Dual Sales Channel Management with Service Competition

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We study a manufacturer’s problem of managing his direct online sales channel together with an independently owned bricks-and-mortar retail channel, when the channels compete in service. We incorporate a detailed consumer channel choice model in which the demand faced in each channel depends on the service levels of both channels as well as the consumers’ valuation of the product and shopping experience. The direct channel’s service is measured by the delivery lead time for the product; the retail channel’s service is measured by product availability. We identify optimal dual channel strategies that depend on the channel environment described by factors such as the cost of managing a direct channel, retailer inconvenience, and some product characteristics. We also determine when the manufacturer should establish a direct channel or a retail channel if he is already selling through one of these channels. Finally, we conduct a sequence of controlled experiments with human subjects to investigate whether our model makes reasonable predictions of human behavior. We determine that the model accurately predicts the direction of changes in the subjects’ decisions, as well as their channel strategies in response to the changes in the channel environment. These observations suggest that the model can be used in designing channel strategies for an actual dual channel environment.1

Key words: dual channels; direct channel; service competition; product availability; supply chain contracting; experimental economics

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1. Introduction

Increasingly, manufacturers have been selling through their direct online channels, in addition to traditional (bricks-and-mortar) retail channels (Allen 2000, NPD Group 2004). In this paper, we determine how a manufacturer can effectively manage his direct online channel and an independent bricks-and-mortar retail channel when the channels compete to provide better service to consumers. To do this, we develop an analytical model that incorporates key characteristics of both channels. For example, the manufacturer can reach Web-savvy consumers through the direct online channel. He can also reach time-conscious consumers through the bricks-and-mortar retail channel. The manufacturer earns a higher profit margin in the direct channel, whereas he can share the inventory risk with the retailer in the retail channel. We characterize the manufacturer’s optimal wholesale price when contracting with the retailer, and the resulting channel mix. The key features of the model include availability-based service competition between the channels and a detailed consumer channel choice model based on the service levels at each channel as well as the consumer’s valuation of the product and the shopping experience.

We formulate a dual channel environment by first determining the key drivers in the consumers’ channel choice process. Consumers consider the relative advantages and disadvantages of the channels in their decisions. For example, buying from the direct online channel requires waiting for delivery, whereas the retail channel offers instant gratification. The survey

1 The listing of authors is alphabetical.
results in Gupta et al. (2004) confirm that delivery lead time is an important factor in consumers’ channel choices. Gilly and Wolfinbarger (2000) report that focus group participants cite instant gratification as a major advantage of retail shopping over online shopping. Visiting the retailer’s store, however, does not guarantee ownership because the consumer may face a stockout. Hence, consumers also consider product availability in their channel choices. Stearns et al. (1981) and Gilly and Wolfinbarger (2000) report that stockouts are the most frequently mentioned reason for consumer dissatisfaction in the retail channel. Fitzsimons (2000) finds that when consumers are exposed to a stockout, they are significantly less likely to return to that store on a subsequent visit. Hence, retailers and independent consumer organizations, such as bizrate.com, provide retailer ratings on the measure “availability of product you wanted” (see also Bernstein and Federgruen 2004).

Retail inconvenience is another driver for why consumers shop online (Bhatnagar et al. 2000, Gilly and Wolfinbarger 2000). A consumer incurs monetary as well as time- and effort-related costs when buying from the retail store. Given these observations and consumer behavior research, we identify the following three drivers of consumer channel choice as our focus: (1) delivery lead time in the direct channel, (2) product availability in the retail channel, and (3) relative inconvenience of buying from the retail channel.

We develop a consumer channel choice model that incorporates the aforementioned channel choice drivers. Consumer demand in each channel is determined endogenously, as a function of the service levels in both channels. The direct channel’s service level is the delivery lead time, which is defined as the time a consumer needs to wait for delivery after making an online order from the direct channel. The manufacturer determines the delivery lead time in the direct channel with his operational decisions. He can provide a shorter delivery lead time and hence better service by keeping more components (or, finished goods) in inventory or by using expedited shipping. Such actions, however, result in a high direct channel cost. The retail channel’s service level is product availability, which is defined as the probability that an arriving consumer finds the product. The retailer’s product availability level is related to her stocking (inventory) level. We also capture the relative inconvenience of the retail channel with the retailer inconvenience cost. This cost includes the monetary as well as the time- and effort-related costs of visiting the retail store. It may also depend on the product characteristics. For example, the inconvenience cost may be higher for a bulky product.

Our consumer choice model is also consistent with the results of the market research conducted by Research International in 2003 and Forrester Research in 2005. The results are based on Internet and phone surveys of more than 2,500 consumers and business technology decision makers. The research uncovers key factors in choosing between the direct channel and the retailer as follows. The first factor is “get the lowest price” (ranked #1). The next two are “get the brand I want” (ranked #2) and “get the product quickly” (ranked #3). These two factors suggest the importance of delivery lead time in the direct channel and product availability in the retail channel. The report also lists the factors “quick and efficient purchase process” (ranked #4), “easily return or exchange the product” (ranked #5), “easy to get to” (ranked #6), “speak to a knowledgeable salesperson” (ranked #7), “demo/interact with product” (ranked #8), and “have flexible shopping hours” (ranked #9). We capture some of these factors in our retailer inconvenience cost.

The same marketing study also states that a major part of sales is through the retailer. The report concludes that a “must-win” strategy is to work with the #1 retailer whose consumer account for technology is the largest. The study states, “Integration of the online and retail experience (e.g., same pricing) is critical.” In fact, most manufacturers avoid undercutting the retailer’s price in the direct channel (see Cattani et al. 2006 for other examples). For the purpose of our study, the same retail price is offered in both the direct and retail channels so the channels do not compete on price. Instead, our focus is on the service competition between the channels.

Our model helps quantify the benefits of a dual channel strategy and identify optimal channel strategies for a given channel environment. In particular,
we determine three types of effective dual sales channel strategies for the manufacturer depending on the channel environment. (1) When the direct channel cost is low, the manufacturer should optimally capture the entire consumer population with his direct channel by setting a short delivery lead time. This strategy induces the retailer to opt out by not stocking the product. (2) When the direct channel cost is above a certain threshold, and if the retailer inconvenience cost is high, the manufacturer should optimally sell through both channels by setting the delivery lead time such that the consumer population is segmented into two. The first segment buys only from the direct channel. The second segment buys only from the retailer, if they find the product available. Otherwise, they leave the system without buying. The manufacturer captures all profit from the retailer by setting a high wholesale price. The manufacturer can follow this strategy because the retailer has to set at least a minimum availability level, independent of the manufacturer’s decisions, to remain in business. (3) When the direct channel cost is high and the retailer inconvenience cost is low, the manufacturer has no choice but to sell through both channels and share profits with the retailer. In this case, the manufacturer separates the market into three segments such that the third segment considers buying from the direct channel if it cannot find the product at the retail store. Note that each channel strategy encompasses three important aspects: channel configuration, market segmentation strategy, and profit sharing strategy. We also provide product examples that are candidates for each channel strategy as discussed later in §5.

We also conduct a sequence of controlled experiments with human decision makers to investigate whether our model makes reasonable predictions of human behavior. Decision makers in practice may not behave as predicted by the Nash equilibrium. To study the effects of behavioral factors on our model’s predictions, we recruited human subjects to play the roles of manufacturer and retailer in computer-simulated experiments. The subjects were paid according to how well they performed in the experiments. The results indicate that the model accurately predicts the direction of changes in the human subjects’ decisions. It also accurately predicts their channel strategies given a channel environment. Hence, the model and the analytical results can be used in designing contracts and channel strategies for an actual dual channel environment. However, we also find that the analytical model is less successful in predicting subjects’ quantitative decisions. Hence, one should perhaps be cautious in using the model to predict the exact quantities or values corresponding to actual decisions.

2. Literature Review

This paper is the first to model availability-based service competition in a dual channel setting. In addition, we integrate a detailed consumer channel choice model to determine the manufacturer’s and the retailer’s optimal operational decisions. The relevant literature can be divided into three areas: dual channel management, general competition in manufacturer-retailer systems, and availability-based competition.

There is a growing literature on dual channel management, reviewed by Tsay and Agrawal (2004a) and by Cattani et al. (2004). Most papers in this area study competition in price and/or marketing effort (Bell et al. 2002, Chiang et al. 2003, Tsay and Agrawal 2004b, Cattani et al. 2006). We add to this literature by studying availability-based service competition. Another factor that distinguishes our model is the consideration of stochastic demand. With the exception of Boyaci (2005) and Seifert et al. (2006), most research in this area assumes deterministic demand and ignores the effects of inventory. Hendershott and Zhang (2006) consider a consumer choice model, similar to our approach in determining end-customer demand; however, they do not study availability-based competition. See also Netessine and Rudi (2006).

The second area is the study of product and price competition between manufacturers and retailers (McGuire and Staelin 1983, Trivedi 1998 and references therein). Trivedi (1998) considers the effect of product and channel substitution on profits. Lal and Sarvary (1999) consider manufacturers that compete in multiple channels, where each manufacturer controls both an online channel and a retail store. This literature focuses on product competition but not on the competition between the manufacturer-owned channel and the retailer.

The last area is the study of availability-based service competition. Representative studies include service competition between firms (Hall and Porteus
2000, Bernstein and Federgruen 2004) and between single-channel supply chains (Boyaci and Gallego 2004). Similar to their models, both channels’ demand in our model depends on the retailer’s availability level. However, in our model availability determines demand through the consumers’ channel choice process, rather than as a parameter of an exogenously determined demand function. Dana (2001) and Mahajan and Van Ryzin (2001) consider a model of consumer choice behavior with availability-based competition. None of the work in this area considers the dual channel setting, in which the manufacturer is both a supplier and a competitor to the retailer.

Developing models for consumer product and channel choice processes is also a growing area of research in marketing. Models of consumer product choice often have five stages: (i) recognition of need, (ii) information search, (iii) evaluation of product alternatives, (iv) purchase decision, and (v) post-purchase stage (Engel et al. 1973). In this paper we focus on the fourth stage, the purchase decision. We study consumers who have already decided which product to purchase and are in the process of making the channel choice. Researchers also study how consumers choose between online and bricks-and-mortar channels. For example, Bhatnagar et al. (2000) calculate a perceived convenience parameter for the online channel. The authors find that larger, expensive, and more-involved products (e.g., home electronics, hardware) have high online convenience, whereas products that require touch and feel (e.g., sunglasses, clothing) have low online convenience. In our model, the online convenience is captured by the relative retailer inconvenience cost that appears in the consumer’s utility function.

