Frequency Reuse and the Cellular Concept
Old Wireless Systems

- A high powered transmitter with an antenna mounted on a tall tower achieves good coverage

- **Problem**: These systems were not capable of meeting the increasing demand for mobile services

- **Challenge**: To resolve the above problem by achieving high capacity with limited radio spectrum, while at the same time covering very large area
The Cellular Concept

- The cellular concept is a major breakthrough in solving the problem of spectral limitation and user capacity
  - It offers a very high capacity in a limited spectrum allocation without any major technological changes

- Under this concept a single, high power transmitter (large cell) is replaced with many low power transmitters (small cells), each providing coverage to only a small portion of the service area
The Cellular Concept (continued)

- Each cellular base station is allocated with a group of radio channels (a portion of the total number of channels available to the entire system) to be used within a small geographic area called a cell
  - For clarity, concentrate on existing AMPS (which is a FDMA) system, where a channel is a frequency bin
The Cellular Concept (continued)

- By limiting the coverage area to within the boundaries of a cell, the same group of channels may be used to cover different cells
- Cells are separated from one another by distances large enough to keep interference levels within tolerable limits
- **Frequency Reuse/Frequency Planning:** The design process of selecting and allocating channel groups for all the cellular base stations within a system
The Cellular Concept (continued)

- Neighboring base stations are assigned with different groups of channels so that all the available channels are assigned to a relatively small number of neighboring base stations
- The base station antennas are designed to achieve the desired coverage within the particular cell
Frequency Reuse Concept

Cellular Frequency Reuse
Frequency Reuse Concept (continued)

- Cells labelled with the same letter use the same group of channels
- The frequency reuse plan is overlaid upon a map to indicate where different frequency channels are used
- The hexagonal cell shape is conceptual and is the simplistic model of the radio coverage for each base station
- The actual radio coverage of a cell is known as the footprint and is amorphous in nature
Why Hexagonal Cell?

1. To cover an entire region without overlap and with equal area.

2. To serve the weakest mobiles within the footprint which are located at the edge of the cell.
   
   - For a given distance between the center of a polygon and its farthest perimeter points, the hexagon has larger area than those of a square and an equilateral triangle.
Why Hexagonal Cell? (continued)

3 By using the hexagon geometry, the fewest number of cells can cover a geographic region

4 The hexagon closely approximates a circular radiation pattern which would occur for an omni-directional base station antenna and free space propagation
Definitions

- **Center-excited Cells:** Base station transmitters are depicted as being in the center of the cell
  - Omni-directional antennas are used

- **Edge-excited Cells:** Base station transmitters are depicted as being on three of the six cell vertices
  - Sectored directional antennas are used

- **Note:** Practically, base stations are positioned up to one-fourth the cell radius away from the ideal location
Capacity of a Cellular System

- Given:
  - $S = \text{Total number of duplex channels}$
  - $k = \text{Number of channels in a cell, } k < S$
  - $N = \text{Total number of cells among which } S \text{ channels are divided}$
    * **Note:** $N$ cells which collectively use the complete set of available frequencies is called a *cluster*
    * The term $N$ is called the *cluster size*

- $M = \text{Number of times a cluster is replicated within the system}$
Capacity of a Cellular System (continued)

- Capacity can be measured by the total number of duplex channels, $C$, and is given by

$$C = MkN = MS$$

- **Remark:** The capacity of a cellular system is directly proportional to the number of times a cluster is replicated in a fixed service area.
Capacity vs Cluster Size

- **Remark:**
  - If $N \downarrow \rightarrow$ more clusters are required to cover a given area, i.e., $M \uparrow \rightarrow C \uparrow$
  - **Trade-off:** A small cluster size indicates that co-channel cells are located much closer together $\Rightarrow$ more interference

- **Definition:** The *frequency reuse factor* is given by $1/N$, since each cell within a cluster is only assigned $1/N$ of the total available channels in the system
\textbf{Number of Cells per Cluster}

- \textbf{Theorem:} In order to tessellate - to connect without gaps between adjacent cells - the geometry of hexagons is such that the number of cells per cluster, \( N \), can only have values which satisfy the following equation

\[
N = i^2 + ij + j^2
\]

where \( i \) and \( j \) are non-negative integers
Nearest Co-channel Neighbors

- To find the nearest co-channel neighbors of a particular cell
  - Move \( i \) cells along any chain of hexagons
  - Turn 60 degrees counter-clockwise and move \( j \) cells

- **Example:** For \( i = 2 \) and \( j = 3 \), \( N = 19 \)
Example of $i$ and $j$
Proof

• Given:
  - $R =$ Cell radius and $D =$ Distance between centers of the nearest co-channel cells
  - $\alpha =$ Distance between adjacent cells

