EE 3350
Exam 2 Solutions
Fall '02
11/22/02
**A.1 True or False? (5 points)**

1. The intermediate frequency of an FM receiver is 10.7 MHz.  \( \text{\checkmark} \)

2. The bandwidth of a narrowband FM signal is approximately 200 Hz, if the message signal has a bandwidth of 200 Hz.  \( \times \)

3. DPCM can provide a greater SNR\(_Q\) than PCM for the same number of bits/sample.  \( \times \)

4. Phase Lock Loop can be used as a FM demodulator.  \( \text{\checkmark} \)

5. In a wideband FM, doubling the peak value of the message signal approximately doubles the bandwidth of the FM signal.  \( \text{\checkmark} \)
2A.1 Fill in the blanks (5 points)

1. The intermediate frequency used in a stereo FM receiver is $10.7 \text{ MHz}$. 

2. The broadcast FM band is $88 - 108 \text{ MHz}$. 

3. The Nyquist frequency for a signal of bandwidth 8 kHz is $16 \text{ kHz}$. 

4. In a uniform quantizer, each additional bit provides an increase of $6$ dB in signal to quantization noise ratio. 

5. In FM stereo broadcast transmission, the pre-emphasized L – R signal is modulated using DSB-SC AM. A pilot signal of frequency $19 \text{ kHz}$ is transmitted for purposes of coherently demodulating the pre-emphasized L-R signal.
2A.2 A signal \( m(t) \) has a bandwidth of 1 kHz, and exhibits a maximum rate of change of 2 volt/second. The signal is sampled at a sampling frequency of 20 kHz and quantized using a delta modulator. What should be the minimum step size to avoid slope overload? (5 points)

\[
\frac{S}{I_S} \geq \frac{d m(t)}{dt}_{\text{max}}
\]

\[
0.2 \times 10^3 \geq 2 \implies S \geq 10^4
\]

Minimum step size = 0.1 mV/\mu s

2A.3 A PCM system, at 10 kbits/second with 10 bits/sample, has a satisfactory SNR of 50 dB. Binary signaling is required and the available transmission bandwidth is 4 kHz. A DPCM system has to be used to transmit the signal in the bandwidth specified. What is the required prediction gain (in dB)? (5 points)

Data rate has to be reduced to 8 kbps for the available bandwidth, or, DPCM has to provide a 2-bit advantage \( \implies 12 \text{ dB prediction gain} \)

2A.4 From the magnitude spectrum of a given signal \( q(t) \) we cannot determine whether it is NBFM or DSB-TC AM signal. The phase spectrum is not available. How will you make the determination? (5 points)

Given signal waveform determines whether it is DSB-TC AM (varying amplitude) or FM (constant amplitude)

2A.5 Under what conditions does aliasing occur in the sampling of signals? (5 points)

- Signal \( g(t) \) with bandwidth \( B \) Hz is undersampled, i.e. \( f_s < 2B \) Hz.
- Signal \( g(t) \) is not bandlimited. Any rate of sampling will result in aliasing.
A.2 A signal $m(t)$ has a bandwidth of 1 kHz, and exhibits a maximum rate of change of 1 volt/second. The signal is sampled at a sampling frequency of 10 kHz and quantized using a delta modulator. What should be the minimum step size to avoid slope overload? (5 points)

$$\frac{S}{T_s} > \left| \frac{dm(t)}{dt} \right|_{\text{max}}$$

$$\frac{S}{10^{-4}} > 1 \Rightarrow S \geq 10^4 \text{ Volts}$$

Minimum step size: 0.1 mV

A.3 A PCM system, at 8 kbits/second with 8 bits/sample, has a satisfactory SNR of 50 dB. Binary signaling is required and the available transmission bandwidth is 3 kHz. A DPCM system has to be used to transmit the signal in the bandwidth specified. What is the required prediction gain (in dB)? (5 points)

Data rate must be reduced to 6 kbps for the available bandwidth. DPCM needs to provide a 2-bit advantage

$$\text{Required Prediction gain} = 12 \text{ dB}$$

A.4 From the magnitude spectrum of a given signal $\varphi(t)$ we cannot determine whether it is NBFM or DSB-TC AM signal. The phase spectrum is not available. How will you make the determination? (5 points)

If the given signal waveform, $\varphi(t)$, has a constant amplitude, it is FM; if the amplitude is varying, it is DSB-TC AM.

A.5 Let $x(t)$ be a periodic square wave with a fundamental period of 1 second. Is it possible to sample this signal and reconstruct it exactly from the sampled signal? Explain your answer. (5 points)

$x(t)$ is not bandlimited. Any rate of sampling will produce aliasing. Therefore, it is not possible to reconstruct exactly from the sampled signal.
1B.1. (25 points)
An angle modulated signal is given by the following expression:

\[ \varphi_{EM}(t) = \cos(\omega_c t + 40 \sin 500\pi t + 20 \sin 1000\pi t) \]

a. Determine the frequency deviation \( \Delta f \), in Hz. (6 points)
b. Estimate the bandwidth, in Hz, of the angle modulated signal by Carson’s rule. (6 points)
c. If the angle modulated signal is a phase modulated signal with the phase deviation constant, \( k_p \), is 5 radians per volt, determine the message signal \( m(t) \). (6 points)
d. If the angle modulated signal is a frequency modulated signal with a frequency deviation constant, \( k_f \), is 20,000 \( \pi \) radians/sec per volt, determine the message signal \( m(t) \). (7 points)

\[ \omega_c(t) = \omega_c + \frac{d\varphi(t)}{dt} = \omega_c + (40 \cdot 5000\pi) \cos 5000\pi t + (20 \cdot 10000\pi) \cos 10000\pi t \]

\[ \Delta \omega = \omega_{\max} - \omega_c = 40,000 \pi \text{ rad/sec} \]

