Instructor:

WRITE CLEARLY and NEATLY. Messy and illegible writing will result in ZERO credit.

1. Examination Duration: 1 hour 15 minutes. If you come early (up to 15 minutes), you can have that extra time. This exam counts towards 20% of the total course numerical grade.

2. You can use a calculator, and the Help Sheet that has been verified by the instructor and the Help Sheet provided by the instructor.

3. There are 2 parts, A and B. Part A: 40 points – (3 problems, 13.33 points each) and Part B: 60 points (3 problems, 20 points each). A bonus problem for 10 points is at the end. DO NOT RELY ON PARTIAL CREDITS, which will be given only for proper steps/logic, and solely at the discretion of the instructor. SHOW ALL YOUR STEPS. Highlight your answers.

4. Answer in the space/sheets provided. Additional sheets are provided at the end for scratch work and/or for space needs. Do not un-staple; if you do, staple back with page numbers in order.

5. Any copying or cheating will result in appropriate action as per university regulations.

Score Tabulation (For Grading Purposes by the Instructor)

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A.1

Assume that the op amp circuit below is ideal. Determine the voltage gain, \( v_0/v_i \), in terms of \( R_1 \) and \( R_2 \).
(Nota: the non-inverting input node is at the top.)

\[
\begin{align*}
\text{Ideal op Amp} & \quad v_n = v_i, \quad i_o = i_n = 0. \\
\text{At non-inverting node input, KCL:} & \\
\frac{v_p - v_i}{R_1} + \frac{v_p}{R_2} = 0 & \Rightarrow v_p \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_i}{R_1} \\
& \Rightarrow v_p = \left( \frac{R_1 R_2}{R_1 + R_2} \right) \left( \frac{v_i}{R_1} \right) = \left( \frac{R_2}{R_1 + R_2} \right) v_i \\
& \Rightarrow v_p = \left( \frac{R_2}{R_1 + R_2} \right) v_i
\end{align*}
\]

\[
\begin{align*}
\text{At the inverting input node, KCL:} & \\
v_i + \frac{v_n - v_0}{R_2} = 0 & \Rightarrow v_i \left[ \frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{v_0}{R_2} \\
& \Rightarrow v_0 = \left( \frac{R_2}{R_1 + R_2} \right) v_i \left[ \frac{(R_1 + R_2)}{R_1 R_2} \right] \\
& = \left[ \frac{R_1 + R_2}{R_1 R_2} \right] R_L = \left[ \frac{v_i}{R_1} \right]
\end{align*}
\]

\[
\Rightarrow \quad \frac{v_0}{v_i} = \frac{R_2}{R_1}
\]
The current and voltage at the terminals of the capacitor in the circuit shown are given by:

\[ i(t) = 3 e^{-2500t} \text{mA}, \quad t \geq 0 \]

\[ v(t) = (40 - 24 e^{-2500t}) \text{V}, \quad t \geq 0 \]

Determine the values of R, C and I<sub>o</sub>.

\[ i'(t) = C \frac{dv(t)}{dt} = C \cdot (-24)(-2500)e^{-2500t} \text{mA} \]

Equating (1) \& (3), we get \[ C = 50 \text{nF} \]

From (1) \& (2), time constant \( \tau = \frac{1}{2500} = RC \Rightarrow R = \frac{1}{2500 \times 50 \times 10^{-9}} = \frac{16}{125} = 12.8 \text{k} \Omega \]

\[ R = 8 \text{k} \Omega \]

At \( t = \infty \), the capacitor is open circuited and all current flows through the resistor.

\[ v(\infty) = 40 = I_0 R = I_0 \times 10^3 \Rightarrow I_0 = 5 \text{mA} \]

**Answer**

\[ R = 8 \text{k} \Omega \]
\[ C = 50 \text{nF} \]
\[ I_0 = 5 \text{mA} \]
In the circuit shown below, the loop current \( i(t) \) for \( t \geq 0 \) is given by

\[
i(t) = B_1 e^{-2000t} \cos(1500t) + B_2 e^{-2000t} \sin(1500t).
\]

The capacitor has a value of 80 nF, the initial value of the current is 7.5 mA, and the initial voltage across the capacitor is -30 V. Determine the values of \( R \), \( L \) and \( B_1 \).