• The area of a small hexagon (cell) is
  
  $$A_{\text{small}} = kR^2$$

• The area of a large hexagon (that joins the center of all co-channel cells) is
  
  $$A_{\text{large}} = kD^2$$

where $k$ is a constant
Proof (continued)
Proof (continued)

- Using the geometry of triangles, we get the following two equations

\[ \alpha^2 = R^2 + R^2 - 2R^2 \left( -\frac{1}{2} \right) = 3R^2 \]

\[ D^2 = i^2 \alpha^2 + j^2 \alpha^2 - 2ij \alpha^2 \cos 120^\circ \]

\[ = \alpha^2 (i^2 + ij + j^2) \]

\[ = 3R^2 (i^2 + ij + j^2) \]
Proof (continued)

- From the previous slide

\[ D^2 = 3R^2 (i^2 + ij + j^2) \]

\[ \frac{D^2}{R^2} = 3 (i^2 + ij + j^2) \]

- The ratio between the area of the large hexagon and the area of the small hexagon is

\[ \frac{A_{\text{large}}}{A_{\text{small}}} = \frac{kD^2}{kR^2} = \frac{D^2}{R^2} = 3 (i^2 + ij + j^2) \]
Proof (continued)

- From the figure

\[ A_{\text{large}} = NA_{\text{small}} + \frac{1}{3}NA_{\text{small}} \times 6 = 3NA_{\text{small}} \]

\[ \frac{A_{\text{large}}}{A_{\text{small}}} = 3N \]

- Therefore

\[ 3N = 3 \left( i^2 + ij + j^2 \right) \]

\[ \Downarrow \]

\[ N = i^2 + ij + j^2 \]
Channel Assignment Strategies

- **Objective:**
  - To minimize interference $\rightarrow$ capacity improvement
- Two types of channel assignment strategies
  - Fixed Channel Assignment Strategy
  - Dynamic Channel Assignment Strategy
Fixed Channel Assignment Strategy

- Each cell is assigned with a predetermined set of voice channels
- Any call attempt within the cell can only be served by the unused channels in that particular cell
- If all the channels in that cell are occupied, the call is blocked
Fixed Channel Assignment Strategy (continued)

- **Borrowing Strategy:** A cell is allowed to borrow channels from a neighboring cell if all of its channels are already occupied and if it does not disrupt or interfere with any of the calls in progress in the donor cell.
Dynamic Channel Assignment (DCA) Strategy

- Channels are not allocated to different cells permanently
- Each time a call request is made, the serving base station requests a channel from the mobile switching center (MSC)
- The switch then allocates a channel to the requested cell following an algorithm
DCA (continued)

- The DCA algorithm takes into account
  - the likelihood of future blocking within the cell
  - the frequency of use of the candidate channel
  - the reuse distance of the channel
  - other cost functions
DCA Strategy (continued)

- DCA reduces the likelihood of blocking, thus increases the trunking capacity of the system

- DCA requires the MSC to collect real-time data on channel occupancy, traffic distribution, and radio signal strength indications (RSSI) of all channels on a continuous basis

- DCA increases the storage and computational load on the system but provides the advantage of increased channel utilization and decreased probability of a blocked call
Interference and System Capacity

- Sources of interference include
  1. another mobile in the same cell
  2. a call in progress in the neighboring cell
  3. other base stations operating in the same frequency band
  4. any non-cellular system which inadvertently leaks energy into the cellular frequency band
Impact of Interference

- On voice channels:
  - it causes cross talk

- On control channels:
  - it leads to missed and blocked calls
Types of Interference

- Two major types of system-generated cellular interference:
  1. Co-channel Interference
  2. Adjacent Channel Interference
Co-channel Interference

- **Co-channel Cells**: The cells in a given coverage area that use the same set of frequencies

- **Co-channel Interference**: Interference between signals from co-channel cells
Co-channel Interference Management

- Co-channel interference cannot be reduced by increasing the carrier transmit power, because it increases interference to neighboring co-channel cells

- It can be reduced by separating the co-channel cells by a minimum distance to provide sufficient isolation due to propagation
Co-channel Reuse Ratio

- Co-channel interference is a function of co-channel reuse ratio
- Co-channel reuse ratio $Q$ is expressed as
  
  $$Q = \frac{D}{R}$$

  where $R$ is the radius of the cell and $D$ is the distance between centers of the nearest co-channel cells
Co-channel Reuse Ratio (continued)

- Co-channel reuse ratio can also be expressed as a function of the cluster size
  - Since

\[
D^2 = \alpha^2 (i^2 + ij + j^2) \\
= 3R^2 N
\]

- we have

\[
Q = \frac{D}{R} = \sqrt{3N}
\]
Co-channel Reuse Ratio and System Capacity

- The value of $Q$ is important because it affects both the traffic carrying capacity of a cellular system and co-channel interference.

