considered. The tippe-top is a dynamically and geometrically symmetric rigid body
that moves along a horizontal plane. If one spins it fast enough about its vertical axis of symmetry with
the center of mass below, then it undergoes a 180-degree and spins about its vertical axis of symmetry
with the center of mass overhead. The simplest model of the “tippe-top” is a dynamically symmetric
non-homogenous sphere with the center of mass lying on the symmetry axis but not coinciding with
its geometric center. The friction is described by a two-parametric model proposed by the author [3].
When one of this two parameters vanishes 1 this model reduces to the Contensou-Zhuravlev model [4, 5],
that describes only sliding and spinning friction, and the power of the sliding friction is much less
than the power of the spinning friction. The unperturbed system describes the “tippe-top” on a plane
with sliding friction (spinning and rolling friction vanish [2]). The global analysis of this system is
provided. After that the analysis of the “tippe-top” dynamics with all kinds of friction are considered
(see [2, 6]).

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00290.

References:

References:
Two-cable System “Dumbbell-load” in Central Gravitational Field
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Motion of a two-cable system in central gravitational field is considered. The system is consisted of a non-symmetrical dumbbell modeled by two mass points, which are connected by weightless inextensible rod, and a point load connected with ends of the dumbbell by weightless inextensible cables with known lengths. Depending on tension kinds of the cables three types of the system motion are determined. Depending on parameters of the system stability and branching of the obtained solutions are analyzed. The results are presented on bifurcational diagrams.

The system consisted of symmetrical dumbbell and cables of equal length are considered separately. In the case of interconnected system motion the geometric interpretation motion in satellite approximation is obtained. Phase system portrait is plotted. The regions of impossibility of interconnected motion are found.

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Unstable, continuous PWL differential systems whose all linear part are Hurwitz: two different examples.
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We will emphasize how instability can arise when matching in a continuous way several Hurwitz linear or affine differential systems. The examples are in some sense counter-intuitive, since all the matrices involved are Hurwitz matrices, that is, they have eigenvalues with strictly negative real parts.

Two cases will be reviewed. First, the case of 3D piecewise linear semi-homogeneous differential systems made up by matching two linear systems. Such continuous piecewise linear differential systems possess one equilibrium point at the origin, and when its two linear parts from both sides correspond with a stable focus-node the appearance of cone-shaped, invariant manifolds can lead to unstable dynamics, see [1].

Another interesting case arises by considering control systems whose only nonlinearity is a saturation. We will show that, when modeled by means of symmetric piecewise linear systems with three linear zones, the stability of the origin is compatible with the existence of stable limit cycles that
preclude global stability for the operating point. Thus, here we could speak of instability from the point of view of robustness against perturbations, see [2].

References:

Degenerate bifurcations and border-collision in one- and two-dimensional piecewise smooth maps
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We first recall three well-known theorems related to the simplest codimension-one bifurcations occurring in discrete time dynamical systems, such as the fold, flip and Neimark-Sacker bifurcations. The purpose of our work is to analyse these bifurcations in presence of certain degeneracy conditions, that is, when the above mentioned theorems are not applied. Occurrence of such degenerate bifurcations is particularly important in piecewise smooth maps, for which it is not possible to specify in general the effect of the bifurcation, as its result strongly depends on the global properties of the map. In fact, the degenerate bifurcations mainly occur in piecewise smooth maps defined in some subspace of the phase space by a linear or linear fractional function (although not necessarily only such functions, as we show in the examples). The degenerate bifurcations in piecewise smooth maps and related properties are strongly connected with the border collision bifurcations.

Method of Lyapunov functions for differential equations with piecewise constant delay and its application to Neural networks model
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In this talk we investigate the stability of differential equations with piecewise constant argument of generalized type [1-4] based on the second Lyapunov method. Although, these equations include delay, it deserves to be mentioned that stability conditions are only given in terms of Lyapunov functions; that is no functionals are used. Moreover, we establish several stability criteria by employing Lyapunov functions to Recurrent neural networks [5-7] with numerical simulations to illustrate the results.