\[ \Delta f = \frac{\Delta \omega}{2\pi} = \frac{40,000\pi}{2\pi} = 20 \text{ kHz} \]

\[ \text{BW}_{EM} = 2(\Delta f + B) \approx B = 500 \text{ kHz}, \quad \Delta f = 20 \text{ kHz} \]

\[ \text{For PM,} \quad 4(t) = k_p \cdot m(t). \]

\[ k_p = 5 \text{ rad/mV}, \quad 4(t) = 40 \sin 5000\pi t + 20 \sin 1000\pi t \]

\[ m(t) = 8 \sin 5000\pi t + 4 \sin 1000\pi t \]

\[ \text{For FM,} \quad \omega_c(t) = \omega_c + k_f \cdot m(t) \]

\[ \text{with } k_f = 20,000\pi \text{ rad/sec/volt in Eqn. 1 on this page.} \]

\[ m(t) = \cos 5000\pi t + \cos 1000\pi t \]
2B.1. (25 points)
An angle modulated signal is given by the following expression:

\[ \varphi_{EM}(t) = 5 \cos (\omega_c t + 20 \sin 1000\pi t + 10 \sin 2000\pi t) \]

a. Determine the frequency deviation \( \Delta f \), in Hz. (6 points)
b. Estimate the bandwidth, in Hz, of the angle modulated signal by Carson’s rule. (6 points)
c. If the angle modulated signal is a phase modulated signal with the phase deviation constant, \( k_p \) is 5 radians per volt, determine the message signal \( m(t) \). (6 points)
d. If the angle modulated signal is a frequency modulated signal with a frequency deviation constant, \( k_f \) is 20,000 \( \pi \) radians/sec per volt, determine the message signal \( m(t) \). (7 points)

\[ \Phi_{EM}(t) = A \cos (\omega_c t + \phi(t)) \]

\[ \omega_i(t) = \omega_c + \frac{d\phi(t)}{dt} = \omega_c + (20 \cdot 1000\pi) \cos 1000\pi t + (10 \cdot 2000\pi) \cos 2000\pi t \]

\[ \Delta \omega = \omega_{i_{\text{max}}} - \omega_c = 40,000\pi \text{ rad/sec} \]

\[ \Delta f = \frac{\Delta \omega}{2\pi} = \frac{40,000\pi}{2\pi} = 20 \text{ kHz} \] — Freq. Dev.

\[ B_{WEM} = 2(\Delta f + B) = 2(20 + 1) = 42 \text{ kHz} \] — BW\_EM

For FM, \( \phi(t) = k_p m(t) \), \( k_p = 5 \) rad/sec/volt

\[ m(t) = 4 \sin 1000\pi t + 2 \sin 2000\pi t \]

For FM, \( \omega_i(t) = \omega_c + k_f \cdot m(t) \)

using \( k_f = 20,000\pi \) rad/sec/volt, and Eqn (1), we get

\[ m(t) = \cos 1000\pi t + \cos 2000\pi t \]
3B.1 (25 points)

Given \( m(t) = \sin 2000\pi t \), the frequency deviation constant equals 200,000\( \pi \) radians/sec/volt and that the phase deviation constant equals 10 radians/volt.

a. Estimate the bandwidths of \( \phi_{FM}(t) \) and \( \phi_{PM}(t) \) in kHz. (7 points)
b. Repeat part (a) if the amplitude is doubled. (7 points)
c. Repeat part (a) if the message frequency is doubled. (7 points)
d. Estimate the FM bandwidth if \( m(t) = \sin 2000\pi t + \sin 10,000\pi t \) (4 points)

\[ B = 1 \text{ kHz for the signal.} \]

\[ m_p = 1. \]

\[ \Delta f = \frac{k_f \cdot m_p}{2\pi} = \frac{200,000\pi \cdot 1}{2\pi} = 100 \text{ kHz} \]

\[ BW_{FM} = 2(\Delta f + B) = 2(100 + 1) = 202 \text{ kHz} \]

\[ \frac{dm(t)}{dt} = (2000\pi) \cos 2000\pi t \]

\[ \Rightarrow \frac{dm}{dt}_{\text{max}} = 2000\pi \]

\[ \Delta f_{FM} = \frac{k_f}{\pi} \left| \frac{dm}{dt} \right|_{\text{max}} = \frac{100}{2\pi} \cdot 2000\pi = 10 \text{ kHz} \]

\[ BW_{FM} = 2(\Delta f_{FM} + B) = 2(10 + 1) = 22 \text{ kHz} \]

Amplitude is doubled.

\[ m_p \text{ is doubled, } \left| \frac{dm}{dt} \right|_{\text{max}} \text{ is doubled} \frac{dm}{dt} = 4000\pi t \]

\[ \Delta f_{FM} \text{ and } \Delta f_{PM} \text{ are doubled from part(a).} \]

\[ \Delta f_{FM} = 200 \text{ kHz}, \quad \Delta f_{PM} = 20 \text{ kHz} \]

\[ BW_{FM} = 2(200 + 1) = 402 \text{ kHz} \]

\[ BW_{PM} = 2(20 + 1) = 42 \text{ kHz} \]

\[ m(t) = \sin 4000\pi t \Rightarrow m_p = 1 \left| \frac{dm}{dt} \right|_{\text{max}} = 4000\pi \]

\[ B = 2 \text{ kHz}. \]

\[ \Delta f_{FM} = 100 \text{ kHz (unchanged)} \]

\[ BW_{FM} = 2(100 + 2) = 204 \text{ kHz} \]

\[ BW_{PM} = 2(20 + 2) = 44 \text{ kHz} \]