Since Chamberlin (1948) and Smith (1962), there has been growing interest in using experiments with human decision makers to understand the behavioral factors affecting decisions. For example, Charness and Chen (2002) study minimum advertised price policies. Croson and Donohue (2002) review experimental studies of the beer distribution game, focusing on the behavioral causes of the bullwhip effect. Schweitzer and Cachon (2000) and Bolton and Katok (2008) study behavioral issues associated with the newsvendor problem. Ho and Zhang (Forthcoming) and Katok and Wu (2006) study the effects of behavioral factors on contracting strategies between a manufacturer and a retailer. This literature focuses mainly on games against nature (e.g., the newsvendor problem) or stage games (e.g., wholesale price setting games), where one player (the retailer) moves after observing the action of another player (the manufacturer).

This paper addresses new aspects with respect to supply chain behavioral research. First, the human subjects in our experiments were competing in operational decisions (delivery lead time and stocking level) as opposed to price. Second, the consequences of the subjects’ decisions, which are results of an underlying consumer choice model, cannot be represented by simple rules. They need to be shown to the subjects using a software tool. Therefore, the purpose of our behavioral experiments is to rigorously test whether complex decisions and competition render analytical game theory irrelevant to real-world behavior.

Next we clarify the boundaries of our research by discussing what we do not study. First, we consider brand-sure consumers who have already decided what product they want to purchase. Hence, we do not address information search issues, such as search costs, or product competition issues, such as substituting products. In fact, Jupiter research finds that 77% of online shoppers have a specific purchase in mind (Gilly and Wolfinbarger 2000). Hewlett-Packard marketing research reports that when shopping for printer supplies, 73% of consumers are brand sure. The figure is 55% for printers and 54% for computers. Second, we address a dual channel structure with a direct online channel and a retail store. Other channel structures are also observed in practice. The manufacturer may operate a retail store together with its direct channel, such as Apple computer’s company stores. Alternatively, the retailer may have her own direct channel, a practice referred to as e-tailing. Third, we focus on a linear wholesale price contract between the manufacturer and retailer. There are other contract possibilities such as rebates, revenue sharing, and buyback contracts. Hence, the study of dual channel management is a fertile research area.

The rest of the paper is organized as follows. In §3, we describe the overall model. In §4, we analyze the model in three stages. In §5, we identify optimal dual channel strategies for the manufacturer and characterize how these strategies change with respect to the
channel environment. In §6, we present our experimental study. In §7, we conclude and provide possible future research directions.

3. Overall Model
Consider a manufacturer that sells a product through his direct online channel and a traditional retail store during a sales season. The sequence of events is as follows (and summarized in Figure 1). During the contracting stage, the manufacturer sets the wholesale price $w$. Given the wholesale price, both firms make operational decisions without observing each other’s actions. In particular, the retailer chooses her service level $\alpha$, the probability of not stocking out, without observing the manufacturer’s decision for the direct channel. The manufacturer sets the delivery lead time $t$ in the direct channel without observing the retailer’s decision. The retailer then orders the required stocking quantity from the manufacturer, who delivers prior to the selling season. During the season, consumers decide which channel to buy from (consumers’ channel choice). To do this, they consider the delivery lead time in the direct channel, the retailer’s service level, and the retailer inconvenience cost $k$. Depending on the sales price $p$, product availability, and the value $v$ the consumer derives from the product, each consumer either buys the product or leaves the system. At the end of the season, the manufacturer’s and the retailer’s profits and consumers’ utilities are realized.

Each consumer may buy the product from either the direct channel or the retailer, or may not buy at all. Consumers differ in their willingness to wait before receiving the product. To model this heterogeneity, we assume that the consumers are uniformly distributed along a unit-length line and indexed by the time-sensitivity index $d \in [0, 1]$. The total number of consumers in the market (the market size), denoted by $X$, is a uniformly distributed random variable between 0 and $a$. Randomness in the market size corresponds to shifts in demand caused by exogenous factors such as the overall state of the economy. For ease of reference, we summarize the notation in Appendix A.

The consumer with index $d$ derives utility $u_d$ when buying from the direct channel. This utility depends on the delivery lead time $t$, which is set by the manufacturer. The direct channel satisfies all orders because the delivery lead time $t$ provides sufficient processing time for the manufacturer. Each consumer also derives an expected utility $E[u_s]$ from visiting the retailer. This expectation is due to the uncertainty in product availability, which depends on the retailer’s service level $\alpha$. The retailer offers instant ownership, so the consumer’s utility does not reduce because of waiting. But the inconvenience of visiting the retailer reduces the consumer’s utility. The explicit utility functions are provided in the following section.

The consumer with index $d$ decides from which channel to buy after comparing her utility from both channels. The outcome of this comparison made by each consumer constitutes four streams of demand for the product (as illustrated in Figure 1). When the consumer is willing to wait, i.e., when $u_d \geq 0$, she considers buying from the direct channel as an alternative. In this case, when $u_d \geq E(u_s)$, the consumer buys from the direct channel. Such consumers constitute the primary demand $D_d^1$ in the direct channel. When $u_d < E(u_s)$, the consumer visits the retailer. If the product is available at the retailer, she buys it. These consumers constitute part of the retailer demand $D_d$. If the consumer faces a stockout, she buys from the direct channel. These consumers constitute the secondary demand $D_d^2$ in the direct channel. When the consumer is sensitive to waiting and receiving the product, i.e., when $u_d < 0$, she does not consider buying from the direct channel at all. In this case, when $E(u_s) \geq 0$,
the consumer visits the retailer. If the product is available at the retailer, she buys it. These consumers constitute the rest of the retailer demand $D_r$. If the product is not available at the retailer, the consumer leaves the store without purchasing. If the expected utilities from both channels are negative, then the consumer does not buy from either channel. These last two groups of consumers constitute the lost demand $D_l$.

To offer a delivery lead time $t$, the manufacturer incurs the direct channel cost $m/t^2$, where $m$ is the direct channel cost parameter. We use the direct channel cost to capture the inventory and shipping costs related to the direct channel operation, which we do not model explicitly. To offer a short delivery lead time (i.e., high service) to consumers, the manufacturer may keep a finished goods inventory and use expedited shipping. These actions would cause a high direct channel cost.

4. Analysis
We solve the three-stage model discussed in the previous section with backwards induction. First, we characterize the expected demand satisfied through the direct and retail channels. Next, we study the Nash equilibrium of the operational decisions game and establish the retailer’s and the manufacturer’s best response functions. Finally, we solve for the manufacturer’s optimal wholesale price $w$.

4.1. Consumers’ Channel Choice
The percentage of consumers served through each channel depends on the service level $\alpha$ in the retail channel and the delivery lead time $t$ in the direct channel. To characterize this split, we introduce the utility that a consumer derives from visiting either channel. Next, we identify the resulting market segments.

The consumer with index $d$ derives utility

$$u_d(d) = v - p - dt$$

from the direct channel. Recall that $v$ is the value that a consumer derives from the product and $p$ is the product’s sales price (where $p < v$). The term $dt$ denotes the reduction in consumer $d$’s utility from waiting $t$ time units before receiving the product.

Each consumer also derives an expected utility from visiting the retailer, i.e.,

$$E[u_i] = \phi(\alpha)(v - p) - k.$$

The expectation is due to the uncertainty in product availability. The term $\phi(\alpha)$ denotes the retailer’s product availability level defined as the probability that a consumer finds the product in store. Consumers infer the availability level $\phi(\alpha)$ from the retailer’s service level $\alpha$. We derive the functional form of $\phi(\alpha)$ when we solve for the retailer’s problem in Lemma 2. We define the minimum service level at the retailer such that $E[u_i] \geq 0$. This inequality defines the retailer’s minimum service level as

$$\alpha_{\text{min}} \equiv \{\alpha \in [0, 1] \mid \phi(\alpha) = \frac{k}{v - p}\}. \quad (1)$$

Note that as long as the retailer’s expected profit is nonnegative, she sets her service level to be at least $\alpha_{\text{min}}$; otherwise consumers do not visit the retailer.

Next we characterize the market segments formed as a result of the heterogeneity in consumers’ willingness to wait and receive the product. Let

$$d_1 = \min\{\{d \mid u_d(d) = E[u_i]\}, 1\}$$

$$d_2 = \min\{\{d \mid u_d(d) = 0\}, 1\}$$

$$d = \frac{\min\{\{(v - p)(1 - \phi(\alpha) + k)/t, 1\}\}}{\min\{\{(v - p)/t, 1\}\}}.$$ \quad (2)

Figure 2 illustrates how $d_1$ and $d_2$ divide the consumer population into three segments.

The consumer’s utility from the direct channel is large when her time-sensitivity index $d$ is low. Consumers with a time-sensitivity index lower than $d_1$ buy from the direct channel without visiting the retailer (because $u_d(d) \geq E[u_i]$). Consumers with a time-sensitivity index higher than $d_1$ visit the retailer. If the product is available at the retailer, they buy it. If consumers face a stockout, they buy from the direct channel only when $d \leq d_2$ (because $u_d(d) \geq 0$). Note also that not all consumers derive positive utility from the direct channel. Hence, consumers with index $d > d_2$ do not consider buying from the direct channel.
channel at all (because \( u_d(d) < 0 \)). Incidentally, the Hewlett-Packard marketing research states that when shopping for technology products, 9% of consumers purchased products through an online channel, 60% considered both retail and online activities, and 31% did not consider any online purchase.

Given the above market segmentation, the following lemma summarizes demand in each channel for all possible combinations of market segments as a function of the delivery lead time. Proofs are deferred to Appendix D.