References:
Anomalous phenomena in a simple nonsmooth system
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We present several surprising phenomena that occur in an extremely simple nonsmooth system of a single inelastic, spherical particle falling under gravity and colliding with walls of a symmetric funnel. One might naively expect that, on average, particles would fall through funnels with steeper sides more quickly, exert a smaller total impulse on the funnel walls, and lose less energy. However, we show that there are special ranges of angles of the funnel walls for which exactly the opposite occurs. Typically, the particle will experience a sequence of collisions that is highly sensitive to the location at which it enters the funnel and nearby particle trajectories become widely dispersed. However, in the special angular ranges this is not the case and the particle can experience sequences of collisions that have a highly coherent structure. We provide a theoretical analysis that can predict and explain this surprising behavior. We also show that such anomalous phenomena occur in both frictionless and frictional particle systems and the frictional force dramatically enhances the anomalous phenomena. This is due to the stability of the highly coherent structure in these nonsmooth systems.

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Analytical and Numerical Investigation of the double-spherical Tippe-Top Dynamics.
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Dynamics of a two-spherical tippe-top on a rough horizontal plane is considered. We suggest that friction forces can be described by the Contenson-Zhuravlev model. The tippe-top is bounded by a non-convex surface that consists of two-spheres and a cylinder; the axis of cylinder coincides with the common spheres’ axes of symmetry. Being fast spinned around its axis of symmetry the tippe-top overturns from the bottom (the big sphere) to the leg (that is modeled by a small sphere). Note, firstly, that dynamics of a non-convex solid body is described by the equations with the non-smooth right-hand side; secondly, multiple impacts appear. The dynamics of the body is investigated by method of generalized Smale’s bifurcation diagrams. The stability and bifurcations of steady-state motions are considered. The method allows to describe qualitatively non-steady motions of the tippe-top. Analytical investigation is illustrated by numerical results. This work is supported by Russian Foundation for Basic Research (project 07-01-00290)
Resonance oscillations in nonsmooth systems with symmetry

Reversible and non-reversible non-smooth dynamical systems
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Many smooth dynamical systems issued from applications, for instance in mechanics, are often time-reversible or Hamiltonian. The difficulty is to identify the specific influence of each property on the behaviour of the system. One can also observe that rare examples of non-reversible Hamiltonian smooth dynamical systems are present in the literature (see L). After a slight adaptation of the definitions, analogous situations and similar questions arise in the non-smooth dynamical systems world.

We shall start the discussion with Anosov semi-linear normal form for discontinuous vector fields (see A). The most interesting case occurs in dimension 4, where this form is linked to the Fuller problem in Control Theory (see ZB). For this reason, HR Sussman asked about the existence of periodic orbits of such semi-linear (uncontrolled) systems and their deformations.

We first give an algorithmic answer to this problem, by means of Effective Algebraic Geometry, that is, standard bases computations and decompositions of semi-algebraic sets.

As a consequence, we can identify which deformations of Anosov normal form present one-parameter families of periodic orbits, and discuss their singular orbits. In particular, periodic orbits of a Hamiltonian non-reversible non-smooth sub-family can be readily analysed.

We shall finally generalise our algorithm to larger classes of non-smooth dynamical systems, including non-autonomous non-smooth differential equations.

References:

Symmetric oscillations in non-smooth mechanical systems
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Necessary and sufficient conditions for periodic motions (oscillations) in non-smooth reversible mechanical systems with space and time symmetry are presented. In the case of linear oscillating system with piecewise-linear coefficients the oscillations are constructed explicitly by sewing the solutions from different linear regions.

The non-smoothness property in engineering applications usually results from the presence of elastic obstacles, tethers, etc. A two-degree-of-freedom oscillator which allows an elastic impact against an obstacle is analyzed. The system under consideration models the building behavior during the earthquake.
Numerical bifurcation analysis of nonsmooth systems

Unfolding the dynamics of a nonsmooth model of gear rattle
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This poster focuses on unfolding the dynamics of a simple model of gear rattle. A second-order nonsmooth ordinary differential equation model is used to describe a pair of meshing gears, where the nonlinearity arises from the backlash between the gear teeth. Despite the model’s simplicity, rich and complex dynamics are still observed. To gain insight into the underlying dynamics a combination of basin of attraction computations, explicit solution construction, one- and two-parameter bifurcation diagrams and manifold computations are employed. We find that there is a complex interplay between both smooth and discontinuity-induced bifurcations, and an explanation of how these locally organise the global dynamics is provided.
Resonance oscillations in mechanical systems with elastic obstacles, dry friction or impacts

