Problem on Link Budget Design using Path Loss Models (10 points): A transmitter provides 15 W to an antenna having 3 dB gain. The receiver antenna has a gain of 0 dB. If the carrier frequency is 1800 MHz, find the T-R separation that will ensure that a received power of -90 dBm is provided for 95% of the time. Assume \( n = 4, \sigma = 8 \) dB, and \( c = 3 \times 10^8 \) m/sec. Note that power decays according to free space propagation model from the transmitter to the reference distance \( d_0 = 1 \) km.

\[
\begin{align*}
\mathcal{P}_r (d) \text{ [dBm]} & = \mathcal{P}_t \text{ [dBm]} - PL (d) \text{ [dB]} \\
PL (d) \text{ [dB]} & = PL (d_0) + 10 n \log \left( \frac{d}{d_0} \right) \\
PL (d_0) \text{ [dB]} & = -10 \log \left[ \frac{G_t G_r \lambda^2}{(4\pi)^2 d_0^2} \right] \\
G_t & = 3 \text{ dB} = 2, \quad G_r = 0 \text{ dB} = 1 \\
\lambda & = \frac{c}{f c} = \frac{3 \times 10^8}{18 \times 10^3} = \frac{1}{6} \text{ m} \\
PL (d_0) \text{ [dB]} & = -10 \log \left[ \frac{2 \times 1}{(4\pi)^2 \times 10^6 \times 36} \right] = 94.54 \text{ dB} \\
PL (d) \text{ [dB]} & = 94.54 + 10 \times 4 \log \left( \frac{d}{1000} \right) \\
& = 40 \log d - 25.46 \\
\mathcal{P}_r (d) \text{ [dBm]} & = 10 \log \left( \frac{15}{0.001} \right) - 40 \log d + 25.46 \\
& = 67.22 - 40 \log (d)
\end{align*}
\]
\[ P \left[ P_r(d) > -90 \right] = Q \left( \frac{-90 - P_r(d)}{\sigma} \right) \]

\[ \Rightarrow \quad 0.95 = Q \left( \frac{-90 - 67.22 + 40 \log d}{8} \right) \]

\[ \Rightarrow \quad Q \left( \frac{40 \log d - 157.22}{8} \right) = 0.95 \]

\[ 40 \log d - 157.22 = 8 \times Q^{-1}(0.95) \]

\[ \log d = \frac{8 \times Q^{-1}(0.95) + 157.22}{40} \]

\[ d = \frac{8 \times Q^{-1}(0.95) + 157.22}{40} \]

\[ d = 10 \]
Problem on Hata Model (10 points): If the received power at a reference distance $d_0 = 5$ km is equal to 1 micro-watt, find the received power at a distance of 10 km from the same transmitter in an urban area using Hata model, where the standard path loss in urban areas is given by

$$L_{50(urban)}(dB) = 69.55 + 26.16 \log f_c - 13.82 \log h_{te} - a(h_{re}) + (44.9 - 6.55 \log h_{te}) \log d$$

Assume $f_c = 1500$ MHz, $h_{te} = 40$m, $h_{re} = 3$m, $G_t = G_r = 0$ dB and

$$a(h_{re}) = 3.2 (\log 11.75 h_{re})^2 - 4.97 \text{ dB}$$

$$P_t = P_r (d = 5 \text{ km}) = 10 \log \left( \frac{10^{-6}}{0.001} \right) = -30 \text{ dBm}$$

$$P_L (d = 5 \text{ km}) = \frac{69.55 + 26.16 \log f_c - 13.82 \log h_{te} - a(h_{re})}{L_f} + (44.9 - 6.55 \log h_{te}) \log d$$

$$P_L (d = 10 \text{ km}) = L_f + (44.9 - 6.55 \log h_{te}) \log (2 \times d_1)$$

$$= L_f + (44.9 - 6.55 \log h_{te}) (\log 2 + \log d_1)$$

$$= L_f + (44.9 - 6.55 \log h_{te}) \log d_1$$

$$P_L (d = 0 \sim 5 \text{ km}) + (44.9 - 6.55 \log h_{te}) \log 2$$

$$P_L (d = 5 \sim 10 \text{ km})$$
\[ P_r (d_2 = 10 \text{ km}) = P_t - PL (d = 5 \sim 10 \text{ km}) \]

\[ = -30 - (44.9 - 6.55 \log(40)) \log 2 \]

\[ = -40.3574 \text{ dBm} \]