**Lemma 1.** Random demand in the direct channel and in the retailer are as follows:

<table>
<thead>
<tr>
<th>Delivery lead time range</th>
<th>( t \leq t^* )</th>
<th>( t \in (t^*, v - p) )</th>
<th>( t \in (v - p, \infty) )</th>
<th>( t \to \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer’s status</td>
<td>Inoperative</td>
<td>Operative</td>
<td>Operative</td>
<td>Operative</td>
</tr>
<tr>
<td>Direct channel coverage</td>
<td>Full</td>
<td>Full</td>
<td>Partial</td>
<td>None</td>
</tr>
<tr>
<td>Retailer demand ((D_1))</td>
<td>0</td>
<td>((1 - d_1)X)</td>
<td>((1 - d_1)X)</td>
<td>(X)</td>
</tr>
<tr>
<td>Primary demand ((D_2))</td>
<td>(X)</td>
<td>(d_1X)</td>
<td>(d_1X)</td>
<td>(n/a^*)</td>
</tr>
<tr>
<td>Secondary demand ((D_3))</td>
<td>(n/a)</td>
<td>([D_r - q]^+)</td>
<td>(d_2 - d_1)</td>
<td>([D_r - q]^+)</td>
</tr>
<tr>
<td>Lost demand ((D_4))</td>
<td>(n/a)</td>
<td>(0)</td>
<td>(1 - d_1)</td>
<td>([D_r - q]^+)</td>
</tr>
</tbody>
</table>

\(^*t^* = (v-p)(1-\phi(\alpha)) + k\) and \(n/a\): not applicable.

For any given operational decisions pair \( \alpha \) and \( t \), the random primary demand in the direct channel is \( D_2 = d_1X \), and the random demand at the retailer is \( D_1 = (1 - d_1)X \). If \( D_1 < q \), then the retailer satisfies all demand in her store. If \( D_1 > q \), then the retailer cannot satisfy \([D_r - q]^+\) units of demand. Among these unsatisfied consumers, \((d_2 - d_1)/(1 - d_1)\)% have \( u_d(d) \geq 0 \); hence they buy from the direct channel. These consumers constitute the secondary demand in the direct channel. The rest, i.e., \((1 - d_2)/(1 - d_1)\)% of these consumers, have \( u_d(d) < 0 \), and they leave the system without buying the product; they are lost demand.

Note that for a given service level \( \alpha \), by setting the delivery lead time \( t > v - p \) (hence, \( d_2 < 1 \)), the manufacturer separates the market into three segments and serves part of the market through his direct channel, allowing the retailer to sell the product, and lets some consumers leave the system without buying the product.

The manufacturer can also separate the market into two segments by setting \( t \leq v - p \) (hence, \( d_2 = 1 \)). In this case, the manufacturer uses the direct channel to serve all consumers. In other words, he ensures that all consumers derive positive utility from buying the product either through the retailer or through the direct channel.

The manufacturer can decide not to segment the market by setting a short delivery lead time. In this case, he serves all consumers only through his direct channel (hence, \( d_1 = 1 \)). In particular, when \( t \leq t^* \), all consumers choose to buy from the direct channel, and the retailer is inoperative. The market can also be left unsegmented by setting \( t \to \infty \) (hence, \( d_1 = d_2 = 0 \)). In this case, the manufacturer sells only through the retailer and essentially shuts down the direct channel operation.

The market segmentation also depends on the retailer’s availability level. Note that we have only two segments if \( \phi(\alpha) = k/(v - p) \) and \( t > v - p \) (because \( d_1 = d_2 < 1 \)). In this case, no consumer finds it optimal to visit the direct channel if she does not find the product at the retailer. That is, there is no secondary demand in the direct channel.

### 4.2. Operational Decisions

#### 4.2.1. Retailer’s Problem

We characterize the retailer’s best response service level \( \alpha^*(t) \) to the manufacturer’s delivery lead time \( t \) at the direct channel. To do this, we first obtain the retailer’s order quantity, the availability level, and the expected sales for a given service level.

**Lemma 2.** For a given service level \( \alpha \),

(i) The retailer optimally orders \( q(\alpha) = a\alpha(1 - d_1(\alpha)) \) units of product:

(ii) The corresponding availability level is \( \phi(\alpha) = \alpha(1 - \ln(\alpha)) \):

(iii) The expected sales in the retailer are \( E[\min(D_r, q)] = a(1 - d_1(\alpha))(\alpha - \alpha^2/2) = q(1 - \alpha/2) \).

Part (i) provides the optimal order quantity for a given service level \( \alpha \). Part (ii) illustrates the one-to-one relationship between the retailer’s service level \( \alpha \) and the corresponding availability level \( \phi(\alpha) \). Recall that the service level is defined as the probability that
the retailer does not experience stockout during the sales season. This definition corresponds to the type-I service level in inventory management (Nahmias 2001). The availability level is defined as the probability that a particular consumer finds the product in stock. In our setting, this definition is equivalent to the fill rate (type-II) service level. The equivalence holds because the consumers are homogenous with respect to their chances of finding the product available. To illustrate the difference between these two measures, consider $\phi(\alpha) = 0.846$ for $\alpha = 0.6$. A service level of 0.6 means that the retailer satisfies all consumer demand in her store with probability 0.6. With probability 0.4, the retailer will experience stock-out during the sales season. From the perspective of a consumer, the probability of finding the product available in the store is 0.846.

The retailer’s expected profit as a function of the service level $\alpha$ is given by

$$\Pi_r(\alpha) = pE[\min(D_r, q)] - wq.$$ (3)

The retailer’s service-level decision $\alpha$ determines her stocking level $q$ and affects the demand $D_r$ in her store through the consumers’ channel choice process.

By demand at the retail channel turns out to be higher than the retailer’s stocking level, the retailer loses sales (i.e., there is no backordering), whereas if demand turns out to be less than the stocking level, the retailer ends up having excess inventory that has zero salvage value. Recall from Lemma 1 that $D_r$ is a function of the market segmentation. Substituting $q$ and $E[\min(D_r, q)]$ from Lemma 2,

$$\Pi_r(\alpha) = \alpha(1 - d_1(\alpha))(p - w - p\frac{\alpha}{2}).$$ (4)

Substituting $d_1$ from Equation (2), we write the retailer’s problem as

$$\max_{\alpha} \Pi_r(\alpha) = \frac{a\alpha (t + (v - p)(1 - \alpha(1 - \ln(\alpha))))}{t} \cdot (p - w - p\frac{\alpha}{2}),$$ (4)

subject to $\alpha \in [\alpha_r, [\alpha_{min}, 1]]$,

where $\alpha_{min}$ is defined in Equation (1) and $\alpha_r$ is defined such that $d_1(\alpha_r) = 1$. We introduce $\alpha_r$ instead of $\alpha = 0$ to avoid an undefined profit function due to the term $\ln(\alpha)$. By choosing $\alpha_r$, the retailer orders $q(\alpha_r) = 0$ and makes zero expected profit. The following proposition characterizes the retailer’s best response.

**Proposition 1.** The retailer’s expected profit function has a unique local maximizer in the domain $(0, \infty)$. Let this local maximizer be $\alpha_r(t)$, which is decreasing in the wholesale price $w$. The retailer’s best response is

$$\alpha^*(t) = \begin{cases} \alpha_{min}, & \text{for } \alpha_r(t) \leq \alpha_{min}, \\ \alpha_r(t), & \text{for } \alpha_r(t) \in (\alpha_{min}, 1), \\ 1, & \text{for } \alpha_r(t) \geq 1, \end{cases}$$

if $\Pi_r(\alpha^*) \geq 0$ holds. Otherwise, the retailer sets $\alpha^*(t) = \alpha_r$.

The best response service level decreases in the wholesale price. A high wholesale price $w$ may force the retailer to offer the minimum service level because ordering a high quantity of products would be costly. A very high wholesale price may cause the retailer’s maximum expected profit to be negative, in which case the retailer sets $\alpha^* = \alpha_r$ and does not order any products. We have the following corollary.

**Corollary 1.** If the manufacturer shuts down his direct channel by setting $t \to \infty$, then the retailer’s best response is to set $\lim_{t \to \infty} \alpha^*(t) = \max\{\alpha_{min}, (p - w)/p\}$.

In the absence of competition from the direct channel, the retailer need not consider the effect of her service level on demand determination. All consumers visit the retailer as long as she provides at least the minimum service level $\alpha_{min}$. Hence, the retailer optimally sets the critical fractile service level $(p - w)/p$ unless this level is below the minimum service level.

### 4.2.2. Manufacturer’s Problem

Here we characterize the manufacturer’s best response delivery lead time $t^*(\alpha)$ to the retailer’s service level $\alpha$ choice. Recall from Lemma 1 that sales in the direct channel is equal to the sum of the primary and the secondary demand. Hence, the manufacturer solves the following problem.

$$\max_t \Pi_m(t) = (w - c)q + (p - c)E[D_1 + D_2] - \frac{m}{T},$$ (5)

This function illustrates the manufacturer’s trade-off between the channels. On one hand, the direct
channel offers a higher profit margin \((p - c)\) than the margin in the retail channel \((w - c)\). On the other hand, the manufacturer is exposed to market uncertainty while selling through his direct channel, whereas his sales to the retailer are risk free because the retailer cannot return unsold products. The manufacturer considers this trade-off as well as the cost of the direct channel.

The following lemma characterizes the expected sales in the direct channel and the manufacturer’s expected profit function.

**Lemma 3.** The expected sales in the direct channel is \(E[D_1^2 + D_2^2] = (a/2)[(a(\alpha - 2)(d_2(\alpha) - d_1(\alpha)) + d_2(\alpha)] \). The manufacturer’s expected profit is a continuous function defined as

\[
\begin{align*}
\Pi^m_1(t) &= \frac{a}{2}(p - c) - \frac{m}{t^2}, & \text{for } t \leq t^*, \\
\Pi^m_2(t) &= \frac{a(p - c)(1 - \alpha)^2}{2} + \frac{1}{t}G^\alpha(t) - \frac{m}{t^2}, & \text{for } t \in (t^*, v - p], \\
\Pi^m_3(t) &= \frac{m}{t^2}, & \text{for } t > v - p,
\end{align*}
\]

where \(G^\alpha(t) = (a\alpha/2)[(v - p)(1 - \alpha(1 - \ln(\alpha))) + k] \cdot [(p - c)(2 - \alpha) - 2(w - c)] \) and \(G^\alpha(t) = [a(p - c)(1 - \alpha)^3 \cdot (v - p)/2 + (a\alpha/2)[(v - p)(1 - \alpha(1 - \ln(\alpha))) + k] \cdot [(p - c)(2 - \alpha) - 2(w - c)] \).