Bifurcations in systems with dry friction

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The concept “bifurcation” was introduced by H. Poincare [3] at the end of 19-th century and was used since then in numerous works of specialists in various branches of science. To the most extent the bifurcation theory has been developed in connection with dynamical system

\[ x' = F(x), \quad x \text{ belongs to } \mathbb{R}^k, \]

where \( x' \) denotes derivative with respect to time \( t \), and vector function \( F : \mathbb{R}^k \rightarrow \mathbb{R}^k \) is smooth in a region \( W \) (i.e., belongs to a class \( C_v(W), \ v \geq 1 \)). The notion of bifurcation unites analytical and geometrical properties and can be introduced by different ways (cf.[5-7]), roughly speaking, it means the appearance of new qualitative properties in response to small variations of certain parameters.

In some applications, it is convenient to relate \( t \) as discrete variable and consider Poincare mapping instead of ODE system (1). Another generalization are piece-wise dynamical systems, also related as Filippov systems [8]: the region \( W \) is divided into a finite number of parts \( W_j \) (\( j = 1, \ldots, m \)) while \( F \) is smooth within each component, including boundary. In such systems, most analytical difficulties arise for those trajectories, which go partly along the boundaries of sub-regions \( W_j \) - so called “sliding modes” [12]. A vast review on bifurcation in non-smooth dynamical systems [1-2] covers works on piece-wise smooth (PWS) systems as well as on impacting systems.

Mechanical systems with dry (Coulomb) friction can be treated as PWS owing to discontinuity of friction force \( T \) as function of relative velocity \( v \) at \( v = 0 \). A known example of friction oscillator was proposed by Andronov with co-workers [11] and became a popular model for testing of new analytical techniques [1-2, 13]. However, this example represents relatively small part of dynamical behavior, typical to systems with friction, since no equilibrium exist here. Moreover, most natural frictional contacts are three-dimensional, i.e. vectors of possible relative velocities form a plane. Above all, there exist situations where the motion in the presence of friction can not be defined uniquely. This paradoxes were detected in case of sliding contacts by P. Painleve [10] and in case of equilibrium by J. Jellett [9]. It seems reasonable to include all known types of irregular behavior to the subject of bifurcation theory and to elaborate a unified approach to their analysis.

The talk is organized as follows. First, the friction law is formulated and general classification of friction contacts is proposed. The simple examples are presented to illustrate this classification, they can be considered as basic models for real systems, which can be used in qualitative analysis.

Then we will study equilibria and periodic orbits in systems with associated friction laws. In contrast with smooth case(1.1), systems with friction possess in general families of equilibrium positions. The evolution of such families may lead to non-local bifurcations. Another specific case is stick-slip transition, which can lead to specific bifurcations of periodic orbits.

Systems with non-associated friction law are considered in the last part of the talk. Along with all peculiarities inherent in systems of previous type, specific bifurcations, or friction catastrophes, are possible here. Appropriate examples are Painleve paradoxes [10] and detachment conditions in systems with non-ideal unilateral constraints [14-15]. This section if followed by concluding remarks.

References:
In this talk nonlinear dynamics of impact oscillators with one sided motion constraints will be discussed. First the oscillator with an elastic constraint is studied experimentally and the results are compared with the predictions obtained using its mathematical model. A particular attention is paid to the chaos recorded near grazing frequency when a non-impacting orbit becomes impacting one under increasing excitation frequency. Extensive experimental investigations have been undertaken on the rig developed at the Aberdeen University [1]. Different bifurcation scenarios under varying excitation frequency near grazing were recorded for a number of values of the excitation amplitude [2]. It was found, that the evolution of the attractors is governed by a complex interplay between smooth and non-smooth bifurcations. In some cases the occurrence of coexisting attractors is manifested through a discontinuous transition from one orbit to another through boundary crisis.

In the second part of the talk, the oscillator with a shape memory alloy (SMA) restraint is considered [3]. This impact oscillator has the secondary support made from a SMA and the thermo-mechanical description of the SMA element follows the formulation proposed by Bernardini et. al. [4,5]. The thermo-mechanical coupling terms included in the energy balance equation allow to undertake the non-isothermal analysis. The undertaken numerical investigations suggest that the system can exhibit complex dynamic responses, which if appropriately controlled can be used for vibration reduction. A comparison with an equivalent elastic oscillator is made and it is found out that the low amplitude regimes are not affected by the SMA element. On contrary, for the large amplitude responses, a significant vibration reduction is achieved due to the phase transformation hysteresis loop. Various bifurcation scenarios are constructed and the influence of the SMA element is discussed.

