Yates and Goodman 3e Solution Set:  3.2.1, 3.2.3, 3.2.10, 3.2.11, 3.3.1, 3.3.3, 3.3.10, 3.3.18, 3.4.3, and 3.4.4

Problem 3.2.1 Solution

(a) We wish to find the value of $c$ that makes the PMF sum up to one.

\[ P_N(n) = \begin{cases} c(1/2)^n & n = 0, 1, 2, \\ 0 & \text{otherwise}. \end{cases} \tag{1} \]

Therefore, \( \sum_{n=0}^{2} P_N(n) = c + c/2 + c/4 = 1 \), implying \( c = 4/7 \).

(b) The probability that \( N \leq 1 \) is

\[ P [N \leq 1] = P [N = 0] + P [N = 1] = 4/7 + 2/7 = 6/7. \tag{2} \]

Problem 3.2.3 Solution

(a) We choose $c$ so that the PMF sums to one.

\[ \sum_x P_X(x) = \frac{c}{2} + \frac{c}{4} + \frac{c}{8} = \frac{7c}{8} = 1. \tag{1} \]

Thus $c = 8/7$.

(b)

\[ P [X = 4] = P_X(4) = \frac{8}{7 \cdot 4} = \frac{2}{7}. \tag{2} \]
(c) \[ P[X < 4] = P_X(2) = \frac{8}{7 \cdot 2} = \frac{4}{7}. \] (3)

(d) \[ P[3 \leq X \leq 9] = P_X(4) + P_X(8) = \frac{8}{7 \cdot 4} + \frac{8}{7 \cdot 8} = \frac{3}{7}. \] (4)

**Problem 3.2.10 Solution**

From the problem statement, a single is twice as likely as a double, which is twice as likely as a triple, which is twice as likely as a home-run. If \( p \) is the probability of a home run, then

\[
P_B(4) = p \quad P_B(3) = 2p \quad P_B(2) = 4p \quad P_B(1) = 8p
\] (1)

Since a hit of any kind occurs with probability of .300, \( p + 2p + 4p + 8p = 0.300 \) which implies \( p = 0.02 \). Hence, the PMF of \( B \) is

\[
P_B(b) = \begin{cases} 
0.70 & b = 0, \\
0.16 & b = 1, \\
0.08 & b = 2, \\
0.04 & b = 3, \\
0.02 & b = 4, \\
0 & \text{otherwise}.
\end{cases}
\] (2)

**Problem 3.2.11 Solution**

(a) In the setup of a mobile call, the phone will send the “SETUP” message up to six times. Each time the setup message is sent, we have a Bernoulli trial with success probability \( p \). Of course, the phone stops trying as soon as there is a success. Using \( r \) to denote a successful response, and \( n \) a non-response, the sample tree is
(b) We can write the PMF of $K$, the number of “SETUP” messages sent as

$$P_K(k) = \begin{cases} 
(1 - p)^{k-1}p & k = 1, 2, \ldots, 5, \\
(1 - p)^5p + (1 - p)^6 = (1 - p)^5 & k = 6, \\
0 & \text{otherwise.}
\end{cases}$$

(1)

Note that the expression for $P_K(6)$ is different because $K = 6$ if either there was a success or a failure on the sixth attempt. In fact, $K = 6$ whenever there were failures on the first five attempts which is why $P_K(6)$ simplifies to $(1 - p)^5$.

(c) Let $B$ denote the event that a busy signal is given after six failed setup attempts. The probability of six consecutive failures is $P[B] = (1 - p)^6$.

(d) To be sure that $P[B] \leq 0.02$, we need $p \geq 1 - (0.02)^{1/6} = 0.479$.

**Problem 3.3.1 Solution**

(a) If it is indeed true that $Y$, the number of yellow M&M’s in a package, is uniformly distributed between 5 and 15, then the PMF of $Y$, is

$$P_Y(y) = \begin{cases} 
1/11 & y = 5, 6, 7, \ldots, 15 \\
0 & \text{otherwise}
\end{cases}$$

(1)

(b)

$$P[Y < 10] = P_Y(5) + P_Y(6) + \cdots + P_Y(9) = 5/11. \quad (2)$$

(c)

$$P[Y > 12] = P_Y(13) + P_Y(14) + P_Y(15) = 3/11. \quad (3)$$

(d)

$$P[8 \leq Y \leq 12] = P_Y(8) + P_Y(9) + \cdots + P_Y(12) = 5/11. \quad (4)$$
Problem 3.3.3 Solution

(a) Each paging attempt is an independent Bernoulli trial with success probability $p$. The number of times $K$ that the pager receives a message is the number of successes in $n$ Bernoulli trials and has the binomial PMF

$$P_K(k) = \begin{cases} \binom{n}{k}p^k(1-p)^{n-k} & k = 0, 1, \ldots, n, \\ 0 & \text{otherwise}. \end{cases} \quad (1)$$

(b) Let $R$ denote the event that the paging message was received at least once. The event $R$ has probability

$$P[R] = P[B > 0] = 1 - P[B = 0] = 1 - (1-p)^n. \quad (2)$$

To ensure that $P[R] \geq 0.95$ requires that $n \geq \ln(0.05)/\ln(1-p)$. For $p = 0.8$, we must have $n \geq 1.86$. Thus, $n = 2$ pages would be necessary.

Problem 3.3.10 Solution

Since an average of $T/5$ buses arrive in an interval of $T$ minutes, buses arrive at the bus stop at a rate of $1/5$ buses per minute.