Now, \( m(t) = \sin 4000\pi t + \sin 10,000\pi t \)

\[ m_p = 2, \quad B = 5 \text{ kHz} \]

\[ \Delta f_{FM} = \frac{k_f}{2\pi} \cdot 200,000\pi \cdot 2 = 200 \text{ kHz} \]

\[ BW_{FM} = 2(200 + 5) = 410 \text{ kHz} \]
4B.1 (20 points)

Given \( m(t) = \sin 4000\pi t \), the frequency deviation constant equals 400,000\( \pi \) radians/sec/volt and that the phase deviation constant equals 20 radians/volt.

a. Estimate the bandwidths of \( \phi_{FM}(t) \) and \( \phi_{PM}(t) \). (7 points)

\[ B = 2 \text{ kHz} \]

\[ m_p = 1 \quad \Delta f_{FM} = \frac{k_p \cdot m_p}{2\pi} = \frac{400,000\pi}{2\pi} = 200 \text{ kHz} \]

\[ B_{w_{FM}} = 2(\Delta f_{FM} + \beta) = 2(200 + 2) = \boxed{404 \text{ kHz}} \]

\[ F_m \]

\[ \frac{\Delta m}{\Delta t} = 4000\pi \cos 4000\pi t \quad \Rightarrow \quad \left| \frac{\Delta m}{\Delta t} \right|_{\text{max}} = 4000\pi \]

\[ \Delta f_{PM} = \frac{1}{2\pi} k_p \frac{\Delta m}{\Delta t} \quad \Rightarrow \quad \Delta f = \frac{1}{2\pi} \cdot 4000 = \boxed{200 \text{ kHz}} \]

\[ B_{w_{PM}} = 2(\Delta f_{FM} + \beta) = 2(200 + 2) = \boxed{840 \text{ kHz}} \]

b. Repeat part (a) if the amplitude is doubled. (7 points)

Amplitude is doubled \( \Rightarrow m_p \) is doubled; \( \left| \frac{\Delta m}{\Delta t} \right| \) is doubled

\[ B_{w_{FM}} = 2(\Delta f_{FM} + \beta) = 2(400 + 2) = \boxed{864 \text{ kHz}} \]

\[ B_{w_{PM}} = 2(\Delta f_{FM} + \beta) = 2(800 + 2) = \boxed{1640 \text{ kHz}} \]

c. Repeat part (a) if the message frequency is halved. (7 points)

Frequency is halved \( \Rightarrow m(t) = \sin 2000\pi t \)

\[ B = 4 \text{ kHz} \], \[ m_p = 1 \]

\[ \Delta f_{FM} = \frac{1}{2\pi} k_p \frac{\Delta m}{\Delta t} = \frac{1}{2\pi} \cdot 4000 = 200 \text{ kHz} \]

\[ B_{w_{FM}} = 2(200 + 1) = 402 \text{ kHz} \]

\[ \Delta f_{PM} = \frac{1}{2\pi} k_p \frac{\Delta m}{\Delta t} \quad \Rightarrow \quad \left| \frac{\Delta m}{\Delta t} \right|_{\text{max}} = 2000\pi \quad \Rightarrow \quad 2000\pi \cdot 20 = 200 \text{ kHz} \]

\[ B_{w_{PM}} = 2(202 + 1) = 42 \text{ kHz} \]

d. Estimate FM bandwidth if \( m(t) = \sin 2000\pi t + \sin 4000\pi t \) (4 points)

\[ m_p = 2, \quad B = 10 \text{ kHz} \]

\[ \Delta f_{FM} = \frac{1}{2\pi} k_p \frac{\Delta m}{\Delta t} = \frac{1}{2\pi} (4000 \pi) \cdot 2 = 400 \text{ kHz} \]

\[ B_{w_{FM}} = 2(\Delta f_{FM} + \beta) = 2(400 + 10) = \boxed{820 \text{ kHz}} \]
1B.2 (25 points)
A wideband FM signal is generated from a narrowband FM generator by Armstrong's indirect method as shown below.

![Diagram of FM signal generation](image)

The desired (target) carrier frequency, \( f_c \) is 100 MHz and frequency deviation \( \Delta f = 75 \) kHz. The message signal to be broadcast is given by

\[
m(t) = 10 \sin \left( 20,000 \pi t \right) + \cos \left( 30,000 \pi t \right)
\]

The following information is given:

\[ f_0 = 100 \text{ kHz}, \Delta f_0 = 10 \text{ Hz} \text{ and } k_1 = 150. \]

Determine:
(a) Frequency Multiplier \( k_2 \) (4 points)
(b) Carrier frequencies \( f_1 \) and \( f_2 \) and the corresponding frequency deviations \( \Delta f_1 \) and \( \Delta f_2 \) (8 points)
(c) The oscillator frequency, \( f_{LO} \) (4 points)
(d) The center frequency and bandwidth of the bandpass filter (9 points)

**Bandwidth of \( m(t) \) = \( 15 \text{ kHz} \) [from Fourier Transform]

\[ a. \quad \frac{\Delta f}{\Delta f_0} = \frac{k_1 \cdot k_2}{k_2} \Rightarrow k_2 = \frac{1}{k_1} \cdot \frac{\Delta f}{\Delta f_0} = \frac{1 \cdot 75 \times 10^3}{150} = 50 \]

\[ b. \quad f_2 = \frac{f_c}{k_2} = \frac{100 \text{ MHz}}{50} = 2 \text{ MHz} \]

\[ \Delta f_2 = \frac{\Delta f}{k_2} = \frac{75 \times 10^3}{50} = 1.5 \text{ kHz} \]

\[ c. \quad f_1 = f_c \cdot k_1 = 100 \times 10^3 \times 150 = 15 \text{ MHz} \]

\[ \Delta f_1 = k_1 \cdot \Delta f_0 = 150 \times 10 = 1.5 \text{ kHz} \]

\[ d. \quad f_{LO} = \Delta f_2 = 1.5 \text{ MHz} \]

The BPF must be able to pass the FM signal at its input, with carrier freq. = \( f_2 \) MHz and

\[ BF = 2(\Delta f_2 + \Delta f_0) = 2(1.5 + 15) \text{ kHz} = 33 \text{ kHz} \]

\[ CF = 2 \text{ MHz} \]

\[ BW = 33 \text{ kHz} \]
2B.2 (25 points)
A wideband FM signal is generated from a narrowband FM generator by Armstrong’s indirect method as shown below.