This is the underdamped natural response of

\[
i(t) = B_1 e^{-2000t} \cos(1500t) + B_2 e^{-2000t} \sin(1500t)
\]

where

\[
2 \alpha = \frac{R}{L}, \quad \omega^2 = \omega_0^2 - \alpha^2, \quad \omega_0 = \frac{1}{\sqrt{LC}}.
\]

From (1) and (2), with \( C = 80 \text{ nF} \), we identify

\[
\alpha = \frac{R}{2L} = 2000 \implies R = 4000 \text{ L}.
\]

\[
\omega_0^2 = (1500)^2 + (2000)^2 \implies \omega_0^2 = 625 \times 10^4.
\]

\[
\omega_0 = 25 \times 10^3 \text{ rad/s}
\]

\[
\frac{1}{\omega_0^2} = \frac{1}{L \times 80 \times 10^{-9}} = 625 \times 10^4
\]

\[
L = \frac{1}{625 \times 10^4 \times 80 \times 10^{-9}} = 2 \text{ H}
\]

\[
R = \frac{4000 \times 2 \times 8 \times 10^{-9}}{2} = 8 \text{ k}\Omega
\]

\[
i(0) = 7.5 \times 10^{-3} = B_1 \cos(0) + 0
\]

\[
B_1 = 7.5 \text{ m}\Omega
\]

\[
R = 8 \text{ k}\Omega
\]

\[
L = 2 \text{ H}
\]

\[
B_1 = 7.5 \text{ m}\Omega
\]
B1
In the circuit shown below, the switch has been in position "a" for a long time, and is moved to position "b" at \( t = 0 \) instantaneously. Determine the current \( i_o(t) \), and the voltage across the inductor \( v_L(t) \) for \( t \geq 0 \).

![Circuit Diagram]

\[ [a] \ t < 0 \]

KVL equation at the top node:

\[
50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}
\]

Multiply by 40 and solve:

\[
2000 = (5 + 1 + 4)v_o; \quad v_o = 200 \text{ V}
\]

\[
\therefore i_o(0^-) = \frac{v_o}{10} = 200/10 = 20 \text{ A}
\]

\[ t > 0 \]

Use voltage division to find the Thévenin voltage:

\[
V_{Th} = v_o = \frac{40}{40 + 120}(800) = 200 \text{ V}
\]

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

\[
R_{Th} = 10 + 120||40 = 10 + 30 = 40 \Omega
\]
The simplified circuit is:

\[
\begin{align*}
\tau &= \frac{L}{R} = \frac{40 \times 10^{-3}}{40} = 1 \text{ ms}; \quad \frac{1}{\tau} = 1000 \\
i_o(\infty) &= \frac{200}{40} = 5 \text{ A} \\
i_o &= i_o(\infty) + \left[ i_o(0^+) - i_o(\infty) \right] e^{-t/\tau} \\
&= \frac{5 + (20 - 5)e^{-1000t}}{15} e^{-1000t} = 5 + 15e^{-1000t} \text{ A}, \quad t \geq 0
\end{align*}
\]

The voltage across the inductor is

\[
V_L(t) = L \frac{di_o(t)}{dt}, \quad t \geq 0
\]

\[
\Rightarrow \quad V_L(t) = 40 \times 10^{-3} \times 15 \times (-1000) \times e^{-1000t} \text{ V}, \quad t \geq 0
\]

\[
\Rightarrow \quad V_L(t) = -600e^{-1000t} \text{ V}, \quad t \geq 0
\]

\[
i_o(0^+) = 5 + 15e^{-1000t} \text{ A}, \quad t \geq 0
\]
B2
In the circuit shown below, the initial voltage across the capacitor is 24 V, and the initial current through the inductor is zero. The component values are given by \( R = 8 \, \text{k}\Omega \), \( C = 0.1\mu\text{F} \), and \( L = 40 \, \text{H} \). Determine the resulting voltage response, \( v(t) \) for \( t \geq 0 \).

\[
L \frac{di_L(t)}{dt} + i_L(t) + \frac{i_C(t)}{C} = 0
\]

\[
i_C(t) = 0 \quad \Rightarrow \quad \frac{V_0}{C} \quad \Rightarrow \quad V_C(0^+) = V_0 = 24 \, \text{V}.
\]

Initial Condition

\[
L \frac{di_L(t)}{dt} = V_0 = 24 \, \text{V}
\]

\[
\frac{d^2 i_L(t)}{dt^2} + \frac{2}{L} \cdot \frac{di_L(t)}{dt} + \frac{i_L(t)}{ RC} = 0
\]

\[
\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{4 \times 10^{-6} \times 1 \times 10^4}} = \sqrt{2.5 \times 10^{-4}} \Rightarrow \omega_0 = 5 \, \text{rad/s}
\]

\[
\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10^{-6} \times 1 \times 10^4} = 5 \, \text{rad/s}
\]

\[
\alpha^2 - \omega_0^2 = (2.5^2 - 5^2) > 0
\]

\[
A_1 = -\alpha + \sqrt{(\alpha^2 - \omega_0^2)} = -2.5 \quad \text{and} \quad A_2 = -\left(\frac{\omega_0^2 - \alpha^2}{\omega_0}\right) = -1000
\]