The three delivery lead time domains follow from Lemma 1. Recall that the direct channel covers the whole consumer population for \(t = v - p\). By setting the delivery lead time below \(v - p\), the manufacturer can increase the market share of the direct channel at the expense of the retail channel. At the extreme, the manufacturer can set \(t \leq t^*\) and eliminate the retailer. The Lemma below further characterizes the profit functions \(\Pi^m_1(t)\) and \(\Pi^m_2(t)\).

**Lemma 4.** (i) The function \(\Pi^m_1(t)\) is increasing in \(t\) when \(G^\alpha(\alpha) \leq 0\). It is unimodal with a maximum at \(t^* = 2m/G^\alpha(\alpha)\) when \(G^\alpha(\alpha) > 0\).

(ii) The function \(\Pi^m_2(t)\) is increasing in \(t\) when \(G^\alpha(\alpha) \leq 0\). It is unimodal with a maximum at \(t^* = 2m/G^\alpha(\alpha)\) when \(G^\alpha(\alpha) > 0\).

(iii) For \(\alpha = 1\), we have \(\Pi^m_2(t) = \Pi^m_3(t)\).

(iv) For \(\alpha < 1\), \(\Pi^m_2(t) = \Pi^m_3(t)\) only for \(t = v - p\). We have \(\Pi^m_1(t) > \Pi^m_2(t)\) for \(t < v - p\), and \(\Pi^m_1(t) > \Pi^m_3(t)\) for \(t > v - p\).

Lemmas 3 and 4 help characterize the manufacturer’s best response function, as summarized in the following proposition.

**Proposition 2.** Given the wholesale price \(w\), the manufacturer’s best response to the retailer’s service level \(\alpha\) choice is

\[
t^* = \frac{2m}{G^\alpha(\alpha)}, \quad \text{if } G^\alpha(\alpha) > 0 \text{ and } t^* \leq t^*,
\]

\[
t^*_j = \frac{2m}{G^\alpha(\alpha)}, \quad \text{if } G^\alpha(\alpha) > 0 \text{ and } t^*_j \in (t^*, v - p],
\]

\[
t^*_j = \frac{2m}{G^\alpha(\alpha)}, \quad \text{if } G^\alpha(\alpha) > 0 \text{ and } t^*_j > v - p \text{ or } G^\alpha(\alpha) \leq 0,
\]

\[
t^*_j = 0, \quad \text{if } G^\alpha(\alpha) \leq 0.
\]

The manufacturer’s best response consists of five types of delivery lead times. Recall from Lemma 1 that when the delivery lead time is less than or equal to \(t^*\), the retailer is essentially inoperative. In other words, the manufacturer may choose to eliminate the retailer by setting a very short delivery lead time and serve the entire consumer population through his direct channel. However, this action would be costly. The other extreme is when the manufacturer shuts down his direct channel by setting an arbitrarily long delivery lead time \(t^* \rightarrow \infty\). In this case, only the retailer is operative. Part of the consumer demand would be satisfied depending on the retailer’s service level. The other three types of delivery lead time result in interior solutions. In particular, when \(t^* = t^*_j\) or when \(t^* = v - p\), both the retail channel and the direct channel are operative. In this case, the manufacturer sets the delivery lead time such that all consumers consider the direct channel as an option. When \(t^* = t^*_j\) the manufacturer acts less aggressively and satisfies part of the consumer population through his direct channel, and some consumers are lost.
4.3. Manufacturer’s Optimal Wholesale Price
To find the manufacturer’s optimal wholesale price, we perform a grid search over the wholesale price values $w \in [c, p]$. We choose the wholesale price for which the Nash equilibrium we find yields the highest expected profit for the manufacturer. In this search, we do not explicitly consider the voluntary participation constraint of the retailer because the retailer can always make zero profit by setting service level $\alpha = \alpha_0$. The algorithm, provided in Appendix B, is an application of the best response dynamics methodology (Matsui 1992). The algorithm searches for pure-strategy Nash equilibria by iteratively finding each agent’s best response to the current action of the other agent, until a joint strategy is reached from which neither of the agents has an incentive to deviate. We ignore mixed strategy Nash equilibria. In general, computing the Nash equilibria of a game is a hard problem. Several algorithms, such as the Lemke-Howson algorithm (Lemke and Howson 1964) for two-player games, have been developed to address this problem. We run the algorithm starting with 10 different $\alpha$ seed values spanning the (0,1] domain. In our numerical experiments, reported in the next section, we did not encounter multiple equilibria.

5. Dual Channel Results
We present the results in three parts. First, we show that there are three types of equilibria that depend on the parameters describing the environment. Each equilibrium corresponds to an optimal dual channel strategy for the manufacturer. Second, we illustrate how the manufacturer’s optimal dual channel strategy changes with respect to the direct channel cost $m$ and the retailer inconvenience cost $k$. Third, we illustrate how the retailer’s and the manufacturer’s decisions and the resulting profits change with respect to the changes in the retailer inconvenience cost $k$.

5.1. Partition into Three Equilibrium Regions
To cover the parameter space, we choose low, medium, and high values for each parameter. Thus, we solve the model for $3^5 = 243$ parameter combinations that correspond to different dual channel environments. We report a representative subset of our numerical results. The complete set is deferred to an addendum available from the authors. Table 1 illustrates the manufacturer’s optimal wholesale price $w^*$, the resulting equilibrium decisions $t^*$ and $\alpha^*$, the sales based on consumers’ optimal channel choice, and the expected profits for sample parameter combinations. We identify three types of equilibria as outlined next.

- **Eliminate retailer (ER):** In this equilibrium, $d_1 = d_2 = 1$. The manufacturer optimally sells only through

---

**Table 1: Sample Results**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Decision variables</th>
<th>Profits</th>
<th>Sales</th>
<th>Equilibrium type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$v$</td>
<td>$p$</td>
<td>$k$</td>
<td>$c$</td>
</tr>
<tr>
<td>1,000</td>
<td>4</td>
<td>1</td>
<td>0.38</td>
<td>0.00</td>
</tr>
<tr>
<td>1,000</td>
<td>12</td>
<td>3</td>
<td>4.50</td>
<td>0.75</td>
</tr>
<tr>
<td>5,000</td>
<td>12</td>
<td>6</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>5,000</td>
<td>4</td>
<td>3</td>
<td>0.75</td>
<td>1.50</td>
</tr>
<tr>
<td>5,000</td>
<td>8</td>
<td>6</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>10,000</td>
<td>4</td>
<td>1</td>
<td>0.38</td>
<td>0.50</td>
</tr>
<tr>
<td>5,000</td>
<td>8</td>
<td>6</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10,000</td>
<td>4</td>
<td>3</td>
<td>0.50</td>
<td>0.75</td>
</tr>
</tbody>
</table>

$1^d_1(\alpha^*) \equiv 1$; hence $q^* = 0$. 

---

3 Best response dynamics is related to the fictitious play process introduced by Brown (1951). Best response dynamics is effective in finding a Nash equilibrium when the complexity of the best response functions makes other algorithms difficult to use (Sureka and Wurman 2005), as is the case with our model.

4 We use $u, v, p/v, k/(v-p)$, and $c/p$ to cover the parameter space because the values of $p$, $k$, and $c$ are constrained by the conditions $p < v, k \leq v - p$, and $c < p$. We keep the maximum market size parameter fixed at $a = 1,000$, without loss of generality.
the direct channel. He offers a high wholesale price \( w \) relative to the sales price \( p \) and a short delivery lead time \( t \). The retailer opts out of the market voluntarily by stocking zero units (or equivalently sets a service level such that \( d_1(a) = 1 \)). The manufacturer serves all consumer types, i.e., with any index \( d \). The consumer population is not segmented.

- **Capture all profit (CP):** In this equilibrium, \( d_1 = d_2 < 1 \). The manufacturer uses both channels, yet he captures all profits from the retailer. The manufacturer sets the wholesale price such that the retailer barely makes a positive profit. The small positive profit for the retailer in Table 1 is due to the coarseness of the grid search. The retailer sets the minimum service level \( \alpha_{\text{min}} \). Consumers are segmented into two groups. The first group with time-sensitivity index \( d \leq d_1 \) buys only from the direct channel. The second group buys only from the retail channel if the product is available. Otherwise, these consumers leave the system without buying the product. Hence, the manufacturer decides not to serve all consumer types.

- **Share profit (SP):** In this equilibrium, \( d_1 < d_2 < 1 \). The manufacturer uses both channels and shares the profit with the retailer. The manufacturer optimally charges a low wholesale price \( w \) relative to the sales price \( p \); hence, the retailer’s expected profit is positive. The consumers are segmented into three groups. The first group with \( d \leq d_1 \) buys only from the direct channel. The second group with \( d \in (d_1, d_2] \) chooses to visit the retailer and buys from the direct channel if they cannot find the product at the retailer. The third group with \( d > d_2 \) buys only from the retailer and leaves the system without buying if the product is not available at the retailer.

We note that each equilibrium has three important aspects: channel configuration, market segmentation strategy, and profit-sharing strategy. Note also that given a channel environment represented by different regions in the parametric space, the manufacturer can induce the type of the equilibrium with his wholesale price decision at the contracting stage. Hence, each equilibrium corresponds to an optimal dual channel strategy for the manufacturer.

### 5.2. Manufacturer’s Optimal Channel Strategy

Here we illustrate the manufacturer’s optimal channel strategy in the direct channel cost \( m \) and the retailer inconvenience cost \( k \) space. Table 2 illustrates the optimal strategy we find for different \( m \) and \( k \) values.

Note from Table 2 that the \( m/k \) plane is partitioned into three strategy regions. We obtain similar partitionings with other unit production cost \( c \), selling price \( p \), and consumer valuation \( v \) values as well. Figure 3 summarizes our results.