The desired (target) carrier frequency, $f_c$ is 100 MHz and frequency deviation $\Delta f = 75$ kHz. The message signal to be broadcast is given by

$$m(t) = 1000 \cdot \sin^2(15,000 \pi t)$$

The following information is given:

$$f_0 = 100 \text{ kHz}, \quad k_1 = 150 \quad \text{and} \quad k_2 = 50.$$  

Determine the following quantities:

(a) NBFM frequency deviation $\Delta f_0$ (4 points)
(b) Carrier frequencies $f_1$ and $f_2$ and the corresponding frequency deviations $\Delta f_1$ and $\Delta f_2$ (8 points)
(c) The oscillator frequency, $f_{LO}$ (4 points)
(d) The center frequency and bandwidth of the bandpass filter (9 points)

From Fourier Transform of $m(t)$, bandwidth of $m(t)$ is 15 kHz.

\[ a. \quad \Delta f_0 = \frac{\Delta f}{k_1 k_2} = \frac{75 \text{ kHz}}{150 \cdot 50} = \frac{10 \text{ kHz}}{50} = \Delta f_0 \]

\[ b. \quad f_1 = k_1 f_0 = 150 \times 100 \text{ kHz} = 15 \text{ MHz} \quad f_2 = \frac{f_c}{k_2} = \frac{100 \text{ MHz}}{50} = 2 \text{ MHz} \quad \Delta f_1 = k_1 \Delta f_0 = 1.5 \text{ kHz} \quad \Delta f_2 = \frac{\Delta f}{k_2} = 1.5 \text{ kHz} \]

\[ c. \quad f_1 - f_{LO} = f_2 \quad \Rightarrow \quad f_{LO} = f_1 - f_2 = (15 - 2) \text{ MHz} = 13 \text{ MHz} \quad f_{LO} \]

\[ d. \quad C = \frac{2 \text{ MHz}}{33 \text{ MHz}} \quad BW = 2(\Delta f_2 + \beta) = 2(1.5 + 15) \text{ kHz} = 33 \text{ kHz} \]
3B.2 (25 points)
A wideband FM signal is generated from a narrowband FM generator by Armstrong’s indirect method as shown below.

The desired (target) carrier frequency, $f_c$ is 100 MHz and frequency deviation $\Delta f = 75$ kHz. The message signal to be broadcast is given by

$$m(t) = 1000 \sin c(30,000\pi t)$$

The following information is given:

$$f_0 = 100 \text{ kHz}, \Delta f_0 = 10 \text{ Hz} \text{ and } k_1 = 150.$$ 

Determine: (a) Frequency Multiplier $k_2$ (4 points)
(b) Carrier frequencies $f_1$ and $f_2$ and the corresponding frequency deviations $\Delta f_1$ and $\Delta f_2$ (8 points)
(c) The oscillator frequency, $f_{10}$ (4 points)
(d) The center frequency and bandwidth of the bandpass filter (9 points)

Bandwidth of $m(t)$ is 15 kHz from $M(w)$

$$k_2 = \frac{1}{k_1} \cdot \frac{\Delta f}{f_0} = \frac{1}{150} \cdot \frac{75 \times 10^3}{10} = 50 \quad k_2$$

$$f_1 = k_1 f_0 = 150 \times 10 \times 10^3 = 15 \text{ MHz}$$

$$\Delta f_1 = k_1 \Delta f_0 = 150 \times 10 = 1.5 \text{ kHz}$$

$$f_2 = \frac{f_c}{k_2} = \frac{100 \text{ MHz}}{50} = 2 \text{ MHz}$$

$$\Delta f_2 = \frac{\Delta f}{k_2} = \frac{75 \text{ kHz}}{50} = 1.5 \text{ kHz}$$

$$f_{10} = f_1 - f_{10} = f_2 \Rightarrow f_{10} = f_1 - f_2 = (15 - 2) \text{ MHz} = 13 \text{ MHz}$$

$$C.F. = 2 \text{ MHz}$$

BW is the bandwidth of the input FM signal 

$$BW = 2(\Delta f_2 + B) = 2(1.5 + 1.5) = 6 \text{ kHz}$$

$$C.F. = 33 \text{ kHz}$$
4B.2 (25 points)
A wideband FM signal is generated from a narrowband FM generator by Armstrong's indirect method as shown below.

The desired (target) carrier frequency, $f_0$, is 100 MHz and frequency deviation $\Delta f = 75$ kHz. The message signal to be broadcast is given by

$$m(t) = \cos(15,000\pi t) + \cos(30,000\pi t)$$

The following information is given:

$$f_0 = 100 \text{ kHz}, k_1 = 150 \text{ and } k_2 = 50.$$ 

Determine the following quantities:

(a) NB-FM frequency deviation $\Delta f_0$ (4 points)
(b) Carrier frequencies $f_1$ and $f_2$ and the corresponding frequency deviations $\Delta f_1$ and $\Delta f_2$ (8 points)
(c) The oscillator frequency, $f_{LO}$ (4 points)
(d) The center frequency and bandwidth of the bandpass filter (9 points)