(a) From the definition of the Poisson PMF, the PMF of $B$, the number of buses in $T$ minutes, is

$$P_B(b) = \begin{cases} (T/5)^b e^{-T/5}/b! & b = 0, 1, \ldots, \\ 0 & \text{otherwise}. \end{cases} \quad (1)$$

(b) Choosing $T = 2$ minutes, the probability that three buses arrive in a two minute interval is

$$P_B(3) = (2/5)^3 e^{-2/5}/3! \approx 0.0072. \quad (2)$$

(c) By choosing $T = 10$ minutes, the probability of zero buses arriving in a ten minute interval is

$$P_B(0) = e^{-10/5}/0! = e^{-2} \approx 0.135. \quad (3)$$
(d) The probability that at least one bus arrives in $T$ minutes is

$$P[B \geq 1] = 1 - P[B = 0] = 1 - e^{-T/5} \geq 0.99. \quad (4)$$

Rearranging yields $T \geq 5 \ln 100 \approx 23.0$ minutes.

**Problem 3.3.18 Solution**

(a) Let $S_n$ denote the event that the Sixers win the series in $n$ games. Similarly, $C_n$ is the event that the Celtics in $n$ games. The Sixers win the series in 3 games if they win three straight, which occurs with probability

$$P[S_3] = (1/2)^3 = 1/8. \quad (1)$$

The Sixers win the series in 4 games if they win two out of the first three games and they win the fourth game so that

$$P[S_4] = \binom{3}{2}(1/2)^3(1/2) = 3/16. \quad (2)$$

The Sixers win the series in five games if they win two out of the first four games and then win game five. Hence,

$$P[S_5] = \binom{4}{2}(1/2)^4(1/2) = 3/16. \quad (3)$$

By symmetry, $P[C_n] = P[S_n]$. Further we observe that the series last $n$ games if either the Sixers or the Celtics win the series in $n$ games. Thus,

$$P[N = n] = P[S_n] + P[C_n] = 2P[S_n]. \quad (4)$$

Consequently, the total number of games, $N$, played in a best of 5 series between the Celtics and the Sixers can be described by the PMF

$$P_N(n) = \begin{cases} 2(1/2)^3 = 1/4 & n = 3, \\ 2\binom{3}{1}(1/2)^4 = 3/8 & n = 4, \\ 2\binom{4}{2}(1/2)^5 = 3/8 & n = 5, \\ 0 & \text{otherwise}. \end{cases} \quad (5)$$
(b) For the total number of Celtic wins \( W \), we note that if the Celtics get \( w < 3 \) wins, then the Sixers won the series in \( 3 + w \) games. Also, the Celtics win 3 games if they win the series in 3, 4, or 5 games. Mathematically,

\[
P[W = w] = \begin{cases} 
P[S_{3+w}] & w = 0, 1, 2, \\
\end{cases} 
\]  

(6)

Thus, the number of wins by the Celtics, \( W \), has the PMF shown below.

\[
P_W(w) = \begin{cases} 
P[S_3] = 1/8 & w = 0, \\
P[S_4] = 3/16 & w = 1, \\
P[S_5] = 3/16 & w = 2, \\
1/8 + 3/16 + 3/16 = 1/2 & w = 3, \\
0 & \text{otherwise}. 
\end{cases} 
\]  

(7)

(c) The number of Celtic losses \( L \) equals the number of Sixers’ wins \( W_S \). This implies \( P_L(l) = P_{W_S}(l) \). Since either team is equally likely to win any game, by symmetry, \( P_{W_S}(w) = P_W(w) \). This implies \( P_L(l) = P_{W_S}(l) = P_W(l) \). The complete expression of for the PMF of \( L \) is

\[
P_L(l) = P_W(l) = \begin{cases} 
1/8 & l = 0, \\
3/16 & l = 1, \\
3/16 & l = 2, \\
1/2 & l = 3, \\
0 & \text{otherwise}. 
\end{cases} 
\]  

(8)

Problem 3.4.3 Solution

(a) Similar to the previous problem, the graph of the CDF is shown below.

\[
F_X(x) = \begin{cases} 
0 & x < -3, \\
0.4 & -3 \leq x < 5, \\
0.8 & 5 \leq x < 7, \\
1 & x \geq 7. 
\end{cases} 
\]  

(1)
(b) The corresponding PMF of $X$ is

$$P_X(x) = \begin{cases} 
0.4 & x = -3 \\
0.4 & x = 5 \\
0.2 & x = 7 \\
0 & \text{otherwise}
\end{cases}$$

Problem 3.4.4 Solution

Let $q = 1 - p$, so the PMF of the geometric ($p$) random variable $K$ is

$$P_K(k) = \begin{cases} 
 pq^{k-1} & k = 1, 2, \ldots, \\
0 & \text{otherwise}.
\end{cases}$$

For any integer $k \geq 1$, the CDF obeys

$$F_K(k) = \sum_{j=1}^{k} P_K(j) = \sum_{j=1}^{k} pq^{j-1} = 1 - q^k.$$ 

Since $K$ is integer valued, $F_K(k) = F_K([k])$ for all integer and non-integer values of $k$. (If this point is not clear, you should review Example 3.22.) Thus, the complete expression for the CDF of $K$ is

$$F_K(k) = \begin{cases} 
0 & k < 1, \\
1 - (1 - p)^{[k]} & k \geq 1.
\end{cases}$$