When the direct channel cost and the retailer inconvenience cost are both high, the analysis shows that capture-all-profit (CP) is an optimal channel strategy. The manufacturer sells through both channels by separating the consumer population into two segments. The first segment buys only from the direct channel, and the second segment buys only from the retailer. Yet the manufacturer captures all profit from the retailer by setting a high wholesale price. Intuitively, the manufacturer can capture the profit because he knows that a high inconvenience cost implies a high minimum service level \( \alpha_{\text{min}} \) at the retailer. Recall that the retailer has to set a minimum service level to
stay in business. The manufacturer, knowing this fact, can squeeze the retailer’s profit by setting a high wholesale price because the retailer cannot decrease her stocking level below this minimum service level. Note, however, that this strategy would not be effective when the retailer inconvenience cost \( k \) is low because her minimum service level will be low as well. Bulky products, such as plasma TVs, are candidates for the application of this strategy, having high shipping and handling costs (high \( m \)). Such products may also be inconvenient for consumers to carry home (high \( k \)).

When the direct channel cost is high and the retailer inconvenience cost is low, the model shows that share-profit (SP) is an optimal channel strategy. The manufacturer should optimally sell through both channels by segmenting the market into three and then share the profit with the retailer. In this case, the manufacturer is at a comparative cost disadvantage. Hence, increasing the wholesale price to capture the retail channel profit would not be as profitable. High value (high \( m \)) caused by inventory costs and small-sized products (low \( k \)), such as digital cameras, are candidates for the application of this strategy.\(^5\)

When the direct channel cost is low, eliminate-retailer (ER) is an optimal channel strategy. In this case, the direct channel is so efficient that the manufacturer can cover the whole market without the retail channel. Small products with low direct channel management costs are candidates for this strategy. A candidate product is notebook PCs. These products have a highly modular product architecture, which reduces the cost of maintaining a direct channel (low \( m \)).

Figure 3 also illustrates how the other model parameters affect the boundaries between these channel strategy regions. In particular, we observe that decreasing the unit production cost \( c \) and the selling price \( p \) values cause the eliminate-retailer region to grow. Hence, as a product matures, the manufacturer is more likely to sell direct only. This is because one expects the direct channel cost, the unit production cost, and the selling price to decrease over the lifecycle of a product.

\(^5\) Recently, Hewlett-Packard expanded its retail presence by adding Best Buy as an outlet for its digital cameras, in addition to a strong direct channel presence.

5.3. Effects on Decision Variables and Resulting Profits

Here we illustrate how the decision variables \( \{w, \alpha, t\} \) change as the manufacturer moves from one dual-channel strategy to the next. Figure 4 exhibits the decision variables as a function of \( k \) for a sample parameter set.

With low retailer inconvenience cost, the retailer is a significant competitor to the manufacturer’s direct channel. Hence, the manufacturer sets a short delivery lead time and uses only the direct channel. For moderate retailer inconvenience cost, the retailer responds to the increasing retailer inconvenience by increasing her service level to attract consumers. For high retailer inconvenience cost, the retailer’s minimum service level constraint is binding. In this case, the retailer chooses the minimum service level \( \alpha_{\text{min}} \). The manufacturer, taking advantage of this fact, sells increasingly through the retailer and captures all profit with a wholesale price that leaves zero profit to the retailer. The manufacturer also begins to reduce his direct channel presence, as illustrated by the increasing delivery lead time. Note from Figure 4 that as the retailer inconvenience cost increases, the manufacturer supports the retailer by reducing the wholesale price; but the switch to the capture-all-profit strategy causes an upward jump in the wholesale price.

For the same parameter set, Figure 5 illustrates the expected profits and the expected sales in each channel. The manufacturer’s expected profit is unchanged with respect to \( k \) for the eliminate-retailer strategy (when \( k \) is low) and it is slightly increasing for the share-profit strategy. The manufacturer’s profit
When the manufacturer switches to the capture-all-profit strategy, he diverts business from the direct channel to the retailer by increasing the delivery lead time. The increased sales in the retail channel, however, do not benefit the retailer because the manufacturer captures all profit.

6. Experimental Study

We conducted a sequence of laboratory-based, human subject experiments to test whether the model makes reasonable predictions of human decisions in the environment it describes. Field tests with an actual retailer would be a more effective way of studying their behavior. However, trying different contracts with an actual retailer is costly, lengthy, and risky; hence, it may be impractical. The two main goals for these experiments are (1) to test whether subjects play the Nash equilibrium and (2) to determine if learning plays an important role in this process.

6.1. Experimental Design and Procedures

In the experiments, we focused on the second stage of our dual channel model, specifically the operational decisions, given an exogenous wholesale price. We recruited 18 subjects from the Stanford student body for the experiments. To participate, subjects had to read instructions for the experiments and pass a Web-based quiz. We conducted two sessions in the Hewlett-Packard Experimental Economics Laboratory in Palo Alto, California. Each session was divided into three experiments that had different parameter sets corresponding to the three types of equilibria identified by the analytical model. For each experiment we ran 25 periods or games, as summarized in Table 3. This experimental design allowed us to determine whether the subjects responded to the changes in the environment in a manner consistent with the model. It also allowed us to observe whether the model is quantitatively accurate in predicting human decisions.

The subjects were informed before each new parameter set took effect. In each period, each subject was randomly matched with another subject. One

\[ \text{Note. } m = 10,000, \nu = 8, \rho = 4, \sigma = 1. \]
of them, again chosen randomly, was assigned to be the manufacturer and the other the retailer. The subjects did not know who they were playing against. Thus, repeated game effects were unlikely, and we treated each game in each period as an independent observation. In each game, the manufacturer chose a delivery lead time and the retailer chose a stocking level. After the decisions were made, the computer-generated market size $X$ is realized, the demand in each channel and the respective manufacturer and retailer’s profits are calculated. The results are displayed to subjects at the end of each period.

Before conducting the actual games, we provided training. An experimenter explained the details of the computer interface before the training periods started. We also provided a decision support tool to help the subjects make their decisions. During each period, a subject could run trial decisions and guess the other player’s decision. The computer displayed, in a table format, sales and payoff for 11 possible realizations of the total market demand $X$. We provide a sample screenshot of the retailer’s screen in Appendix C. During the actual periods, the subjects were given 45 seconds to make a decision. At the end of each session, the subjects were paid according to their performance in the game, measured by their profit level.

To check for order effects, we used a different sequence of experiments in each session. In Session 1, we conducted the experiments in the order 1a, 1b, 1c. In Session 2, we changed the sequence of the experiments and conducted Experiment 2b first, followed by 2c and 2a. We did not find any evidence for order effects.

Note: $m = 10,000$, $v = 8$, $p = 4$, $c = 1$.

---

7 For a given delivery lead time $t$, there is a one-to-one correspondence between the service level $a$ and the stocking level $q$; hence, these decisions are equivalent. In the experiments, the retailer subjects were asked to make a stocking level decision, which is relatively more intuitive than the service level decision. We also conducted a numerical experiment, similar to the one reported in §5. We observe similar results and did not encounter any new equilibria type. The results are available from the authors.

8 Order effects refer to the possibility that the subjects’ experience in an experiment might bias their decisions in the following experiments (Camerer 2003).
6.2. Experimental Study Results

We draw three conclusions from the experimental data.

First, subjects responded differently to the different parameter settings in the three experiments in each session. In other words, decisions in one experiment were significantly different from decisions in a different experiment. Figures 6(a) and (b) illustrate this result for Session 1. Each triangle or circle represents the outcome of one game, played by a manufacturer-retailer pair. The corresponding equilibria are shown as squares. The separation observations are supported by two-dimensional Kolmogorov-Smirnov tests. We use this two-dimensional test because the manufacturer’s and the retailer’s decisions are not independent. In all our statistical analysis, we used a significance level of 0.05. The results verify that decisions (as a pair) in different experiments came from different two-dimensional distributions (all p values are less than $10^{-6}$). This finding is strong evidence that the differences in the underlying economics of the three experiments drove significantly different behavior. In addition, the directional changes in the experiment results are consistent with the model’s predictions. For example, the model predicts that the stocking levels in Experiment 1a should be higher than those in Experiment 1b, which should be higher than those in Experiment 1c. On average, the human subjects’ decisions also reflect this separation. These findings indicate that the structural predictions of the model are robust with respect to behavioral issues. Thus, qualitative recommendations from the model are likely to be applicable in actual business settings.

Second, the data show deviations from the model’s numerical equilibrium predictions. Table 4 compares the equilibrium predictions and the means of the observed data. The table also presents the $p$ values of the Wilcoxon signed rank test results. This test is used to measure the statistical significance of the deviations (Wilcoxon 1945). In particular, the manufacturers set substantially longer delivery lead times than the model’s prediction. In all experiments, the means of observed delivery lead time were higher than the theoretical equilibrium values. In Experiments 2a and 2b

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Experimental Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session</td>
<td>No. Subjects</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: All experiments had $v = 20$, $p = 10$, $a = 1,000$, $c = 0$. 

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Comparing the Observed Data and Predicted Equilibrium Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delivery lead time $t$</td>
<td>Stocking level $q$</td>
</tr>
<tr>
<td>Exp</td>
<td>Equilibrium</td>
</tr>
<tr>
<td>1a</td>
<td>13.95</td>
</tr>
<tr>
<td>1b</td>
<td>23.55</td>
</tr>
<tr>
<td>1c</td>
<td>10.00</td>
</tr>
<tr>
<td>2a</td>
<td>15.67</td>
</tr>
<tr>
<td>2b</td>
<td>34.32</td>
</tr>
<tr>
<td>2c</td>
<td>4.00</td>
</tr>
</tbody>
</table>
(as indicated in the $p$ values in Table 4), the differences were not significant at the 0.05 level. In the other four experiments, the differences were significant. One possible explanation for this behavior is that human subjects are loss and/or risk averse. In the model’s environment, the cost of short delivery lead times for the manufacturer is deterministic, whereas the benefit is uncertain. Another explanation is that even risk-neutral manufacturers use longer delivery lead times if they believe the retailer competes less aggressively. Unlike the manufacturer, the retailer’s behavior appears to depend on the equilibrium type. In particular, the retailers significantly understocked in the experiments with share-profit type equilibria (1a and 2a). In the eliminate-retailer type equilibria (1c and 2c), understocking was not possible because the theoretical prediction is zero. In the capture-all-profit type equilibria, the observed mean stocking level for Experiment 1b does not show significant difference from the theory. In contrast, Experiment 2b shows significant difference and the observed stocking level is 4.5% higher than the theoretical equilibrium.