Bandwidth of $m(t)$ is $B = 15 \text{ kHz}$, highest frequency present

\[ \Delta f_0 = \frac{\Delta f}{k_1 \cdot k_2} = \frac{75 \times 10^3}{150 \cdot 50} = 10 \text{ kHz} \]

\[ f_1 = k_1 \cdot f_0 = 150 \times 100 \times 10^3 \cdot \frac{1}{3} = 15 \text{ MHz} \]

\[ f_2 = \frac{f_0}{k_2} = \frac{100 \text{ MHz}}{50} = 2 \text{ MHz} \]

\[ \Delta f_1 = k_1 \cdot \Delta f_0 = 1.5 \text{ kHz} \]

\[ \Delta f_2 = \frac{\Delta f_0}{k_2} = \frac{75 \text{ kHz}}{50} = 1.5 \text{ kHz} \]

\[ f_{LO} = f_1 - f_2 = (15 - 2) \text{ MHz} = 13 \text{ MHz} \]

\[ \text{BPF center freq. is } f_2 \text{ and bandwidth must equal or exceed the input FM signal bandwidth.} \]

\[ CF = 2 \text{ MHz} \]

\[ BW = 2(\Delta f_2 + B) = 2(1.5 + 15) \text{ kHz} = 33 \text{ kHz} \]
1B.3 (25 points)

Five telemetry signals, \( m_i(t) \), \( i = 1, 2, \ldots, 5 \), each of bandwidth 4 kHz, are to be transmitted simultaneously by PCM. The maximum tolerable error in the sample amplitude of the first signal \( m_1(t) \), is 0.1% of the peak amplitude; for other signals, it is 0.2% of the peak amplitude. The first signal is sampled at 50% over the Nyquist rate and the others at 25% over Nyquist rate. Framing and synchronizing requires an additional 1% extra bits. **Determine the minimum possible data rate (bits/second) and the minimum bandwidth (in Hz) required to transmit the composite signal by 4-ary signaling.**

\[
\begin{align*}
\text{m_1:} & \quad f_{s_1} = 1.5 \times f_{N_1} = 1.5 \times 8 \text{ kHz} = 12 \text{ kHz} \\
& \quad \Delta v = \frac{0.1}{100} m_p \Rightarrow \frac{m_p}{L} \Rightarrow L = 1000 \\
& \quad \Rightarrow B_{n_1} = 10 \text{ bits/10m} \\
& \quad \Rightarrow D_R_1 = f_{s_1} \cdot B_{n_1} = 12 \times 10^3 \times 10 \text{ bps} = 120 \text{ kbps}
\end{align*}
\]

\[
\begin{align*}
\text{m_2-m_5:} & \quad i = 2, 3, 4, 5 \\
& \quad f_{s_i} = 1.25 \times f_{N_i} = 1.25 \times 8 \text{ kHz} = 10 \text{ kHz} \\
& \quad \Delta v = \frac{0.2}{100} m_p \Rightarrow \frac{m_p}{L} \Rightarrow L = 500 \\
& \quad \Rightarrow B_{n_i} = 9 \text{ bits/10m} \\
& \quad \Rightarrow D_R_i = f_{s_i} \cdot B_{n_i} \text{ bps} \\
& \quad = 10 \times 10^3 \times 90 = 90 \text{ kbps}
\end{align*}
\]

**Raw Data Rate:** \[ \frac{5}{5} \cdot D_R_C = (120 + 4.90) \text{ kbps} = 124.9 \text{ kbps} \]

**Min. Transmission Data Rate:**

\[ D_R = \frac{D_R_{Raw}}{1} + \text{Overhead} = \frac{1}{100} \times D_R_{Raw} \]

\[ D_R = 1.01 \times 480 \text{ kbps} \]

\[ D_R = 484.8 \text{ kbps} \]

**Min. Data Rate in BPS:**

\[ B_{T_{min}} = \frac{1}{2} \cdot (484.8) \text{ kbps} = 121.2 \text{ kHz} \]

**M-ary Signaling:**

\[ B_{T_{min}} = \frac{1}{2} \cdot \log_2 M \text{ kbps} = \frac{1}{2} \cdot (484.8) \text{ kbps} = 121.2 \text{ kHz} \]

\[ B_{T_{min}} \]
2B.3 (25 points)

Five telemetry signals, \( m_i(t) \), \( i = 1, 2, \ldots, 5 \), each of bandwidth 8 kHz, are to be transmitted simultaneously by PCM. The maximum tolerable error in sample amplitude of the first signal \( m_1(t) \), is 0.2\% of the peak amplitude; for other signals, it is 0.1\% of the peak amplitude. The first signal is sampled at 25\% over the Nyquist rate and the others at 50\% over Nyquist rate. Framing and synchronizing requires an additional 1\% extra bits. Determine the minimum possible data rate (bits/second) and the minimum bandwidth required to transmit the composite signal by 8-ary signaling.

\[ f_{s_1} = 1.25, \quad f_N = 1.25 \times 16 \text{ kHz} = 20 \text{ kHz} \]

\[ \frac{\Delta V}{2} \leq \frac{0.2}{100} \frac{m_p}{L} \Rightarrow \frac{m_p}{L} \leq \frac{0.2}{100} \Rightarrow L = 500 \Rightarrow B_{n_1} = 9 \text{ bit/sample} \]

\[ DR_1 = f_{s_1} \cdot B_{n_1} = 20 \times 10^3 \times 9 \text{ bps} = 180 \text{ kbps} \]

\[ m_2, m_3, \ldots, m_5 \]

\[ f_{s_2} = 1.5, \quad f_N = 1.5 \times 16 \text{ kHz} = 24 \text{ kHz} \]

\[ \frac{\Delta V}{2} \leq \frac{0.1}{100} \frac{m_p}{L} \Rightarrow \frac{m_p}{L} \leq \frac{0.1}{100} \Rightarrow L = 1000 \Rightarrow B_{n_2} = 10 \text{ bit/sample} \]

\[ DR_c = f_{s_c} \cdot B_{n_c} = 24 \times 10^3 \times 10 = 240 \text{ kbps} \]

\[ \text{Raw Data Rate} \]

\[ DR_{raw} = \sum_{c=1}^{5} DR_c = (180 + 4 \times 240) \text{ kbps} = 1140 \text{ kbps} \]

\[ \text{Trans. Data Rate} \]

\[ DR = DR_{raw} + 0.1H. = 1140 \text{ kbps} + 0.1 \times 1140 \text{ kbps} = 1151.4 \text{ kbps} \]

\[ B_{T_{\min}} \text{ for 8-ary Signaling} \]

\[ M = 8 \]

\[ B_{T_{\min}} = \frac{1}{2} \cdot \frac{DR \text{ bps}}{\log_2 M} = \frac{1}{2} \cdot \frac{1151.4}{3} = 191.9 \text{ kHz} \]
3B.3 (25 points)