- Assume total RF channels, $S$ are constant. Since $C = MS$,
  - $Q \downarrow \Rightarrow N \downarrow \Rightarrow M \uparrow \Rightarrow C \uparrow$
  - Conversely
    $Q \uparrow \Rightarrow N \uparrow \Rightarrow M \downarrow \Rightarrow C \downarrow$
The Trade-off

- A small value of $Q$ provides larger capacity since the cluster size $N$ is small, whereas a large value of $Q$ improves the transmission quality, due to a smaller level of co-channel interference.
- An actual cellular design demands a trade-off between these two objectives.
**Signal-to-Interference Ratio (SIR)**

- **Consider** the forward channel (base station to mobile)

- The SIR for a mobile receiver in the forward channel can be expressed as

\[
\frac{S}{I} = \frac{S}{\sum_{i=1}^{i_0} I_i}
\]

where

- \( S \) = received power from the desired base station
- \( I_i \) = interference power caused by the \( i \)th interfering co-channel cell base station
- \( i_0 \) = number of co-channel interfering cell
Simple Trigonometry

$$d = \sqrt{H^2 + X^2}$$

$H = \text{Base station height}$

$h = \text{Mobile station height}$

$X = \text{Distance between the base station and mobile}$
• Propagation measurements in a mobile radio channel show that the average received signal strength at any point decays as a power of law of the distance separation between a transmitter and receiver.

• The average received power can be expressed as

\[ P_r \propto \frac{P_T}{(\sqrt{H^2 + X^2})^n} \]

* \( P_T = \) transmitted power
* \( n = \) path loss exponent and \( 2 \leq n \leq 5 \)
• **Remark:** The path loss exponent depends on the terrain environment

• Interference from the $i$th co-channel base station can be expressed as

$$I_i = \frac{P_{T_i}}{D_i^n}$$

where $D_i = \text{distance between the mobile and the } i\text{th co-channel base station}$
Assumptions

- $P_{T_i} = P_T$ for all $i$ and the transmitter power of the desired base station is $P_T$

- Six closest cells are close enough to create significant interference and they are all approximately equi-distant from the desired base station and $D_i = D$ or all $i$
SIR (continued)

- From the assumptions of the previous slide

\[
\frac{S}{I} = \frac{\frac{P_T}{R^n}}{6 \frac{P_T}{D^n}} = \frac{1}{6} \left( \frac{D}{R} \right)^n = \frac{1}{6} Q^n = \frac{1}{6} (\sqrt{3N})^n
\]

- **Example:** The U.S. AMPS cellular system which uses FM and 30 kHz channels requires

\[
\frac{S}{I} \geq 18 \text{ dB} = 63.1
\]
**Example:** When the path loss exponent is 4

\[
\frac{S}{I} = \frac{(\sqrt{3N})^4}{6} \geq 63.1
\]

\[\Rightarrow N \geq 6.49\]

- The above equation implies that a minimum cluster size of 7 is required to meet an SIR requirement of 18 dB

- **Note:** For a seven cell reuse pattern, co-channel reuse ratio is

\[Q = \sqrt{3N} = 4.6\]
SIR (continued)

- **Worst Case Scenario:**
  - The mobile is located at the cell boundary

- Assume $N = 7$ and consider the first tier of co-channel interfering cells
  - Here, the mobile is a distance $(D - R)$ from the two nearest co-channel interfering cells and approximately $(D + R)$ and $D$ from the other interfering cell in the first tier
Worst Case Scenario

First Tier of Co-channel Cells for a Cluster Size of N=7
SIR in the Worst Case

- The signal-to-interference ratio for the worst case can be closely approximated as

$$\frac{S}{I} \approx \frac{\frac{P_T}{R^n}}{2 \frac{P_T}{(D-R)^n} + 2 \frac{P_T}{D^n} + 2 \frac{P_T}{(D+R)^n}}$$

$$= \frac{R^{-n}}{2 (D - R)^{-n} + 2D^{-n} + 2 (D + R)^{-n}}$$

$$= \frac{1}{2 \left(\frac{D}{R} - 1\right)^{-n} + 2 \left(\frac{D}{R}\right)^{-n} + 2 \left(\frac{D}{R} + 1\right)^{-n}}$$

$$= \frac{1}{2 (Q - 1)^{-n} + 2Q^{-n} + 2 (Q + 1)^{-n}}$$
SIR in the Worst Case (continued)

- For $N = 7$, the co-channel reuse ratio $Q$ is 4.6. Using the equation of the previous slide and assuming $n = 4$, the worst case SIR is approximated as

$$\frac{S}{I} = 49.56 = 17\ \text{dB}$$

- Recall that the required SIR should be 18 dB
Remarks

- For a 7-cell cluster, the SIR is slightly less than 18 dB for the worst case.

- To design a cellular system for proper performance in the worst case, it would be necessary to increase $N$ to the next larger size, which is 12.