Third, we observe dispersion with respect to behavior; see for example, Figures 6(a) and (b). There are two possible explanations. First, dispersion may be caused by an inherent heterogeneity in behavior (such as different aptitudes). Second, the subjects may be searching the strategy space for better strategies. If so, learning and experience should reduce dispersion. However, we determine that there is no significant learning effect with respect to behavioral dispersion. As summarized in Table 5, the two-dimensional Kolmogorov-Smirnov tests cannot reject the hypothesis that the decisions in the first and the second halves (earlier and latter periods) of the experiments come from the same distribution in three of six experiments at the 0.05 significance level. We also measure dispersion\(^7\) using the multivariate standard deviation (normalized by mean values). As reported in Table 5, we did not observe a significant reduction in dispersion from the first half to the second half of the experiments for all sessions. This finding supports that learning to play a single pure strategy does not seem to be a significant aspect of the behavioral process.

### 7. Conclusion

Today’s consumers know what they want, and they often want it immediately. Although the manufacturer’s direct online channel offers certain advantages, the inherent delivery lead time makes it unattractive to some consumers. Marketing research shows that an important reason why consumers shop online yet buy at the retail store is that they want the product immediately. Visiting a retail store, however, does not guarantee instant ownership, because of stockouts. In this paper, we present a strategic analysis of manufacturer-retailer interaction in a dual channel setting that integrates a consumer channel choice model with the manufacturer’s and the retailer’s operational decisions. We also evaluate the validity of our strategy recommendations through controlled experiments with human subjects.

In the model, the strategic interaction between the manufacturer and the retailer is driven by a consumer channel choice model that considers consumers’ willingness to wait and product availability concerns, as well as the relative convenience of shopping from the online and retail channels. The analysis illustrates how the manufacturer can use the dual channel structure to his advantage. First, the manufacturer can service discriminate the consumer population, which is heterogeneous with respect to the willingness to wait. The manufacturer sells his product to time-sensitive consumers through the retail channel and to less-time-sensitive consumers through his direct channel. Second, the manufacturer can balance his profit with the

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Kolmogorov-Smirnov (K-S) Test $p$ Values and Dispersion Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>$p$ value</td>
</tr>
<tr>
<td></td>
<td>1st half</td>
</tr>
<tr>
<td>1a</td>
<td>0.02</td>
</tr>
<tr>
<td>1b</td>
<td>0.04</td>
</tr>
<tr>
<td>1c</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\(^7\)The dispersion measure is calculated as

$$
\sqrt{\frac{\sum_{i=1}^{n} ((t_i - \overline{t})/\overline{t})^2 + \sum_{i=1}^{n} ((q_i - \overline{q})/\overline{q})^2}{n-1}},
$$

where $(t_i, q_i)$ represents a data point, $n$ is the number of data points, and $\overline{t}$ and $\overline{q}$ are the means of the delivery lead time and stocking quantity decisions, respectively. Note that we divide the deviations by their respective mean values for normalization, so that stocking quantities $q_i$, which are considerably larger, do not dominate the delivery lead times $t_i$.\n
---

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risk he is exposed to from uncertain demand. The retail channel offers a lower profit margin than the direct channel but allows the manufacturer to push inventory risk to the retailer. The direct channel, in contrast, offers a higher profit margin but exposes the manufacturer to risk from uncertain demand. These trade-offs, together with the cost of the direct channel, are the fundamental drivers behind the manufacturer’s dual channel strategy. The model enables us to quantify these trade-offs.

We determine three types of optimal channel strategies for the manufacturer, depending on the channel environment. (1) When the direct channel cost is low, the optimal strategy for the manufacturer is to capture the entire market with his direct online channel by setting a short delivery lead time. This strategy induces the retailer to opt out by not stocking the product. (2) When the direct channel cost is above a certain threshold, and if the retailer inconvenience cost is high, the optimal strategy for the manufacturer is to sell through both channels and capture retail channel profits by setting a high wholesale price. (3) When the direct channel cost is high and the retailer inconvenience cost is low, the optimal strategy for the manufacturer is to sell through both channels and share the profits with the retailer.

To verify whether the model provides reasonable predictions when actual human decision makers are involved, we conducted a sequence of behavioral experiments, with human subjects playing the roles of the manufacturer and the retailer. We determine that the model provides valid directional predictions with respect to parameter changes when human behavior is taken into account. Hence, the analytical results can be used to improve the wholesale price contract and the operational decisions in an actual dual channel environment. In addition, as indicated by these experiments, the model is useful in comparing alternative strategies and wholesale price contracts because of its robustness in predicting the direction of changes regarding behavioral effects. However, human subjects have bias compared to, and dispersion from, the model’s quantitative predictions. These observations suggest that one should perhaps be cautious in using the model to identify the exact quantities or values corresponding to each decision. One interesting research direction is to study how the behavioral factors affect the optimality of the wholesale price. In the absence of an accurate behavioral model, we can experimentally determine whether the true optimal wholesale price is higher or lower than the one suggested by the pure rational model. We are currently running more laboratory experiments and extending the research to capture the impact of behavioral issues on strategic considerations that manufacturers face in complex environments.

7.1. Should the Manufacturer Establish the Second Sales Channel?

To answer this question, we first study the scenario in which the manufacturer sells only through the retail channel. The analysis is deferred to an online addendum. We compare the results with the results of the present paper’s dual channel scenario. We determine that the introduction of the direct channel always increases the manufacturer’s profit, and we quantify this increase. Using this analysis, the manufacturer can decide whether to establish a direct channel by comparing the related benefits and costs. We also determine that establishing a direct channel reduces the retailer’s profit but increases the total profit. This finding suggests the possibility of a mutually beneficial outcome for the manufacturer and the retailer. Comparing the service levels, we determine that the retailer provides a lower service level (stocking quantity) in the dual channel setting than in the retail-channel-only setting. Hence, the manufacturer cannot use the direct channel as a strategic tool to induce a higher service level from the retailer. Finally, we also consider a scenario in which the manufacturer sells only through his direct channel. We determine that the manufacturer’s profit is higher in the dual channel scenario than in the direct-channel-only scenario. This result signals the importance of the retail channel. In addition, we determine that the manufacturer offers a longer delivery lead time (lower service) in the direct channel in the absence of a competing retail channel.


7.2. Other Channel Structures and Competition
An interesting research direction is to study manufacturer competition. This scenario would address comparative shopping behavior of consumers in addition to their channel choice. In the case of the system with one retailer and multiple manufacturers, the market power of manufacturers would decrease compared to our current model. Thus, one would expect that the retailer would enjoy a larger share of the profits and that a manufacturer would be less likely to engage in the eliminate-retailer or capture-all-profit strategies.

Another research direction is to study a scenario in which the retailer also operates an online channel in addition to a bricks-and-mortar store. The advantages of the direct channel over the retailer are not clear. On one hand, the retailer may face a higher direct channel cost than the manufacturer because she does not have access to the upstream supply chain. On the other hand, the retailer can offer to deliver the product through her online channel if a consumer faces a stockout in the store. The consumer may accept this offer if her utility from the retailer’s online channel is greater than the utility from the manufacturer’s online channel. Hence, the online channel would allow the retailer to capture part of the secondary demand in the manufacturer’s direct online channel.

7.3. Other Contracts
Our modeling framework can also be used to study the impact of other contract forms, such as rebates, revenue sharing, and buyback contracts. We are currently investigating different types of rebate contracts. An interesting question is to whom the manufacturer should offer the rebate. The manufacturer can motivate the retailer to sell more by offering her a rebate per product sold. The rebate to the retailer does not directly alter the consumer preferences. Hence, the expressions to find $d_1$, $d_2$, and $q$ remain unchanged. The retailer’s optimal service level $α^*$, however, will be a function of the rebate amount. The manufacturer can also offer rebates directly to consumers, rather than to the retailer. The rebate affects the consumer choice process as the consumer’s net value from buying the product becomes $v - p + r$. Alternatively, the manufacturer can offer rebates only to direct channel customers. The manufacturer uses the rebate to indirectly undercut the retailer’s selling price. The utility of consumer $d$ from the direct channel becomes $u_d(d) = v - p + r - dt$, and the expression to find $E[u_1]$ remains unchanged.

Additional research is needed to substantiate the impact of other forms of competition, contract forms, and complex dual channel environments faced in practice. Hence, the study of dual channel management is an exciting and fertile research area.

Acknowledgments
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Appendix A. Notation

Exogenous Constants
- $v$: Product’s value to consumers
- $p$: Selling price at both channels
- $a$: Maximum market size for the product
- $k$: Retailer inconvenience cost
- $c$: Unit production cost
- $m$: Direct channel cost parameter

Decision Variables
- $α \in [0, 1]$: Retailer’s service level
- $ϕ(α) \in [0, 1]$: Availability level
- $q(α)$: Stocking level
- $t$: Direct channel’s delivery lead time
- $w$: Wholesale price

Others
- $d \in [0, 1]$: Consumer time-sensitivity index
- $D_1^1$: Primary demand in the direct channel
- $D_2^1$: Secondary demand in the direct channel
- $D_1$: Demand in the retail channel
- $X$: Market size $\sim UNIF[0, a]$

Appendix B. Algorithm
Set $δ = 0.01, ε = 10^{-6}$, $Π_{0\beta} = (small\ number)$
For $w = c \text{ to } w = p$ Do
(Find the Nash equilibrium of the operational decisions game for a given $w$.)
For $i = 1 \text{ to } i = \text{number of initial seeds}$ Do
Set $j = 0$ and $α_j^* = (seed \ i)$
\[ α_{j+1} = l_{j+1}^* = t_j^* = (large\ number) \]
While $(α_j^* - α_j^* > ε \text{ and } t_{j+1}^* - t_j^* > ε)$ Do
\[ l_{j+1}^*(α_j^*) \leftarrow (find\ the\ manufacturer’s\ best\ response\ to\ α_j^*) \]
When $j = \text{number of initial seeds}$ Exit Do