Five telemetry signals, \( m_i(t), i = 1, 2, \ldots, 5 \), each of bandwidth 4 kHz, are to be transmitted simultaneously by PCM. The maximum tolerable error in the sample amplitude of the first signal \( m_1(t) \), is 0.1% of the peak amplitude; for other signals, it is 0.2% of the peak amplitude. The first signal is sampled at 50% over the Nyquist rate and the others at 25% over Nyquist rate. Framing and synchronizing requires an additional 1% extra bits. **Determine the minimum possible data rate (bits/second) and the minimum bandwidth (in Hz) required to transmit the composite signal by 4-ary signaling.**
4B.3 (25 points)

Five telemetry signals, \( m_i(t) \), \( i = 1, 2 \ldots 5 \), each of bandwidth 8 kHz, are to be transmitted simultaneously by PCM. The maximum tolerable error in sample amplitude of the first signal \( m_1(t) \), is 0.2% of the peak amplitude; for other signals, it is 0.1% of the peak amplitude. The first signal is sampled at 25% over the Nyquist rate and the others at 50% over Nyquist rate. Framing and synchronizing requires an additional 1% extra bits. *Determine the minimum possible data rate (bits/second) and the minimum bandwidth required to transmit the composite signal by 8-ary signaling.*

*See 2.6.3*
1B.4 (25 points)

Two signals, m₁(t) and m₂(t), are to be sampled, quantized, and multiplexed for transmission by binary signaling. Each of the signals has a bandwidth of 2 kHz. The samples of the signal m₁(t) is uniformly distributed between -3 volts and 3 volts. The RMS value of the samples of the signal m₂(t) is one tenth the peak amplitude. Both signals are sampled at twice the Nyquist rate. Each of the signals must have a minimum SNR_Q of 46 dB. Two types of quantizers are available: (1) uniform quantizer, and (2) µ-law quantizer with µ=100. Zero overhead is assumed for framing and synchronization. Choose appropriate quantizer for each signal to minimize the data rate and determine the minimum transmission bandwidth in kHz.

Data rate is minimized by choosing the quantizer that provides the smaller # of bits/sample for each signal (while still meeting any requirements) as fₛ is fixed.

$$f_s = 2, f_N = 2.4 \text{ kHz} = 8 \text{ kHz}$$

m₁: Amplitude is uniformly distributed.

$$\left[ \frac{m_1^2}{m^2} \right] = 3$$

**Uniform:**

$$\text{SNR}_A = 6.02 B_n + 4.77 - 20\log_{10}3 \geq 46 \text{ dB}$$

$$\Rightarrow B_n = 8 \text{ bits/sample}$$

**µ-Law:**

$$\text{SNR}_A = 6.02 B_n + 4.77 - 20\log_{10}(\ln(1+\mu)) \geq 46 \text{ dB}$$

$$= 6.02 B_n + 4.77 - 20\log_{10}(110) \geq 46 \text{ dB}$$

$$= 6.02 B_n + 4.77 - 13.28 \geq 46 \text{ dB}$$

$$\Rightarrow B_n = 10 \text{ bits/sample}$$

For m₁, we choose the uniform quantizer; B_n = 8 bits/sample.

m₂: $$\left[ \frac{m_2^2}{m^2} \right] = 100.$$

**Uniform:**

$$\text{SNR}_A = 6.02 B_n + 4.77 - 20\log_{10}(100) \geq 46 \text{ dB}$$

$$\Rightarrow B_n = 11 \text{ bits/sample}$$

**µ-Law:**

$$\text{SNR}_A = 6.02 B_n + 4.77 - 20\log_{10}(\ln(1+\mu)) \geq 46 \text{ dB}$$

$$= 6.02 B_n + 4.77 - 20\log_{10}(110) \geq 46 \text{ dB}$$

$$\Rightarrow B_n = 10 \text{ bits/sample}$$

For m₂, we choose the µ-law quantizer with B_n = 10 bits/sample.

Data Rate:

$$D_{RPS} = f_s (B_n + B_{n_2}) = 8 \times 10^3 \left[ 8 + 10 \right] = 144 \text{ kbps}$$

$$B_{T_{\min}} = \frac{1}{2} D_{RPS} = \frac{1}{2} \text{ 144 kbps} = 72 \text{ kbps}$$
2B.4 (25 points)
Two signals, \(m_1(t)\) and \(m_2(t)\), are to be sampled, quantized, and multiplexed for transmission by binary signaling. Each of the signals has a bandwidth of 2 kHz. The samples of the signal \(m_1(t)\) is uniformly distributed between -4 volts and 4 volts. The RMS value of the samples of the signal \(m_2(t)\) is one seventh the peak amplitude. Both signals are sampled at twice the Nyquist rate. Each of the signals must have a minimum SNR of 46 dB. Two types of quantizers are available: (1) uniform quantizer, and (2) A-law quantizer with \(A=70\). Zero overhead is assumed for framing and synchronization. Choose appropriate quantizer for each signal to minimize the data rate and determine the minimum transmission bandwidth in kHz.