- **Problem:** It results in a significant decrease in capacity, since 12-cell reuse offers a spectrum utilization of $1/12$ within each cell, whereas 7-cell reuse offers a spectrum utilization of $1/7$. 
SIR in the Worst Case (continued)

- In practice, a capacity reduction of $7/12$ would not be tolerable to accommodate for the worst case situation which rarely occurs.

- **Conclusion:** Co-channel interference determines link performance, which in turn dictates the frequency reuse plan and the overall capacity of cellular systems.
Adjacent Channel Interference (ACI)

- Interference resulting from signals which are adjacent in frequency to the desired signal is called *adjacent channel interference*.

- Adjacent channel interference results from imperfect receiver filters which allow nearby frequencies to leak into the passband.

- The problem can be particularly serious if an adjacent channel user is transmitting in very close range to a subscriber’s receiver, while the receiver attempts to receive a base station on the desired. This is referred to as *near-far effect*. 
Minimizing ACI

- Adjacent channel interference can be minimized through careful filtering and channel assignments
  - Channels are allocated such that the frequency separation between channels in a given cell is maximized
• By sequentially assigning successive channels in the frequency band to different cells, many cell allocation schemes are able to separate adjacent channels in a cell by as many as $N$ channel bandwidths, where $N$ is the cluster size.
Minimizing ACI Further

- Some channel allocation schemes also prevent a secondary source of adjacent channel interference by avoiding the use of adjacent channels in neighboring cell sites.
Hand-off

- When a mobile moves into a new cell from a different cell then
  - the operation of transferring a call in progress by the mobile switching center (MSC) to a new channel belonging to a new base station is called Hand-off

- **Note:** Both the voice and control channels associated with the new base station are different from those of the old base station
Hand-off (continued)

- Hand-offs must be
  - performed successfully
  - performed as infrequently as possible
  - imperceptible to the users

- An optimal signal level to initiate a hand-off is specified to meet the above requirements
Hand-off Threshold

- A minimum usable signal level for acceptable voice quality at the base station receiver is normally taken as between -90 dBm and -100 dBm

- A slightly stronger signal level than the minimum usable signal level is used as a threshold at which a hand-off is made
**Hand-off Margin**

- Hand-off margin is given by

\[ \Delta = P_{r, \text{hand-off}} - P_{r, \text{minimum usable}} \]

- \( \Delta \) cannot be too large or too small
- \( \Delta \) is too large \( \Rightarrow \) unnecessary hand-offs burden the MSC
- \( \Delta \) is too small \( \Rightarrow \) insufficient time to complete a hand-off before a call is lost due to weak signal conditions
When to Hand-off?

- **Concerns:** In deciding when to hand-off, it is important to ensure that
  - the drop in the measured signal level is not due to momentary fading
  - the mobile is actually moving away from the serving base station

- **Technique:**
  - the base station monitors the signal level for a certain period of time before a hand-off is initiated
When to Hand-off? (continued)

- The monitoring of signal level should be optimized to
  - avoid unnecessary hand-offs
  - ensure that hand-offs are completed before a call is terminated due to poor signal level
**Dwell Time**

- The time over which a call may be maintained within a cell, without hand-off, is called the *dwell time*.

- The dwell time of a particular user is governed by the following factors:
  - propagation
  - interference
  - distance between the subscriber and the base station
  - other time varying effects
Dwell Time (continued)

- The statistics of dwell time vary greatly depending on
  - the speed of the user
  - the type of radio coverage
- In mature cell which provide coverage for vehicular highway users, the dwell time for an arbitrary user is a random variable with a distribution that is highly concentrated about the mean dwell time
- In cluttered micro-cell environments, the dwell time varies largely around the mean
Hand-off in 1-G Cellular Systems

- Signal strength measurements are made by the base stations and supervised by the MSC
- Each base station constantly monitors the signal strengths of all its reverse voice channels to determine the relative location of each mobile user with respect to the base station tower
Hand-off in 1-G Cellular Systems (continued)

- A spare receiver in each base station, called the *locator receiver*, is used to determine the signal strengths of mobile users which are in neighboring cells.

- The locator receiver is controlled by the MSC and based on the locator receiver signal strength information from each base station, the MSC decides if a hand-off is necessary or not.
Mobile Assisted Hand-off (MAHO)

- It is used in second generation cellular systems that use digital TDMA technology
- Every mobile station measures the received power from surrounding base stations and continually reports the results of these measurements to the serving base station
- A hand-off is initiated when the power received from the base station of a neighboring cell begins to exceed the power received from the current base station by a certain level or for a certain period of time
MAHO (continued)

- The MAHO method enables the call to be handed over between base stations at a much faster rate than in the first generation analog systems, because
  - hand-off measurements are made by each mobile
  - the MSC no longer constantly monitors signal strengths
- MAHO is particularly suited for micro-cellular environments where hand-offs are more frequent
Inter-system Hand-off

- It is necessary if, during the course of a call, a mobile moves from one cellular system to a different cellular system controlled by a different MSC

