ENGR 3301-001
Final Exam
December 9, 2010

Please Print.

Last Name: First Name:

Instructions

WRITE CLEARLY and NEATLY. Messy and illegible writing will result in ZERO credit.

1. Examination Duration: 2 hours
2. You can use a calculator, and the Help Sheet that has been verified by the instructor.
3. There are 2 parts, A and B. Part A: 40 points – (4 problems, 10 points each) and
   Part B: 60 points (Answer any 3 problems, 20 points each). Use the other problem
   in Part B as the bonus problem for 10 points. CLEARLY DESIGNATE YOUR
   CHOICE OF THE BONUS PROBLEM.
4. DO NOT RELY ON PARTIAL CREDITS, which will be given only for proper steps/logic,
   and solely at the discretion of the instructor. SHOW ALL YOUR STEPS. Highlight your
   answers.
5. Answer in the space/sheets provided. Additional sheets are provided at the end for scratch
   work and/or for space needs. Do not un-staple; if you do, staple back with page numbers in
   order.
6. Any copying or cheating will result in appropriate action as per university regulations.

Score Tabulation (For Grading Purposes by the Instructor)

A1 -
A2 -
A3 -
A4 -
B1 -
B2 -
B3 -
B4 -
1. The voltage response for the circuits shown below is given by

\[ v(t) = 6000 e^{-500t} + 8 e^{-500t} V, \quad t \geq 0. \]

The value of the resistance is 1 kΩ. Determine the values of the capacitor, the inductor, and the initial voltage across the capacitor.

\[ R = 1 kΩ \Rightarrow \frac{1}{2 \times 10^5 \times 500} = 1 \mu F \]

\[ C = \frac{1}{2 \times 10^5 \times 500} = 1 \mu F \]

\[ L = \frac{1}{500 \times 10^{-3}} = \frac{1}{0.5} \]

\[ L = 4 H \]

\[ v(0) = v_c(0^+) = v(t) \bigg|_{t=0} = 8 V \]

Answer:

\[ C = 1 \mu F \]
\[ L = 4 H \]
\[ v_c(0^+) = V_0 = 8 V \]
In the circuit given below, determine the range of the input voltage, $v_s$, for which the output voltage, $v_o$, will not saturate.

For non-saturation: $-5 \leq v_o \leq 5 \text{V}$

Range of $v_o$: $[-10 \leq v_o \leq 10 \text{V}]$ corresponds to $v_s = -5 \text{V}$

$V_p = V_n = 0$. (Virtual short).

Node C:

$\frac{v_c - v_n}{4} + \frac{v_c}{4} + \frac{v_b}{3} = 0 \Rightarrow \frac{v_c}{2} + \frac{v_c}{3} = \frac{v_n}{4} \Rightarrow \frac{5}{6} v_c = \frac{v_n}{4} \Rightarrow v_c = \frac{6}{25} v_n = 0.3 \text{V}$

Node B:

$\frac{v_c}{3} + \frac{v_b}{5} = 0 \Rightarrow \frac{v_b}{5} = -\frac{v_c}{3} \Rightarrow v_b = -\frac{5}{3} v_c = -\frac{5}{3} \times 0.3 \text{V} \Rightarrow v_b = -0.5 \text{V}$
A.3
Identify each of the following circuits as a Low Pass filter, High Pass Filter or Band Pass filter. You need not show any steps.

(a) LPF  (b) HPF  (c) HPF  (d) LPF  (e) LPF
A.4
The voltage and current were measured at the terminals of the device shown in Figure (a). The results are shown in a table in Figure (b). Construct a circuit model for this device using an ideal voltage source and a resistor.

<table>
<thead>
<tr>
<th>$v_t$ (V)</th>
<th>$i_t$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>66</td>
<td>2</td>
</tr>
<tr>
<td>82</td>
<td>4</td>
</tr>
<tr>
<td>98</td>
<td>6</td>
</tr>
<tr>
<td>114</td>
<td>8</td>
</tr>
<tr>
<td>130</td>
<td>10</td>
</tr>
</tbody>
</table>

Model:

\[ v_e = i_e \cdot R + v_0 \]

Take 2 sets of values from the table to solve for $R$ and $v_0$ in (1).