Data rate is minimized by choosing the quantizer that provides the smaller \# of bits/sample for each signal (while still meeting all requirements) as \(f_s\) is fixed.

\[
f_s = 2 \times f_n = 2 \times 2.4 \text{ kHz} = 4.8 \text{ kHz}
\]

Amplitude is uniformly distributed

\[
\Rightarrow \frac{m_{\text{max}}}{m_{\text{rms}}} = 3
\]

Uniform quantizer:

\[
\begin{align*}
\text{SNR}_{q} &= 6.02 B_n + 4.77 - 10 \log_2 \left( \frac{m_{\text{max}}}{m_{\text{rms}}} \right) \geq 46 \text{ dB} \\
&= 6.02 B_n + 4.77 - 10 \log_2 3 \geq 46 \text{ dB} \\
&\Rightarrow B_n = 8 \text{ bits/} \text{sample}
\end{align*}
\]

A-law quantizer:

\[
\begin{align*}
\text{SNR}_{q} &= 6.02 B_n + 4.77 - 20 \log_{10} (1 + \text{ln } A) \geq 46 \text{ dB} \\
&= 6.02 B_n + 4.77 - 20 \log_{10} (1 + \text{ln } 70) \approx 46 \text{ dB} \\
&= 6.02 B_n + 4.77 - 14.4 \geq 46 \text{ dB} \\
&\Rightarrow B_n = 10 \text{ bits/} \text{sample}
\end{align*}
\]

For \(m_1\), we choose the uniform quantizer with \(B_{n_1} = 8 \text{ bits/} \text{sample}\).

For \(m_2\), we choose the uniform quantizer with \(B_{n_2} = 10 \text{ bits/} \text{sample}\).

Data rate:

\[
\text{DR}_{\text{bps}} = f_s (B_{n_1} + B_{n_2}) = 8 \times 10^3 \times (8+10) = 144 \text{ kbps} \Rightarrow \text{DR}_{\text{bps}}
\]

Trans. Bw.

\[
B_{T_{\text{min}}} = \frac{1}{2} \text{DR}_{\text{bps}} = \frac{1}{2} \times 144 \times 10^3 = 72 \text{ kHz} \Rightarrow B_{T_{\text{min}}}
\]
3B.4 (25 points)
Two signals, \(m_1(t)\) and \(m_2(t)\), are to be sampled, quantized, and multiplexed for transmission by binary signaling. Each of the signals has a bandwidth of 2 kHz. The samples of the signal \(m_1(t)\) is uniformly distributed between -3 volts and 3 volts. The RMS value of the samples of the signal \(m_2(t)\) is one tenth the peak amplitude. Both signals are sampled at twice the Nyquist rate. Each of the signals must have a minimum SNR of 46 dB. Two types of quantizers are available: (1) uniform quantizer, and (2) \(\mu\)-law quantizer with \(\mu=100\). Zero overhead is assumed for framing and synchronization. Choose appropriate quantizer for each signal to minimize the data rate and determine the minimum transmission bandwidth in kHz.

See 164
4B.4 (25 points)

Two signals, \( m_1(t) \) and \( m_2(t) \), are to be sampled, quantized, and multiplexed for transmission by binary signaling. Each of the signals has a bandwidth of 2 kHz. The samples of the signal \( m_1(t) \) is uniformly distributed between –4 volts and 4 volts. The RMS value of the samples of the signal \( m_2(t) \) is one seventh the peak amplitude. Both signals are sampled at twice the Nyquist rate. Each of the signals must have a minimum SNR of 46 dB. Two types of quantizers are available: (1) uniform quantizer, and (2) A-law quantizer with \( A = 70 \). Zero overhead is assumed for framing and synchronization. Choose appropriate quantizer for each signal to minimize the data rate and determine the minimum transmission bandwidth in kHz.
**Bonus 1 (10 points)**

An FM signal

$$\phi_{FM}(t) = 5 \cos \left( 2\pi \cdot 10^6 t + \frac{5}{\pi} \sin 50,000\pi t \right)$$

is input to a square-law nonlinearity (with the characteristic: $$y = 2x^2$$, where $$x$$ is the input and $$y$$ is the output), and filtered by an ideal bandpass filter. The bandpass filter has a center frequency of 2 MHz and bandwidth of 25 kHz. Determine the output $$z(t)$$, and sketch its magnitude spectrum.

$$\phi_{FM}(t) \xrightarrow{2x^2} y(t) \xrightarrow{BP} z(t)$$

$$y(t) = 2A^2 \cos^2(\omega_c t + \gamma(t)) = A^2 \left( 1 + \cos \left[ 2\omega_c t + 2\gamma(t) \right] \right)$$

For our case, $$A = 5$$, $$\omega_c = 2\pi \cdot 10^6 \text{ rad/s}$$ (1 MHz)

and $$\gamma(t) = \sin 50,000\pi t$$

$$y(t) = 2A^2 + 2A^2 \cos \left( 2\pi \cdot 2.10^6 t + 2 \sin 50,000\pi t \right) \xrightarrow{\text{BP}} y_1(t)$$

**This term is rejected by BP**

$$y_1(t) = \frac{2A^2}{n=\infty} \sum_{n=1}^{\infty} J_n(2) \cos \left( 2\pi \left( 2.10^6 + n \cdot 25.10^3 \right) t \right)$$

$$J_0(2) = 0.224$$

Only the carrier at 2 MHz passes through the filter.

$$z(t) = 5.6 \cos \left[ 2\pi \left( 2.10^6 \right) t \right]$$

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<td>2 MHz</td>
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</table>
Bonus 2 (10 points)