- An MSC engages in an inter-system hand-off when
  - a mobile signal becomes weak in a given cell
  - the MSC cannot find another cell within its system to which it can transfer the call in progress
Inter-system Hand-off (continued)

- The following issues must be addressed when implementing an inter-system hand-off
  - A local call may become a long-distance call as the mobile moves out of its home system and becomes a roamer in a neighboring system
  - Compatibility between two MSCs must be determined before implementing an inter-system handoff
Trunking

- **Objective:** To allow a large number of users to share the relatively small number of channels in a cell by providing access to each user, on demand, from a pool of available channels

- In a trunked radio system,
  - each user is allocated a channel on a per call basis
  - upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels
Trunking (continued)

- In a trunked mobile radio system, when a particular user requests service and all of the radio channels are already in use, the user is **blocked, or denied access** to the system.

- In some systems, a queue is used to hold the requesting users until a channel becomes available.
Grade of Service (GOS)

- The *Grade of Service* (GOS) is a measure of the ability of a user to access a trunked system
- The GOS is a benchmark used to define the desired performance of a particular trunked system by specifying a desired likelihood of a user obtaining channel access given a specific number of channels available in the system
- GOS is typically given as the likelihood that a call is blocked, or the likelihood of a call experiencing a delay greater than a certain queuing time
Definitions

- **Set-up Time:** The time required to allocate a trunked radio channel to a requesting user

- **Holding Time:** Average duration of a typical call and is denoted by $H$ (in seconds)

- **Traffic Intensity:** Measure of channel time utilization, which is the average channel occupancy measured in Erlangs and is denoted by $A$
  - it is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels
Definitions (continued)

- **Erlang**: One Erlang represents the amount of traffic intensity carried by a channel that is completely occupied (i.e. 1 call-hour per hour or 1 call-minute per minute)
  - **Example**: a radio channel that is occupied for thirty minutes during an hour carries 0.5 Erlangs of traffic

- **Load**: It is the traffic intensity across the entire trunked radio system, measured in Erlangs
Traffic Intensity

- Each user generates a traffic intensity of $A_u$ Erlangs given by
  
  $$A_u = \lambda H$$

  where $\lambda$ = average number of call requests per unit time

- For a system containing $U$ users and an unspecified number of channels, the total offered traffic intensity $A$, is given as
  
  $$A = U A_u$$
Traffic Intensity (continued)

- In a $C$ channel trunked system, if the traffic is equally distributed among the channel, then the traffic intensity per channel, $A_c$, is given as

$$A_c = UA_u/C$$
Remarks

• The offered traffic is not necessarily the traffic which is carried by the trunked system, only that which is offered to the trunked system

• When the offered traffic exceeds the maximum capacity of the system, the carried traffic becomes limited due to the limited number of channels
More Remarks

- The maximum possible carried traffic is the total number of channels, $C$, in Erlangs
- The AMPS cellular system is designed for a GOS of 2 percent blocking, i.e.
  - the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour
Blocked Calls Cleared

- It is a type of trunked system which offers no queuing time
- For every user who requests service, it is assumed
  - there is no setup time
  - the user is given immediate access to a channel if one is available
  - if no channels are available, the requesting user is blocked without access and free to try again later
Blocked Calls Cleared (continued)

- Major assumptions of this trunking system are
  - there are an infinite number of users
  - there are memoryless arrivals of requests ⇒ all users, including block users may request a channel at any time
  - calls arrival process is Poisson process
  - the probability of a user occupying a channel is exponentially distributed
  - there are a finite number of channels available in the trunking pool⇒ this is known as $M/M/m$ queue
Erlang B Formula

- It determines the probability that a call is blocked and is a measure of the GOS for a blocked calls cleared system.
- Erlang B formula is given by

\[ Pr[\text{blocking}] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}} = \text{GOS} \]

where

- \( C \) = number of trunked channels offered by a trunked radio system
- \( A \) = total offered traffic
Erlang B Formula (continued)

• Comments:
  – for trunked systems with finite users, the resulting expressions are much more complicated than the Erlang B result
  – for typical trunked systems which have users that outnumber available channels by order of magnitude
    * Erlang B formula provides a conservative estimate of the GOS
Blocked Calls Delayed

- This type of trunked system provides queue to hold calls which are blocked
  - if a channel is not available immediately, the call request may be delayed until a channel becomes available

- GOS is defined as the probability that a call is blocked after waiting a specific length of time in the queue
  - GOS is found by first determining the likelihood that a call is initially denied access to the system
**Erlang C Formula**

- It is used to determine the likelihood of a call not having immediate access to a channel and is given by

\[
Pr[\text{delay} > 0] = \frac{A^C}{A^C + C! \left(1 - \frac{A}{C}\right) \sum_{k=0}^{C-1} \frac{A^k}{k!}}
\]
Blocked Calls Delayed (continued)

