The Postulates of Quantum Mechanics
(from Quantum Mechanics by Claude Cohen-Tannoudji, Bernard Diu, and Franck Laloë)

**Introduction** The postulates of quantum mechanics are the theory. Their physical content and their consequences are what we take time to develop in the course. The postulates provide us with an answer to the following questions:

(i) How is the state of a quantum mechanical system at a given time described mathematically?
(ii) Given this state, how can we predict the results of the measurement of various physical quantities?
(iii) How can the state of the system at an arbitrary time \( t \) be found when the state at time \( t_0 \) is known?

**First Postulate**
At a fixed time \( t_0 \), the state of a physical system is defined by specifying a wavefunction \( \psi(x, y, z, t_0) \).

**Second Postulate**
Every measurable physical quantity \( Q \) is described by an operator \( \hat{Q} \); this operator is called an observable.
**Third Postulate**

The only possible result of the measurement of a physical quantity $Q$ is one of the eigenvalues of the corresponding observable $\hat{Q}$.

**Fourth Postulate (non-degenerate case)**

When the physical quantity $Q$ is measured on a system in the normalized state $\psi$, the probability $\mathcal{P}(q_n)$ of obtaining the non-degenerate eigenvalue $q_n$ of the corresponding observable $\hat{Q}$ is

$$\mathcal{P}(q_n) = \int \phi_n^* \psi$$

where $\phi_n$ is the normalized eigenvector of $\hat{Q}$ associated with the eigenvalue $q_n$.

**Fifth Postulate (collapse)**

If the measurement of the physical quantity $Q$ on the system in the state $\psi$ gives the result $q_n$, the state of the system immediately after the measurement is $\phi_n$. 
Sixth Postulate (time evolution)

The time evolution of the wavefunction $\psi(x, y, z, t)$ is governed by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$

(2)

where $\hat{H}$ is the observable associated with the total energy of the system.

Seventh Postulate (symmetrization)

When a system includes several identical particles, only certain wavefunctions can describe its physical states (leads to the concept of bosons and fermions).

The 4th and 5th postulates are problematic. The origin of these problems lies in the fact that the system under study is treated independently from the measurement device, although their interaction is essential to the observation process. One should actually consider the system and the measurement device together as a whole (except that this raises more problems...).