Critically damped natural response.

\[
i_L(t) = A_1 e^{-\alpha t} + A_2 e^{-\omega_0 t}
\]

\[
\Rightarrow \quad i_L(t) = A_1 e^{-2.5t} + A_2 e^{-1000t}, \quad t > 0
\]

\[
i_L(0^+) = I_0 = 0 = A_1 + A_2 \Rightarrow A_1 + 4A_2 = 0 \quad \text{(1)}
\]

\[
\frac{di_L(t)}{dt} = 0.6 = -25 A_1 - 1000 A_2 \Rightarrow A_1 = 0.6 \, \text{mA}
\]

\[
A_2 = -0.8 \, \text{mA}
\]

\[
\therefore \quad i_L(t) = 0.8 \, e^{-25t} - 0.8 \, e^{-1000t}, \quad t > 0
\]

\[
V(t) = L \frac{di_c(t)}{dt} = \frac{24}{40} \left(0.8e^{-25t} - 0.8e^{-1000t}\right) \, \text{V}, \quad t > 0
\]

\[
V(t) = -8 \, e^{-25t} + 32 \, e^{-1000t} \, \text{V}, \quad t > 0
\]
B3.
Determine the current $i_x$ in the following circuit. Assume that the op amp circuit below is ideal. (Note that the non-inverting input node is at the top.)

**Ideal op amp:** $v_p = v_n$, $i_p = i_n = 0$.

**Note:** $v_n = v_o$ (Why?)

$\Rightarrow v_p = v_n = v_o$

**Node 1, KCL:**

$$-4 + \frac{v_1}{3} + \frac{v_1 - v_p}{6} + \frac{v_1 - v_o}{12} = 0$$

$\Rightarrow v_1 \left( \frac{1}{3} + \frac{1}{6} + \frac{1}{12} \right) - v_p \left( \frac{1}{6} + \frac{1}{12} \right) = 4$

$\Rightarrow v_1 \cdot \frac{7}{12} - v_p \cdot \frac{1}{4} = 4 \Rightarrow 7v_1 - 3v_p = 48$ \hfill (1)

**Node 2, (non-inverting input node), KCL:**

$$\frac{v_p - v_1}{6} + \frac{v_p - 0}{6} = 0 \Rightarrow v_p \cdot \frac{1}{3} = \frac{v_1}{6} \Rightarrow v_1 = 2v_p$$ \hfill (2)

Substituting (2) in (1), we get

$$14v_p - 3v_p = 48 \Rightarrow v_p = \frac{48}{11}$$

$$i_x = \frac{v_p}{6 \times 10^3} = \frac{v_p}{6} \text{ mA} = \frac{48}{11 \times 6} \text{ mA} = 0.727 \text{ mA}$$

$$i_x = 0.727 \text{ mA} \quad \text{or} \quad i_x = \frac{8}{11} \text{ mA}$$
C. Bonus (10 points)

Determine the equivalent capacitance across terminals “a” and “b”.

\[
\begin{align*}
\text{(i)} & \quad 16 \mu F \ \text{in series with} \ 48 \mu F \\
& \quad \Rightarrow \quad \frac{1}{C_1} = \frac{1}{16} + \frac{1}{48} \quad \Rightarrow \quad C_1 = 12 \mu F \\
\text{(ii)} & \quad 3 \mu F \ \parallel \ 12 \mu F \\
& \quad \Rightarrow \quad C_2 = 15 \mu F \\
\text{(iii)} & \quad 30 \mu F \ \text{in series with} \ 15 \mu F \\
& \quad \Rightarrow \quad \frac{1}{C_3} = \frac{1}{30} + \frac{1}{15} \\
& \quad \Rightarrow \quad C_3 = 10 \mu F \\
\text{(iv)} & \quad 10 \mu F \ \parallel \ 10 \mu F \\
& \quad \Rightarrow \quad C_4 = 5 \mu F \\
\text{(v)} & \quad C_{eq} \rightarrow 5 \mu F, 20 \mu F \ \text{and} \ 4 \mu F \ \text{in series} \\
& \quad \Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{4} \\
& \quad \Rightarrow \quad C_{eq} = 2 \mu F
\end{align*}
\]