- The probability that a delayed call is forced to wait more than \( t \) seconds is given by

\[
Pr[\text{delay} > t] = Pr[\text{delay} > 0]Pr[\text{delay} > t | \text{delay} > 0] \\
+ Pr[\text{delay} = 0]Pr[\text{delay} > t | \text{delay} = 0] \\
= Pr[\text{delay} > 0] \exp \left( -\frac{(C - A)}{H} t \right)
\]
• The average delay of a blocked calls delayed trunking system is given by

\[
E[\text{delay}] = E[D] \\
= \int_0^\infty Pr[D > t]dt \\
= \int_0^\infty Pr[D > 0] \exp\left(-\frac{(C - A)}{H}t\right) dt \\
= Pr[D > 0] \int_0^\infty \exp\left(-\frac{(C - A)}{H}t\right) dt \\
= Pr[D > 0] \frac{H}{C - A} \\
= Pr[D > 0] E[D|D > 0]
\]
Example

- **Given:**
  - Three competing trunked mobile networks systems A, B and C provide cellular service in an urban area with population of 2 million.
  - System A has 394 cells with 19 channels each
  - System B has 98 cells with 57 channels each
  - System C has 49 cells with 100 channels each
  - Probability of blocking is 2 percent
  - Each user averages 2 calls per hour at an average call duration of 3 minutes
Example (continued)

- **Find:** The percentage of the total number of users supported by the three systems with a 2 percent blocking probability

- **Assumption:** All three systems are operated at maximum capacity
System A

- Traffic intensity per user is

\[ A_u = \lambda H \]

\[ = 2 \times (3/60) \]

\[ = 0.1 \text{ Erlangs} \]
System A (continued)

- For GOS = 0.02 and \( C = 19 \), from Erlang B chart, the total carried traffic, \( A \), is obtained as 12 Erlangs.

- The number of users that can be supported per cell

\[
U = \frac{A}{A_u}
\]

\[
= \frac{12}{0.1}
\]

\[
= 120
\]

- Total number of subscribers that can be supported by system A is equal to \( 120 \times 394 = 47280 \)
System B

- For GOS = 0.02 and C = 57, from the Erlang B chart, the total carried traffic, A, is obtained to be 45 Erlangs.

\[
U = \frac{A}{A_u} \\
= \frac{45}{0.1} \\
= 450
\]

- For 98 cells, total number of subscribers supported by system B is equal to \(450 \times 98 = 44100\)
System C

- Similarly for System C, total carried traffic, $A$, is 88 Erlangs

$$U = \frac{A}{A_u}$$

$$= \frac{88}{0.1}$$

$$= 880$$

- For 49 cells, total number of subscribers supported by cell C is equal to $880 \times 49 = 43120$
Example (continued)

- The percentage market penetration for system A = \( \frac{47280}{2000000} = 2.36\% \)
- The percentage market penetration for system B = \( \frac{44100}{2000000} = 2.205\% \)
- The percentage market penetration for system C = \( \frac{43120}{2000000} = 2.156\% \)
- Total market penetration by the three system is \( \frac{(47280 + 44100 + 43120)}{2000000} = 6.725\% \)
Trunking Efficiency

- It is a measure of the number of users which can be offered a particular GOS with a particular configuration of fixed channels

- The way in which channels are grouped can substantially alter the number of users handled by a trunked system
Example 2

- Given:
  - Radius of a hexagonal cell, \( R = 1.387 \) km
  - Area covered per cell, \( A_{\text{small}} = 2.598 \times (1.387)^2 = 5 \) sq km
  - Number of cells per cluster, \( N = 4 \)
  - Total number of channels = 60
  - Load per user = 0.029 Erlangs
  - Average number of call requests per hour, \( \lambda = 1 \)
  - Grade of Service (GOS) = 5%
Example 2 (continued)

• Find:
  1. Number of users per sq km supported by this system
  2. Probability that a delayed call will have to wait for more than 10 seconds
  3. Probability that a call will be delayed for more than 10 seconds
Solutions

- Number of channels per cell, $C = 60/4 = 15$ channels
- From Erlang C chart, for GOS of 5 percent with $C = 15$, traffic intensity = 8.8 Erlangs
- Therefore, number of users is

\[
A = \frac{\text{total traffic intensity}}{\text{traffic per user}}
\]

\[
= \frac{8.8}{0.029}
\]

\[
= 303 \text{ users/cell}
\]

\[
= 60 \text{ users/sq km}
\]
Solutions (continued)

- Holding time is

\[ H = \frac{A_u}{\lambda} = 0.029 \text{ hour} = 104.4 \text{ seconds} \]

- The probability that a delayed call will have to wait for more than 10 seconds

\[
Pr[\text{delay} > 10|\text{delay}] = \exp\left(- (C - A) \frac{t}{H}\right) \\
= \exp\left(- (15 - 8.8) \frac{10}{104.4}\right) \\
= 0.5502 
\]
Solutions (continued)