References:
Global analysis of impacting systems
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It is since long known that impacting systems can show very rich dynamics, where attractors are born or disappear in grazing bifurcations [1]. This has lead to an explosion of books and research papers on local analysis of the dynamics in the vicinity of such bifurcations [2, 3]. In this talk, however, we will show two completely different methodologies to perform global analysis of systems with impacts. In the first method we will use a classical approach in which limit cycles are found and their corresponding stable and unstable manifolds are explored. Together with basin of attraction calculation we can thus describe why some periodic and chaotic attractors suddenly disappear that is not obvious from the analysis of grazing bifurcations on their own. We here will use a model for Gear rattle [4] to show what types of results one could expect. In the second method we will use a topological approach, originally described by Chillingworth [5], where the curved phase spaces are straightened out and impacting surfaces gets curved. This technique reveals some interesting mechanisms in which different periodic orbits are born or disappear and highlights why a grazing event can cause rapid changes in the dynamics. To explore this topological method a simple periodically forced harmonic oscillator will be used.

References:

Non-smooth dynamics of friction in modeling of earthquakes
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It is generally believed that one of the basic mechanisms of earthquakes is frictional instability arising at the interface of tectonic plates [1]. This viewpoint is roughly supported by the observation that worldwide locations of seismic activity tend to concentrate around the known plate borders. Frictional instability, on the other hand, appears to be a typical manifestation of static-dynamic friction duality causing the jerking motion that often occurs when two surfaces are sliding over each other (also known as the stick-slip phenomenon [2]).
A mathematical description of the stick-slip motion is a long-standing problem in physics, since it requires a description of the dynamics at very different spatial and temporal scales, ranging from microscopic interactions in mechanical contacts between two surfaces to the macroscopic nonlinear dynamics of the moving bodies. Attempts to reduce the complexity in the theory of frictional dynamics resulted in the introduction of the class of so-called spring-block models, which reproduce well the basic features of the frictional phenomena and do not contradict (fit well) to the results of experiments. The basic idea [3] underlying this class of models consists in including the details pertaining to the physics of contacts to the constitutive friction law, while considering the effects of inertia and deformations via appropriately introduced masses and springs. The aforementioned details include such effects as, e.g., dependence of friction on environmental conditions (temperature, humidity, presence of lubricant), history of friction contact (memory effect), wear of materials, surface roughness, etc. Some variants of the friction laws discussed in the literature also include non-smooth functions describing the dependence of the friction force on parameters, that makes the dynamics produced by the corresponding models highly non-trivial [4].

In this talk, I survey some of the recent results in the field of modeling earthquakes with blocks, springs, and several variants of the friction law. First, the apparently simple one-block configuration is considered at different levels of complexity, starting from the simplest description of friction by the Coulomb law to the more comprehensive model of rate- and state- dependent friction introduced by Dieterich, Rice, and Ruina [5]. The necessary conditions for transition from stable sliding to the stick-slip motion are discussed, with a purpose of finding the simplest possible model of earthquakes. Then, I consider the multi-block configurations known as Burridge-Knopoff model of earthquake faults [6] that received much recent attention after the works by Carlson and Langer [7] who demonstrated that block arrays subject to a simple velocity-weakening friction are able to reproduce the well-known Gutenberg-Richter scaling law [8] between the magnitude of earthquakes and their occurrence rate. Finally, I discuss the relationship between the complexity in the behavior demonstrated by the system of blocks and springs with overall dimensionality of the model (number of degrees of freedom) defined by either the friction law or the system geometry. Two examples of minimal spring-block configurations that produce complex behavior will be presented [9,10] that elucidate the main factors responsible for the transition between periodic and chaotic stick-slip motion, as well as clarify the role of asymmetric elastic forces in the appearance of intermittent frictional instability.

References:
An analytical approach to codimension 2 bifurcations in the dry friction oscillator
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The dry-friction oscillator, that is a spring with suspension point and dry-friction, can be modelized by a differential equation which has discontinuous right hand side due to the friction.