Appendix C. Screenshot from the Experiment Software

Figure A.1 Sample Screenshot from the Retailer's Screen

<table>
<thead>
<tr>
<th>Game</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Role</td>
<td>Retailer</td>
</tr>
<tr>
<td>Stage</td>
<td>Stock Level Decision</td>
</tr>
<tr>
<td>Value of Product</td>
<td>20</td>
</tr>
<tr>
<td>Retail Price of Product</td>
<td>10</td>
</tr>
<tr>
<td>Search cost at the Retailer</td>
<td>6</td>
</tr>
<tr>
<td>Wholesale Price / Unit</td>
<td>5</td>
</tr>
</tbody>
</table>

#### Decision Support Tool

<table>
<thead>
<tr>
<th>If manufacturer’s shipping time is</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>and my stock level is</td>
<td>520</td>
</tr>
</tbody>
</table>

#### If the total demand (max possible 1000) turns out to be

<table>
<thead>
<tr>
<th>Units Sold</th>
<th>Direct</th>
<th>Units Sold</th>
<th>Inventory Unsold</th>
<th>Customers Lost</th>
<th>Total Customers Lost</th>
<th>Manufacturer's Profit</th>
<th>My Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2091.6</td>
<td>-100.9</td>
</tr>
<tr>
<td>100</td>
<td>41</td>
<td>59</td>
<td>441</td>
<td>6</td>
<td>9</td>
<td>2461.6</td>
<td>-1910.9</td>
</tr>
<tr>
<td>200</td>
<td>82</td>
<td>118</td>
<td>262</td>
<td>6</td>
<td>9</td>
<td>2875.6</td>
<td>-1320.9</td>
</tr>
<tr>
<td>300</td>
<td>123</td>
<td>177</td>
<td>225</td>
<td>6</td>
<td>9</td>
<td>3283.0</td>
<td>-750.0</td>
</tr>
<tr>
<td>400</td>
<td>163</td>
<td>237</td>
<td>263</td>
<td>6</td>
<td>9</td>
<td>3691.6</td>
<td>-120.0</td>
</tr>
<tr>
<td>500</td>
<td>204</td>
<td>296</td>
<td>204</td>
<td>6</td>
<td>9</td>
<td>4093.6</td>
<td>460.0</td>
</tr>
<tr>
<td>600</td>
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<td>145</td>
<td>6</td>
<td>9</td>
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<tr>
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<td>27</td>
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<td>9</td>
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<td>2239.6</td>
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<tr>
<td>900</td>
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<td>0</td>
<td>32</td>
<td>18</td>
<td>5837.6</td>
<td>2509.6</td>
</tr>
<tr>
<td>1000</td>
<td>448</td>
<td>590</td>
<td>0</td>
<td>92</td>
<td>52</td>
<td>6353.6</td>
<td>2839.6</td>
</tr>
<tr>
<td>2000</td>
<td>192</td>
<td>132</td>
<td>347</td>
<td>6</td>
<td>9</td>
<td>3107.0</td>
<td>-970.0</td>
</tr>
</tbody>
</table>

Your decision

| Units to stock | 0 |

\[
\alpha_j^* (t_{j+1}^*) \leftarrow (\text{find the retailer's best response to } t_{j+1}^*)
\]

End While

Report the Nash equilibrium as the pair \((\alpha_j^*(i), t_j^*(i))\)

End For \(i\) loop

Check whether there are multiple equilibria

\[ t^* \leftarrow t_j^*(1) \text{ and } \alpha^* \leftarrow \alpha_j^*(1) \]

If \(\Pi_m^a \leq \Pi_m^a (t^*)\) is defined in Equation (5),
then \(\Pi_m^a \leftarrow \Pi_m^a (t^*)\) and \(w^* \leftarrow w\)

\(w \leftarrow w + \delta\)

End For \(w\) loop

Report \(w^*, \Pi_m^a\) and the corresponding \((t^*, \alpha^*)\).

Appendix D. Proofs

**Proof of Lemma 1.** The time-sensitivity indices of consumers are uniformly distributed and the consumers' arrival to the system is independent of their time-sensitivity level. Hence, the results follow from the market segmentation discussed in §4.1 before and after the statement of the lemma.

**Proof of Lemma 2.** To prove Part (i), note from Lemma 1 that \(D_r\) is defined as \((1-d_1)X\). Hence, it is uniformly distributed over the interval \([0, a(1-d_1)]\). To offer service level \(\alpha\), the retailer has to order \(q\) units to satisfy \(\Pr(D_r \leq q) = \alpha\). This implies \(q = a \alpha (1-d_1)\).

To prove Part (ii) recall that \(\phi(\alpha)\) is the probability that a consumer finds the product in the retailer, \(P(\text{find})\) to be short. We have

\[
P(\text{find}) = E[P(\text{find} | D_r)] = \int_{z=0}^{\phi(1-d_1)} \frac{1}{a(1-d_1(\alpha))} dz + \int_{z=q}^{\phi(1-d_1)} \frac{1}{z} \frac{1}{a(1-d_1(\alpha))} dz = a(1-\ln(\alpha)).
\]

The last equality is obtained by substituting \(q = a \alpha (1-d_1(\alpha))\) from Part (i).

To prove Part (iii), we have

\[
E[\min[D_r, q]] = \int_{\alpha}^{a(1-d_1(\alpha))} \frac{1}{a(1-d_1(\alpha))} dz + \int_{a(1-d_1(\alpha))}^{a(1-d_1(\alpha))} a \alpha (1-d_1(\alpha)) \frac{1}{a(1-d_1(\alpha))} dz = a(1-d_1(\alpha))(\alpha - \alpha^2/2).
\]

Substituting \(q = a \alpha (1-d_1(\alpha))\), we have \(E[\min[D_r, q]] = q(1-\alpha/2)\). □
Proof of Proposition 1. The proof proceeds in four steps. First, we characterize the $\alpha$ values at which $\Pi_i(\alpha)$ crosses zero. Second, we show that $\Pi_i(\alpha)$ has a unique local maximizer that we refer to as $\alpha_i$. Third, we show that the best response $\alpha^*$ is equal to either $\alpha_i$ or one of the boundary values $\alpha_{\text{min}}$ and 1. Fourth, we show that $\alpha^*$ is decreasing in the wholesale price $w$.

From Equation (3), we observe that $\Pi_i(\alpha)$ crosses zero at $\alpha_1 = 2(p-w)/p < 2$ and at $\alpha$ values such that $d_i(\alpha) = 1$ (note also that we have $\lim_{\alpha \to 0^+} \Pi_i(\alpha) = 0$). Next, we characterize these $\alpha$ values. Substituting $d_i$ from Equation (2), $d_i(\alpha) = 1$ becomes $\alpha(1 - \ln(\alpha)) = (v - p - t + k)/(v - p)$. Let $Z(\alpha) = \alpha(1 - \ln(\alpha))$. We have $\lim_{\alpha \to 0^+} Z(\alpha) = 0$, $\frac{\partial Z(\alpha)}{\partial \alpha} > 0$ for $\alpha \in (0, 1)$, $\frac{\partial Z(\alpha)}{\partial \alpha} = 0$ for $\alpha = 1$, and $\frac{\partial Z(\alpha)}{\partial \alpha} < 0$ for $\alpha \in (1, 1)$. Note that $Z(\alpha) = 0$ when $\alpha = e$. Hence, for $S < 0$, the equation $z(\alpha) = S$ is satisfied by a unique $\alpha$ value $\alpha_2 > e \geq 2.71$. For $S \geq 0$ and $S \neq 1$, the equation $z(\alpha) = S$ is satisfied by two distinct $\alpha$ values: $\alpha_3 < \alpha_4$ and $\alpha_4 \in (1, e)$. For $s = 1$, the equation $z(\alpha) = S$ is satisfied only by $\alpha = 1$.

This observation implies that for $v - p - t + k < 0$, the equation $d_i(\alpha) = 1$ is satisfied by $\alpha_2 > e \geq 2.71$ and for $v - p - t + k \geq 0$ and $t \neq k$, the equation $d_i(\alpha) = 1$ is satisfied by $\alpha_3 < \alpha_4$ and $\alpha_4 \in (1, e)$. We do not consider the case with $t = k$, because the retailer is eliminated with $t = k$.

Next we show that $\Pi_i(\alpha)$ has a unique local maximizer, $\alpha_i$. From Equation (4), we observe $\lim_{\alpha \to 0^+} \Pi_i(\alpha) = \infty$. We also observe $\frac{\partial \Pi_i(\alpha)}{\partial \alpha} = (a/t)[(p - w - ap)[1 - \alpha[(v - p)] - (v - p) \alpha \ln(\alpha)(2(p - w) - 3ap)]].$ This implies that $\lim_{\alpha \to 0^+} \frac{\partial \Pi_i(\alpha)}{\partial \alpha} = a(t)(p - w - ap)/\alpha^2$. Hence, $\frac{\partial \Pi_i(\alpha)}{\partial \alpha}$ is a unique minimizer. This implies $\frac{\partial \Pi_i(\alpha)}{\partial \alpha}$ crosses zero at most three times.

We consider two cases below, depending on the number of $\alpha$ values at which $\Pi_i(\alpha)$ crosses zero. Note that $\lim_{\alpha \to 0^+} \Pi_i(\alpha) = 0$ holds in both cases.

Case 1. When $v - p - t + k < 0$, we have

$$\lim_{\alpha \to 0^+} \frac{\partial \Pi_i(\alpha)}{\partial \alpha} > 0,$$

and $\Pi_i(\alpha)$ crosses zero only at $\alpha_1 = 2(p-w)/p$, and $\alpha_2 > e \geq 2.71$. Hence, we have $\Pi_i(\alpha) \geq 0$ for $\alpha \in (0, \alpha_1)$, $\Pi_i(\alpha) < 0$ for $\alpha \in (\alpha_1, \alpha_2)$, and $\Pi_i(\alpha) \geq 0$ for $\alpha \geq \alpha_2$. This implies that $\Pi_i(\alpha)$ has at least one local maximizer $\alpha_1 \in (0, \alpha_1)$ and one local minimizer $\alpha_2 \in (\alpha_1, \alpha_2)$. In addition, both $\alpha_1$ and $\alpha_2$ are unique. If there were any other local maximum (minimum), there had to be another local maximum (minimum), bringing the total number of positive extremum to four. This contradicts the fact that $\frac{\partial \Pi_i(\alpha)}{\partial \alpha} = 0$ is satisfied at most at three positive $\alpha$ values. Hence, $\Pi_i(\alpha)$ has a unique local maximizer $\alpha_1$. This maximizer satisfies $\alpha_1 \in (0, \alpha_1)$.