- Probability that a call is delayed more than 10 seconds

\[
Pr[\text{delay} > 10] = Pr[\text{delay} > 0]Pr[\text{delay} > 10|\text{delay}]
\]
\[
= 0.05 \times 0.5502
\]
\[
= 0.0276
\]
Improving Capacity in Cellular Systems

- **Objective:**
  - to provide more channels per unit coverage area

- Three techniques are used
  1. Cell Splitting
  2. Sectoring
  3. Coverage Zone Approaches
Cell Splitting

- It is the process of subdividing a congested cell into smaller cells (*micro-cells*), each with its own base station and a corresponding reduction in antenna height and transmitter power.

- *Micro-cells* have smaller radius than the original cells ⇒ capacity increases due to the additional number of channels per unit area.

- Cell splitting keeps the channel allocation scheme unchanged.
Cell Splitting (continued)

- Consider a circle with radius $R$
- The area covered by such a circle is four times as large as the area covered by a circle with radius $R/2 \Rightarrow$ increased number of cells by a factor of 4
  - Thus, number of clusters $\uparrow \Rightarrow$ number of channels $\uparrow \Rightarrow$ capacity $\uparrow$
Cell Splitting (continued)

- In the figure
  - original base stations are surrounded by the new microcell base stations such a way that
    \[ R_{\text{new}} = \frac{R_{\text{old}}}{2} \]
  - the microcell base station \( G \) is placed half way between two larger stations utilizing the same channel set \( G \)
Cell Splitting (continued)

- **Goal:**
  - To find the reduced transmit power of the new microcells
- The transmit power of the new cells can be found by examining the received power $P_r$ at the new and old cell boundaries equal and setting them equal to each other
Cell Splitting (continued)

- We want

\[
\left( \frac{S}{I} \right)_{\text{old}} = \left( \frac{S}{I} \right)_{\text{new}}
\]

\[
\left( \frac{S}{I} \right)_{\text{old}} = \frac{P_r}{\text{co-channel interference + other interference}}
\]

\[
\approx \frac{P_{t,\text{old}}}{R_{\text{old}}^n} \left[ \frac{2}{(D_{\text{old}} - R_{\text{old}})^{-n}} + \frac{2}{D_{\text{old}}^{-n}} + \frac{2}{(D_{\text{old}} + R_{\text{old}})^{-n}} \right] + \sigma^2
\]

\[
= \frac{1}{2 \left[ (Q_{\text{old}} - 1)^{-n} + Q_{\text{old}}^{-n} + (Q_{\text{old}} + 1)^{-n} \right]} + \frac{\sigma^2 R_{\text{old}}^n}{P_{t,\text{old}}}
\]
Cell Splitting (continued)

- Co-channel reuse ratio $Q$ is the same for both the old system and the new system

$$Q_{\text{new}} = \frac{D_{\text{new}}}{R_{\text{new}}} = \frac{D_{\text{old}}/2}{R_{\text{old}}/2} = Q_{\text{old}}$$

- After cell splitting the new SIR can be expressed as

$$\left(\frac{S}{I}\right)_{\text{new}} = \frac{1}{2 \left[(Q_{\text{old}} - 1)^{-n} + Q_{\text{old}}^{-n} + (Q_{\text{old}} + 1)^{-n}\right] + \frac{\sigma^2 R_{\text{new}}^n}{P_{t,\text{new}}}}$$
Cell Splitting (continued)

- Equating the old SIR and the new SIR we get

\[
\left( \frac{S}{I} \right)_{\text{old}} = \left( \frac{S}{I} \right)_{\text{new}}
\]

\[
\Rightarrow \frac{\sigma^2 R^n_{\text{old}}}{P_{t,\text{old}}} = \frac{\sigma^2 R^n_{\text{new}}}{P_{t,\text{new}}}
\]

\[
\Rightarrow \frac{P_{t,\text{new}}}{P_{t,\text{old}}} = \frac{R^n_{\text{new}}}{R^n_{\text{old}}} = \left( \frac{1}{2} \right)^n
\]
Cell Splitting (continued)

- From the previous slide, for \( n = 4 \)

\[
\frac{P_{t,\text{new}}}{P_{t,\text{old}}} = \frac{1}{16} = -12 \text{ dB}
\]

- In other words, the transmit power must be reduced by 12 dB in order to fill in the original coverage area with microcells while maintaining the same SIR requirement
Limitations of Cell Splitting

• In practice, not all cells are split at the same time
  – **Problem 1**: Different cell sizes will exist simultaneously
  – **Problem 2**: Channel assignments become more complicated
  – **Problem 3**: The smaller the cells, the more handoffs will occur
Problem 1