This Filippov system has two parameters related to the friction and the periodic forcing. For a wide range of values of these parameters the existence of a periodic orbit has been proved.

However, it has been seen numerically that, depending on the values of these parameters, the behavior of this periodic orbit is qualitatively different. In this work we analytically study this changes of qualitative behavior around two codimension two points in the parameter space, from which some codimension 1 bifurcation curves emanate. The existence of these curves is proved and the behavior of the periodic orbit for parameters lying in this curves is studied.

Critical Excitation for an Elasto-Plastic Oscillator
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We consider control problems for the variational inequality describing a single degree of freedom elasto-plastic oscillator. We are particularly interested in finding the "critical excitation", i.e., the lowest energy input excitation that drives the system between the prescribed initial and final states within a given time span. The response of the oscillator to an excitation can be described as a sequence of alternating elastic and and plastic phases where the state variables have to satisfy size and sign constraints. Using the appropriate governing equations inside the elastic and plastic regimes, respectively, and continuity conditions between connecting segments we are able to construct a forward process, which combined with an iterative procedure provides approximations of the critical excitation.
Open problems of nonsmooth bifurcation theory coming from control

Bifurcations of feasible and virtual orbits in a ZAD-controlled buck converter
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Based on a recently obtained Lemma about periodic orbits in linear systems with a piecewise-linear non-autonomous periodic control, we describe analytically the bifurcation structures in a ZAD controlled buck converter. This analytical description shows that the period doubling bifurcation in this system may be both subcritical and supercritical. Considering virtual orbits we show how a saddle-node bifurcation becomes feasible and how it is destroyed at a new codimension-2 bifurcation point, where the subcritical period doubling bifurcation becomes supercritical. We also show that this phenomenon does not take place when a piecewise-linear approximation is used in the ZAD condition.
Open mathematical problems and applications of nonsmooth bifurcation theory in engineering

Excitation and control of nonsmooth resonant mode for ultrasonically assisted machining of intractable materials
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The use of the nonsmooth (vibro-impact) resonant vibration mode to develop mechanical work for machining of intractable materials is discussed. This can be achieved by condensing the tool-workpiece interaction into a sequence of micro collisions for impulsive penetration of the tool to the material being treated. This transforms the basically unstable cutting process into incremental, repetitive and controlled. Nonlinear models of cutting process with superposition of ultrasonics are proposed and investigated. It is confirmed that under the influence of high frequency vibration the phenomenological plasticisation of brittle materials and fluidisation of dry friction occurs. The phenomena are well described by averaging of intermittent penetration process. The mechanical characteristics of transformed machining processes are obtained. It is shown that excitation of the high-frequency nonlinear mode of tool-workpiece interaction is the most effective way of ultrasonic influence on dynamic characteristics of machining. The exploitation of this mode needs a new method of nonlinear control called as autoresonant one. To stabilise the nonlinear mode under unpredictable change of processing loads, an approach has been developed to design a cutting machine as an intelligent self-exciting oscillating system. The autoresonant system with supervisory computer control was developed and successfully used for the control of the piezoelectric transducer during ultrasonically assisted cutting.

References:

Gap junctions and emergent brain rhythms: A mathematical study
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The presence of gap junction coupling among inhibitory neurons of the neocortex has been appreciated for some time now. In recent years there has been an upsurge of interest from the mathematical community in understanding the contribution of these direct electrical connections between cells to large-scale brain rhythms. In this talk I will focus on a class of exactly soluble single neuron models, capable of producing realistic action potential shapes, that can be used as the basis for understanding dynamics at the network level. In essence I will discuss piece-wise linear and discontinuous dynamical models that can mimic the firing response of both Type I and Type II cells. Under constant current injection periodic responses can be obtained in closed form, as indeed can the phase response curve (PRC), and conditions for stability (using Floquet theory). From the calculated PRC and the periodic orbit a phase interaction function can be constructed that allows the investigation of phase-locked network states using the theory of weakly coupled oscillators. For large networks with global connectivity I will also show how to develop a theory of strong coupling instabilities of the synchronous and splay state. In this case an increase in the strength of gap junction coupling is shown to lead to large amplitude bursting oscillations in the mean membrane potential of Morris-Lecar networks.