An FM signal

\[ \phi_{FM}(t) = 5 \cos (2\pi \cdot 10^6 t + \sin 20,000\pi t) \]

is input to a square-law nonlinearity (with the characteristic: \( y = 2x^2 \), where \( x \) is the input and \( y \) is the output), and filtered by a bandpass filter. The center frequency of the bandpass filter is 2 MHz and the bandwidth is 10 kHz. Determine the output \( z(t) \), and sketch its magnitude spectrum.

\[ y(t) = 2 \phi_{FM}^2(t) = 2 \left[ A \cos (\omega_c t + \gamma(t)) \right]^2 = 2 \left[ \frac{A^2 + A^2}{2} \cos (2\omega_c t + 2\gamma(t)) \right] = A^2 + A^2 \cos \left[ 2\omega_c t + 2\gamma(t) \right] \]

Let \[ y_1(t) = A^2 \cos (2\omega_c t + 2\gamma(t)) \]

For our case, \[ y_1(t) = 25 \cos (2\pi \cdot 2.10^6 t + 2\sin 20,000\pi t) \]

is an FM signal with \( f_c = 2 \text{ MHz} \), \( \beta = 2 \) and \( f_m = 10 \text{ kHz} \).

\[ y_1(t) = 25 \sum_{n=-\infty}^{\infty} J_n(2) \cos (2\pi (2.10^6 + n \cdot 10^3) t) \]

The BPF is centered at 2 MHz with a BW = 10 kHz. Therefore, \( z(t) = 25 J_0(2) \cos (2\pi \cdot 2.10^6 t) \)

The "Carrier" signal only. With \( J_0(2) = 0.224 \), we have \[ z(t) = 5.6 \cos (2\pi \cdot 2.10^6 t) \]

**Graph:**

- \( z(t) \)
- \( -2\pi \cdot 2.10^6 \)
- 0
- \( 2\pi \cdot 2.10^6 \)
- \( 2\pi \cdot 2 \text{ MHz} \)
- \( 2\pi \cdot 2 \text{ MHz} \)
Bonus 3 (10 points)

An FM signal

\[ \phi_{FM}(t) = 5 \cos (2\pi \cdot 10^6 t + \beta \sin 50,000\pi t) \]

is input to a square-law nonlinearity (with the characteristic: \( y = 2x^2 \), where \( x \) is the input and \( y \) is the output), and filtered by an ideal bandpass filter. The bandpass filter has a center frequency of 2.025 MHz and bandwidth of 25 kHz. Determine the output \( z(t) \), and sketch its magnitude spectrum.

\[ y(t) = 2 \phi_{FM}^2(t) = 2 [A \cdot \cos(\omega_c t + \varphi(t))]^2 = A^2 + A^2 \cos(2\omega_c t + 2\varphi(t)) \]

Rejected by \( BPF \)

For our case,

\[ y_1(t) = 25 \cos (2\pi \cdot 2.10^6 t + 2 \beta \sin 50,000\pi t) \]

is an FM signal with a carrier frequency of 2 MHz, \( \beta = 2 \) and \( f_m = 25 \) kHz.

\[ y_1(t) = 25 \sum_{n=-\infty}^{\infty} J_n(2) \cos \left( 2\pi \left( 2.10^6 + 25 \cdot 10^3 \right) t \right) \]

The \( BPF \) is centered at 2.025 MHz with a bandwidth of 25 kHz. Thus only \( J_1(2) \cos \left( 2\pi \left( 2.10^6 + 25 \cdot 10^3 \right) t \right) \) component (i.e. the first upper sideband tone) will pass through.

\[ z(t) = 25 \cdot J_1(2) \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]

\[ z(t) = 25 \cdot J_1(2) \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]

\[ z(t) = 14.425 \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]

\[ J_1(2) = 0.577 \]

\[ z(t) = 25 \cdot 0.577 \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]

\[ z(t) = 14.425 \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]

\[ z(t) = 14.425 \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]

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\[ z(t) = 14.425 \cos \left( 2\pi \cdot 2.025 \cdot 10^6 t \right) \]
**Bonus 4 (10 points)**

An FM signal

\[ \phi_{FM}(t) = 5 \cos (2\pi \cdot 10^6 t + \sin 20,000\pi t) \]

is input to a square-law nonlinearity (with the characteristic: \( y = 2x^2 \), where \( x \) is the input and \( y \) is the output), and filtered by a bandpass filter. The center frequency of the bandpass filter is 2.03 MHz and the bandwidth is 10 kHz. Determine the output \( z(t) \), and sketch its magnitude spectrum.

\[ y(t) = 2 \left( \frac{\phi_{FM}^2(t)}{2} \right) = 2 \cdot A^2 \cos^2 (\omega_c t + \Phi(t)) \]

Rejected by BPF

\[ = A^2 + A^2 \cos (2\omega_c t + 2\Phi(t)) \]

For our case,

\[ y_1(t) = 25 \cos (2\pi \cdot 2.10^6 t + 2 \sin 20,000\pi t) \]

is an FM signal with \( f_c = 2 \text{MHz} \), \( \beta = 2 \) and \( f_m = 10 \text{kHz} \).

\[ y_1(t) = 25 \sum_{n=-\infty}^{\infty} J_n(2) \cos \left( 2\pi \left( 2.10^6 + n \cdot 10.10^3 \right) t \right) \]

The only tone that will pass through the BPF w.r.t. \( \omega_f = 2.03 \text{MHz} \) and \( \omega_m = 10 \text{kHz} \) will be the tone at 2.03 MHz.

\[ z(t) = 25 \cdot J_3(2) \cos \left( 2\pi \left( 2.10^6 + 30 \cdot 10^3 \right) t \right) \]

\[ J_3(2) = 0.129 \]

\[ z(t) = 3 \cdot 2.25 \cos \left( 2\pi \left( 2.10^6 + 30 \cdot 10^3 \right) t \right) \]

\[ \hat{z}(\omega) \]
\( J_n(\beta) \) Table

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