Case 2. When $v - p - t + k \geq 0$, we have

$$\lim_{\alpha \to 0^+} \frac{\partial \Pi_i(\alpha)}{\partial \alpha} < 0,$$

and $\Pi_i(\alpha)$ crosses zero only at $\alpha_1 = 2(p-w)/p$, $\alpha_3 < 1$, and $\alpha_4 \in (1, e)$. Depending on the value of $\alpha_1$, five cases are possible: (1) $\alpha_1 < \alpha_5 < \alpha_4 < e$, (2) $\alpha_1 = \alpha_5 < \alpha_4 < e$, (3) $\alpha_1 < \alpha_4 < e$, (4) $\alpha_1 < \alpha_4 < e$, (4) $\alpha_1 < \alpha_4 < e$, and (5) $\alpha_3 < \alpha_4 < e$. In all five cases, $\Pi_i(\alpha)$ has a local maximizer $\alpha_1$ and two local minimizers $\alpha_{j1}$ and $\alpha_{j2}$ that satisfy $\alpha_{j1} < \alpha_1 < \alpha_{j2}$. No other extremum exists because the total number of positive extrema of $\Pi_i(\alpha)$ is at most three. Hence, $\alpha_1$ is the unique local maximizer. This maximizer satisfies $\alpha_1 \in (0, \max[\alpha_1, \alpha_3])$.

Next, considering Case 1 and Case 2, we show that the retailer’s best response $\alpha^*$ is equal to either $\alpha_i$ or one of the boundary values $\alpha_{\text{min}}$ and 1. Let $\alpha_{\text{max}} = \max[\alpha_{i}, \Pi_i(\alpha) = 0]$. For Case 1, $\alpha_{\text{max}} = \alpha_2 > e \geq 2.71$. For Case 2, $\alpha_{\text{max}} = \max[\alpha_1, \alpha_4] > 1$. For both cases, we have $\alpha_{\text{max}} > 1$, so we cannot have $\alpha^* \in (\alpha_{\text{min}}, \infty)$. That is, $\alpha^*$ cannot be in the rightmost domain where $\Pi_i(\alpha)$ is increasing, because this domain is outside the relevant domain $[\alpha_{\text{min}}, 1]$. In this case, the solution to the constrained problem in Equation (4) is neither $\alpha_i$ or one of the boundary values $\alpha_{\text{min}}$ and 1, and provided that the retailer’s expected profit is non-negative. If the retailer’s best response profit in the domain $\alpha \in (\alpha_{\text{min}}, 1)$ is negative, then the retailer simply sets $\alpha^* = \alpha_1$, and does not order any product. In this case, we have $\Pi_1(\alpha^*) = 0$.

Finally, we show that $\alpha^*$ decreases in the wholesale price $w$, which requires $\frac{\partial^2 \Pi_i(\alpha)}{\partial \alpha \partial w} < 0$. We have $\frac{\partial^2 \Pi_i(\alpha)}{\partial \alpha \partial w} = (a/t)[(p - w - ap)[1 - \alpha(1 - 2\ln(\alpha))] < 0$ because $t \geq t^* = (v - p)(1 - (1 - 2\ln(\alpha))) + k$. This inequality holds because the manufacturer does not set $t < t^*$ in the equilibrium.

Proof of Corollary 1. From Equation (4), we have $\lim_{\alpha \to 0^+} \Pi_i(\alpha, t) = a(a(p - w - ap)/2)$. This function is maximized at $\alpha = (p - w)/p$. Considering the minimum service level condition, we find $\lim_{\alpha \to \alpha_{\text{min}}} \alpha^*(t) = \max[\alpha_{\text{min}}, (p-w)/p]$. □

Proof of Lemma 3. From Lemma 1, we know the distribution of the primary and secondary demand, that is, $D_j^1$ and $D_j^2$ in the direct channel as well as $D_i$ in the retailer for different market segmentation cases. From Lemma 2, we also have the retailer’s optimal order quantity $q = a(a(1 - d_j(\alpha))$. Substituting these values, we obtain the expected sales in the direct channel. For example, when $t \in [t^*, v-p]$,

$$E[D_j^1 + D_j^2] = E[d_iX] + E[(1 - d_i)X - q]^+ = d_i(a/2) + \int_{q=a(1-d_i(\alpha))}^{(1-d_i(\alpha))} (z-q)[1/a(1-d_i)]dz.$$
Considering all segmentation cases, we find the expected
sales in the direct channel as
\[
E[D^*_2 + D^*] = \begin{cases}
\frac{a}{t'}, & \text{for } t \leq t', \\
\frac{a}{2}(a(1-\alpha) + 1) & \alpha(1-\ln(\alpha)) + k, & \text{for } t \in (t', v - p), \\
\frac{a}{2t}(a-\phi(a)-k) + (v-p), & \text{for } t \in (v-p, \infty), \\
0, & \text{for } t \rightarrow \infty.
\end{cases}
\]

Finally, we substitute \( E[D^*_2 + D^*] \), \( q = \alpha(1-d_1(\alpha)) \) and \( \phi(\alpha) = \alpha(1-\ln(\alpha)) \) into Equation (5) to find the manufacturer’s expected profit function. For example, when \( t \leq t' \), we have \( q = 0 \) because \( d_1(\alpha) = 0 \). Hence, \( \Pi_m(t) = (p-c_2)/2 - m/t^2 \).

**Proof of Lemma 4.** We prove Parts (i) and (ii) together because the functions \( \Pi^*_m(t) \) and \( \Pi^*_u(t) \) are similar. Let \( \Pi_m(t) \) represent either of the two functions. We have \( \partial \Pi_m(t)/\partial t = -G(a)/t^2 + 2m/t^2 = (1/t^2)(2m/t - G(a)) \). The first derivative crosses zero at \( t = 2m/G(a) \).

For \( G(a) < 0 \), we have \( \partial \Pi_m(t)/\partial t > 0 \) for all \( t \in (0, \infty) \), hence, \( \Pi_m(t) \) is strictly increasing in \( t \). For \( G(a) > 0 \), we have \( \partial \Pi_m(t)/\partial t \mid_{t<0} > 0 > \partial \Pi_m(t)/\partial t \mid_{t>0} < 0 \). Hence, \( \Pi_m(t) \) is unimodal and the maximizer is \( t^*_f \).

Parts (iii) and (iv) follow from the definitions of the functions \( \Pi^*_m(t) \) and \( \Pi^*_u(t) \) in Lemma 3.

**Proof of Proposition 2.** From Lemma 3, \( \Pi_m(t) \) is a continuous function defined by three functions defined in three connected regions. To characterize \( t^*_f \), we examine \( \Pi_m(t) \) in each region. First note that \( G^*(a) \geq G^*(\alpha) \). Hence, we consider three main cases.

**Case 1.** When \( G^*(a) \geq G^*(\alpha) > 0 \), we have \( t^*_j \leq t^*_f \). There are six subcases to consider.

When \( t^*_j \in (0, t') \) and \( t^*_j < v-p \), we have \( \Pi_m(t^*_j) \) increasing, hence it achieves its maximum at \( \Pi_m(t^*_j) \). From Lemma 4(i), \( \Pi^*_m(t) \) is decreasing in \( t \in (t^*_j, v-p) \) (because \( t^*_j < t' \)) and from Part (ii) \( \Pi^*_u(t) \) is also decreasing in \( t \in (v-p, \infty) \). Hence, \( \Pi_m(t) \) achieves its maximum at \( t^*_j \).

When \( t^*_j \in (t', v-p) \) and \( t^*_j < v-p \), \( \Pi^*_m(t) \) is increasing; \( \Pi^*_u(t) \) is increasing in \( t \in (t_j, t^*_f) \) and decreasing thereafter; \( \Pi^*_m(t) \) is decreasing in \( t \in (v-p, \infty) \). Hence, \( t^*_f = t^*_j \).

When \( t^*_j \in (v-p, \infty) \) and \( t^*_j < v-p \), \( \Pi^*_m(t) \) is increasing; \( \Pi^*_u(t) \) is increasing in \( t \in (t^*_j, v-p) \) and achieves its maximum at \( t = v-p \); \( \Pi^*_m(t) \) is decreasing in \( t \in (v-p, \infty) \). Hence, \( t^*_f = t^*_j \).

Note that subcases \( t^*_j \in (0, t') \), \( t^*_j > v-p \), and \( t^*_j \in (t', v-p) \), \( t^*_j > v-p \) are not possible because \( t^*_j \leq t^*_f \).

When \( t^*_j \in (v-p, \infty) \) and \( t^*_j > v-p \), \( \Pi^*_m(t) \) is increasing; \( \Pi^*_u(t) \) is increasing in \( t \in (t^*_j, v-p) \) and achieves its maximum at \( t = v-p \); \( \Pi^*_m(t)^* \) is increasing in \( t \in (v-p, t^*_j) \) and decreasing thereafter. Hence, \( t^*_f = t^*_j \).

**Case 2.** When \( G^*(a) > 0 \geq G^*(\alpha) \), we have \( \Pi_m^*(t) \) and \( \Pi_u^*(t) \) increasing in \( t \). Hence, \( \Pi_m(t) \) achieves its maximum at \( t = v-p \) for \( t \leq v-p \). If \( t^*_j > v-p \), then \( t^*_f = t^*_j \). Otherwise, \( t^*_f = v-p \) because \( \Pi_m(t) \) is decreasing in \( t \in (t^*_j, \infty) \).

**Case 3.** When \( 0 \geq G^*(a) \geq G^*(\alpha) \), we have \( \Pi_m^*(t) \), \( \Pi_u^*(t) \), and \( \Pi_m^*(t) \) all increasing in \( t \). Hence, \( \Pi_m(t) \) achieves its maximum at an arbitrarily large \( t \). We denote this maximizer as \( t^*_f \), concluding the proof.

**References**