- When there are two cell sizes in the same region, the same transmit power cannot be used for all cells
  - if the original larger transmit power is used for all new cells, some channels used by the larger cells would not be sufficiently separated from the co-channel cells $\Rightarrow$ damage to the larger cells
  - if the new reduced transmit power is used in original cells, there would be parts of larger cells left unserved
Partial Solution to Problem 1

• Solutions: Channels in the old cell are broken down into two channel groups
  – one of the groups corresponds to the smaller cell reuse requirements and the other corresponds to the larger cell reuse requirements
  – the larger cell is usually dedicated to high speed traffic so that handoffs occur less frequently → cause interference to smaller cells
Sectoring

- *Sectoring* is a technique for decreasing co-channel interference and thus increasing system capacity by replacing a single omni-directional antenna at the base station by several directional antennas, each radiating within a specified sector
  - a given cell will receive interference and transmit with only a fraction of the available co-channel cells
• The factor by which the co-channel interference is reduced depends on the amount of sectoring used

• Two types of sectoring are used
  – three 120° sectors
  – six 60° sectors
Three Sector Case

- Assuming 7-cell reuse, for a three sector case, the number of interferers in the first tier is reduced from 6 to 2
  - the number of interferers in the first tier is reduced from 6 to 2
Three Sector Case (continued)

- SIR for the worst case scenario for a three sector antenna is

\[
\frac{S}{I} = \frac{R^{-n}}{(D + 0.7R)^{-n} + D^{-n}} = \frac{1}{(Q + 0.7)^{-n} + Q^{-n}}
\]

- For a 7-cell reuse pattern \( Q = \sqrt{3N} = 4.6 \)

- Assuming path-loss exponent \( n = 4 \), SIR for the worst case scenario is 24.5 dB
Three Sector Case (continued)

- In reality S/I could be 6 dB weaker than 24.5 dB in a heavy traffic area as a result of irregular terrain condition and imperfect site location
  - the remaining S/I of 18.5 dB is still greater than the required S/I of 18 dB

- Recall that for the omni-directional antennas under the worst case scenario, a 12-cell reuse is required to achieve the minimum required S/I of 18 dB
Six Sector Case

- Assuming 7-cell reuse pattern and path-loss exponent of 4, S/I under the worst case scenario for a six sector case is

\[
\frac{S}{I} = \frac{R^{-4}}{(D + 0.7R)^{-4}}
\]

\[
= (Q + 0.7)^4
\]

\[
= (4.6 + 0.7)^4
\]

\[
= 794
\]

\[
= 29 \text{ dB}
\]

- **Remark:** When heavy traffic occurs, the six sector configuration can be used to reduce co-channel interference
Disadvantages of Sectoring

- It results in
  - an increased number of antennas at each base station
  - a decrease in trunking efficiency due to channel sectoring at the base station
  - an increased number of hand-offs due to the reduced coverage area of a particular group of channels
Disadvantages of Sectoring (continued)

- The decreased trunking efficiency due to sectoring can be shown by comparing the probability of blocking for both the sectored and unsectored case.

- Probability of blocking for a six sector case is

\[
P_B(60^\circ) = \frac{(A/6)^{C/6}}{(C/6)!} \sum_{k=0}^{C/6} \frac{(A/6)^k}{k!}
\]

- Probability of blocking for a three sector case is

\[
P_B(120^\circ) = \frac{(A/3)^{C/3}}{(C/3)!} \sum_{k=0}^{C/3} \frac{(A/3)^k}{k!}
\]
Disadvantages of Sectoring (continued)

- Probability of blocking for an unsectored case is

\[ P_B(360^\circ) = \frac{\binom{A}{C}}{C!} \sum_{k=0}^{C} \frac{A^k}{k!} \]

- From the above expressions we can see that

\[ P_B(360^\circ) < P_B(120^\circ) < P_B(60^\circ) \]
Micro-cell Zone Concept

- This concept is a solution to the problem of increased load on the switching and control link elements due to increased number of hand-offs when sectoring is employed
- It is based on a micro-cell concept for 7-cell reuse
  - each zone site is connected to the single base station and share the same radio equipment
  - the zones are connected by coaxial cable, fiber-optic cable, or microwave link to the base station
  - multiple zones and a single base station make up a cell
Micro-cell Zone Concept (continued)

- As a mobile travels from one zone to another within the cell
  - it retains the same channel ⇒ a handoff is not required, the base station simply switches the channel to a different zone site
  - a given channel is active only in the particular zone in which the mobile is traveling ⇒ the base station radiation is localized and interference is reduced
  - the channels are distributed in time and space by all zones and are also reused in co-channel cells in normal fashion
Advantages of Micro-cell Zone Concept

- Since a large central base station is replaced by several lower powered transmitters (zone transmitters) on the edges of the cell
  - a higher trunking efficiency is achieved over sectoring
  - co-channel interference is reduced while the cell maintains a particular coverage radius ⇒ improved the signal quality and an increase in capacity