Set 1: $v_t = 50$, $i_t = 0$.

\[ 50 = 0 \times R + v_0 \Rightarrow v_0 = 50 \text{V} \]

Set 2: $v_t = 82$, $i_t = 4$; (we know $v_0 = 50$V).

\[ 82 = 4 \times R + 50 \Rightarrow R = 8 \Omega \]

Model:
B1
For the circuit shown below determine (a) the transfer function \( H(s) = \frac{V_0(s)}{V_g(s)} \) (6 points), (b) the corresponding impulse response, \( h(t) \) (4 pts) and (c) the output \( v_0(t) \) when the input is \( v_g(t) = 10 \exp(-10t) \, u(t) \) (10 pts). Assume that the circuit has zero energy.

\[
\begin{align*}
R &= 10 \, k\Omega \\
C &= 10 \, \mu F
\end{align*}
\]
The switch in the circuit has been in position 1 for a long time, and moves instantaneously to position 2 at \( t = 0 \). Determine \( v_0(t) \) for \( t \geq 0^+ \). (Note the polarity on \( v_0(t) \).)
B3. (YOU MUST USE LAPLACE TRANSFORM METHOD TO GET CREDIT.)

In the circuit shown below, switches 1 and 2 operate simultaneously. At t=0 sec, switch 1 closes and switch 2 opens. Prior to that switch 1 has been open and switch 2 closed for a long time. Determine the voltage $v_c(t)$ for $t \geq 0$ sec.

With a non-zero initial voltage on the capacitor, the $s$-domain circuit becomes:

$$\frac{V_o - 18/s}{0.2s + 2800} + \frac{(V_o - 30/s)s}{8 \times 10^6} = 0$$

$$V_o \left[ \frac{5}{s + 14,000} + \frac{s}{8 \times 10^6} \right] = \frac{30}{80 \times 10^6} + \frac{90}{s(s + 14,000)}$$

$$\therefore \quad V_o = \frac{30s^2 + 420,000s + 720 \times 10^6}{s(s + 4000)(s + 10,000)} = \frac{K_1}{s} + \frac{K_2}{s + 4000} + \frac{K_3}{s + 10,000}$$

$$K_1 = \frac{720 \times 10^6}{40 \times 10^6} = 18$$

$$K_2 = \frac{30s^2 + 420,000s + 720 \times 10^6}{s(s + 10,000)} \bigg|_{s = -4000} = 20$$

$$K_3 = \frac{30s^2 + 420,000s + 720 \times 10^6}{s(s + 4000)} \bigg|_{s = -10,000} = -8$$

$$V_o = \frac{18}{s} + \frac{20}{s + 4000} - \frac{8}{s + 10,000}$$

$$v_o(t) = [18 + 20e^{-4000t} - 8e^{-10,000t}]u(t) \text{ V}$$
B4.
Determine the output $v_0(t)$ for $t \geq 0$ sec if the input voltage $v_g(t) = 5 \sin(2t) u(t)$. Assume that the initial current through the inductor is zero.

\[ L = 1 \, \text{H}, \quad R = 1 \, \text{A} \Rightarrow \frac{R}{L} = 1 \]

\[ V_g(t) = 5 \sin(2t) u(t) \Rightarrow V_g(0) = \frac{10}{\sqrt{13}} \]

\[ V_0(0) = \left. \frac{10}{(\beta + 1)(\alpha^2 + 4)} \right|_{\alpha = -1} = \frac{10}{(\beta + 1)(\alpha^2 + 4)} = \left[ \frac{2}{\beta + 1} \right] \]

\[ A = V_0(0)(\alpha + 1) = \frac{10}{(\beta + 1)(\alpha^2 + 4)} = \left[ \frac{2}{\beta + 1} \right] \]

\[ \Rightarrow \quad V_0(\alpha) = \frac{2}{(\beta + 1)} + \left[ \frac{-2 \alpha^2 + 2}{\alpha^2 + 4} \right] \]

\[ = \frac{2}{(\beta + 1)} + (\alpha^2) \frac{A}{(\alpha^2 + 4)} + (1) \frac{3}{(\alpha^2 + 4)} \]

\[ \Rightarrow \quad v_0(t) = \left[ \frac{2 - \frac{t}{2} + 5 \sqrt{5} \cos(2t + \tan^{-1}(\frac{t}{2}))}{(\alpha^2 + 4)} \right] u(t) \]

or

\[ v_0(t) = \left[ 2 e^{-t} + 5 \sqrt{5} e^{-t} (2t + 20.5 \sqrt{2}) \right] u(t) \]