References:
Effects of Dry Friction in Automotive Transmissions
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Dry friction plays an important role in different components of automotive transmissions. In the talk we will start with the classic instabilities caused by negative friction gradient, then we’ll go further to more complex phenomena like flutter-type behavior and memory-type hysteresis in systems with distributed friction contacts. Examples are friction discs in clutches, axial bearings, CVT-chains and springs usually used in torsional dampers.

References:

Nonsmooth Features of Large Dynamical Systems
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During the last three decades significant progress has been made with respect to the mathematical foundations of non-smooth mechanics, especially with respect to non-smooth dynamics. Less progress has been made with respect to apply these tools to large industrial problems though their usefulness is obvious and though the number of such applications are growing from year to year. Examples are powertrains of cars and trucks, especially chains and belts, chimney damping systems, all types of vibration conveyors, large hydraulic systems and many problems from electronics.

The key points in modeling such systems consists in the compact formulation of the system itself and in a second step on the numerical algorithms available. By establishing models with the help of Moreau’s measure differential equations, by introducing Augmented Lagrange Methods in connection with prox-function from set-theory and by applying time-stepping for numerical solutions many of the problems for large systems could be solved in the last years, though computing times are still large. The presentation will give a survey of these problems, as well theoretically as practically presenting large industrial examples.

References:
Complex Attractors in Wave Equations with Friction Compared with Earthquakes
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Earthquakes display a rich set of complex behaviors, and a wealth of quantitative data exists describing these behaviors. Yet the origin of these behaviors remains largely unexplained, and finding a set of equations which can reproduce all the behaviors is still an as yet unachieved goal. However, simple spatially extended dynamical models have shown that a remarkably rich set of earthquake-like behaviors can arise from nonsmooth friction applied to the wave equation. The complex attractors arising in these systems show surprising insensitivities to many aspects of the system, including dimensionality of the bulk and dispersive properties in the bulk. Additional robustness includes structural stability to inhomogeneous spatial perturbations of friction. Projections of the complex attractors onto behaviors which can be compared with earthquakes shows many successes, and some deficiencies. This talk provides an overview of earthquakes and key robust observations associated with them, and complex attractors arising from wave equations with nonsmooth friction, seen in efforts to model them.

References:

Where Piecewise-Linear Systems Appear in Engineering Dynamics
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As one might expect, the concerns of engineering dynamicists are not generally the same as those of mathematicians when thinking about nonlinear dynamical systems. Unfortunately for engineers, many of the nonlinearities encountered in engineering are also non-smooth: impacting phenomena, clearances and friction are all common causes of concern. Very often, analytical results are not available for guidance and then empirical methods are employed. The objective of this presentation
will be to highlight two areas of structural dynamics where non-smooth systems occur, to discuss how
engineerings have tried to accommodate their complex behaviours and (hopefully) to solicit advice
from the mathematical community. The first area of interest will be that of system identification.
This is the problem of inferring equations of motion from measured data. Although many approaches
have proved successful in the case of polynomial nonlinearities; non-smooth systems present extra
difficulties, often in the form of imposing a nonlinear optimisation problem. The second area of
engineering dynamics discussed is that of damage detection. Often, the occurrence of structural
damage will convert an initially linear system into a nonlinear system and the nonlinearities will be
non-smooth. Simply identifying that the system is now nonlinear can sometimes be regarded as a
baseline diagnosis of damage. In the past, features commonly encountered in nonlinear dynamical
systems theory have been used as diagnostic data. The presentation will discuss how successful this
strategy has been in the past and outline the pros and cons of the approach.

References:
Other topic

Bifurcations over a nonsmooth implicit function theorem
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An abstract approach to local solvability of an implicit equation with nonsmooth terms is considered. Sufficient conditions for bifurcation are proposed and branching directions of solutions are obtained in an explicit way. The approach can be applied for studying bifurcations in nonsmooth Poincare maps.

Methods of invariant normalization for Hamiltonian systems
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We discuss new algorithm of calculation of Hamiltonian normal form. A normal form of a Hamilton system has two main properties: a) Tailor expansion of the normal form has the simplest form; b) its linear part commutates with a nonlinear one. Property a is used for the normalization procedure. Property b is used to build asymptotic solutions. For this purpose, instead of the normal form we introduce a definition of a symmetrical form: a form submitting property b. Symmetrization algorithm is reduced to sequential calculations of the quadrature in the approximation of each order and is essentially simpler than all the classical normalization procedures. The following examples are considered: the plane circular restricted three-body problem, three-dimensional oscillations of a heavy point on the spring for resonance, Lagrange top on vibration base, spherical pendulum with three-dimensional vibration of the suspension point.

