The Geometric\( (p) \) Distribution

The model is given by the probability mass function

\[
P(X = k) = (1 - p)^{k-1}p, \quad k = 1, 2, \ldots
\]

The associated tail probabilities are given by

\[
P(X \geq k) = (1 - p)^{k-1}, \quad k = 1, 2, \ldots
\]

This probability distribution has mean \( \frac{1}{p} \), variance \( \frac{1-p}{p^2} \), and standard deviation \( \sqrt{\frac{1-p}{p^2}} \). See http://en.wikipedia.org/wiki/Geometric_distribution

Exercise 1. Suppose that 3% of the items on an assembly line in a factory are defective. What is the average number of items inspected before finding the first defective item?

Answer: 33.3.

Exercise 2. (Continuation) What is the probability that the first defective found is the 5th item inspected?

Answer: 0.0266.

Exercise 3. (Continuation) What is the probability that no defectives are found in the first 5 items inspected? In the first 10 items inspected? In the first 20 items inspected? In the first 30 items inspected? In the first 40 items inspected?

Answers: 0.86, 0.74, 0.54, 0.40, 0.30.

Exercise 4. (Continuation) What is the probability that at least 6 trials are needed before finding a defective?

Answer: 0.86.
The Poisson(λ) Distribution

The model is given by the probability mass function

\[ P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots \]

This probability distribution has mean \( \lambda \), variance \( \lambda \), and standard deviation \( \sqrt{\lambda} \). See http://en.wikipedia.org/wiki/Poisson_distribution

Exercise 1. Suppose that the number of plants of a given species that we expect to find in a one meter square quadrat follows the Poisson(5) distribution. What is the probability of finding exactly 7 plants?

Answer: 0.104.

Exercise 2. (Continuation) What is the probability of finding 3 or fewer plants?

Answer: 0.265.

Exercise 3. For a Poisson(\( \lambda \)) distribution, suppose that the probability attached to 0 is 0.000553. What is \( \lambda \)?

Answer: 7.5.
The Exponential($\mu$) Distribution

The model is given by the probability density function

$$f(x) = \frac{1}{\mu} e^{-x/\mu}, \ 0 < x < \infty$$

The associated tail probabilities are given by

$$P(X > x) = e^{-x/\mu}, \ 0 < x < \infty$$

This probability distribution has mean $\mu$, variance $\mu^2$, and standard deviation $\mu$. See http://en.wikipedia.org/wiki/Exponential_distribution

but note that this article uses the alternate parameterization $f(x) = \lambda e^{-\lambda x}, \ 0 < x < \infty$.

**Exercise 1.** A new lightbulb is packaged in a wrapper with a statement that the “Life” is 1000 hours. Interpreting the lifetime as an Exponential(1000) chance variable, what is the probability that the lifetime of your lightbulb is at least 1000 hours?

Answer: $\frac{1}{e} = 0.368$.

**Exercise 2.** (Continuation) What is the probability that the lifetime is less than 500 hours?

Answer: 0.394.

**Exercise 3.** Suppose that two such new lightbulbs are placed into a lamp. What is the probability that both of them have lifetimes less than 500 hours?

Answer: 0.155.