The algorithm can be useful for solving Hamiltonian systems, which arise after reduction of nonsmooth systems to smooth ones by the means of nonsmooth replacement.

References:

On nonlinear resonance oscillations of a spring supported point particle
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Full research of two-dimensional small nonlinear oscillations of spring pendulum with nonlinear dependence of a tension of a spring on its lengthening is performed. The method of a Hamiltonian normal form is used. For reduction to a Hamiltonian normal form the method of invariant normalization is used, what essentially reduces calculations. Solutions of the normal form equations have shown that periodic reorganization between vertical and horizontal oscillations occurs only in case of resonances 1:1 and 2:1. At a resonance 2:1 this effect is shown in square-law members of the equation and at a resonance 1:1 one should take into account cubic members. In all other cases, both in the presence of a resonance, and at its absence, oscillations have constant frequencies with a little different from frequencies of linear approach. For a resonance 2:1 it is found maximum detuning of frequencies at which the effect of swapping of energy from one kind of oscillation to another disappears. The resonance 1:1 is physically possible only for a spring possessing the negative cubic term in the law.
of deformation. The problem of a spring pendulum is a good way to illustrate the method of invariant normalization, described in A.G. Petrov talk Methods of invariant normalization for Hamiltonian systems. The algorithm can be useful for solving Hamiltonian systems, which arise after reduction of non-smooth systems to smooth ones by the means of non-smooth replacement.

References:

Railway Vehicle Dynamics - a Challenge in Mathematics and Numerical Analysis
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In the modern society the railways recover as a transport system due to its ability to move many persons and heavy loads fast, safely and with low impact on the environment. In order to be competitive the railways must deliver a cheap and reliable product in cooperation with other means of transport.

These challenges force the railway companies in cooperation with the railway industry to take large steps forward in their development of the infrastructure and the rolling stock in contrast to the traditional empirical advances that had hitherto characterized the development of railway technology. In order to preserve safety the progression had taken place in small steps due to the empirical nature of the applied methods.

The large steps forward into unknown territory were made possible by the recognition and rapid development of nonlinear dynamics in the late 20th century in connection with the rise of the computer as a tool for the analysis of nonlinear mathematical problems in a non-trivial number of dimensions.

The dynamics of railway vehicles is modelled as mechanical multibody systems that may contain continuous sub systems. The models are formulated mathematically as dynamic systems. In addition kinematic constraints enter, so the dynamic problem of the determination of the motion and the forces of the system cannot be formulated as a system of ordinary differential equations only.

The total system of equations form a nonlinear and non-smooth, multi-parameter dependent dynamic system, which must be solved for various initial conditions in certain ranges of the parameters. The dominant parameter, which varies most in the solution interval, is the speed of the vehicle. It is chosen as the primary control parameter. The number of degrees of freedom of a model of a realistic railway vehicle is large enough to prevent the use of analytical methods for the solution of the dynamic problem. A numerical analysis of the problem is thus unavoidable.

Due to the nonlinear nature of the models, multiple attractors exist in certain parameter intervals. A bifurcation analysis of the dynamic system and an investigation of the stability of the various solutions of the dynamic problem are therefore fundamental steps in the total analysis. The steady motion of railway vehicles can be stationary in an appropriately chosen frame, it can be periodic or quasi periodic in time or it can be chaotic. Both chaotic attractors and chaotic transients with large amplitudes have been found theoretically and in measurements in road tests of real vehicles.

In the presentation an example of a dynamic model of a vehicle will be presented and typical constraints and non-smoothnesses will be pointed out. The methods used today for the analysis of the dynamic problems will be presented and discussed including the question of a firm mathematical basis for the solution. The presentation will include a survey of mathematical and numerical problems - some where the numerical solution has yielded a result but the fundamental mathematical theory is missing and some open mathematical and numerical questions.

References:
Hoffmann M., Dynamics of European two-axle Freight Wagons, IMM, The Technical University of Denmark, 2006,
http://www2.imm.dtu.dk/pubdb/views/publication_details.php?id=4853