Homework 5

Problem 1. Floyd-Warshall
Recall that an $n \times n$ matrix $W$ represents the edge weights in the Floyd-Warshall algorithm. Suppose that we want to compute a matrix $L[1..n][1..n]$ where $L[i][j]$ is the number of edges in a shortest path from $i$ to $j$. Explain how to modify the Floyd-Warshall algorithm to compute matrix $L[..][..]$ in $O(n^3)$ time.

Problem 2. Max flow
There are $n$ undergraduate students and $k$ departments at some university. The Student Senate must have $k$ students, one from each department. It also should have $k_1$ freshmen, $k_2$ sophomores, $k_3$ juniors, and $k_4$ seniors where $k_1 + k_2 + k_3 + k_4 = k$. The task is to decide if the Student Senate can be formed. If it can be formed, find a solution with $k$ students. Design an algorithm for this task using max flow.

Problem 3. Max flow/min cut
Let $G$ be the graph shown below.
(a) Enumerate all cuts in $G$ and show their capacities.
(b) Find minimum cuts in $G$. Also find a maximum cut (a cut with maximum capacity).
(c) Find max flow in $G$ using Ford-Fulkerson algorithm. Show the flow value and the corresponding (augmenting) paths.

Problem 4. Max flow/min cut
Suppose that the max flow value of a graph $G$ is greater than zero. An edge $(u, v)$ of $G$ is called essential if decreasing $c(u, v)$ by any small amount results in a decrease in the maximum flow value.
(a) Show that $G$ has an essential edge.
(b) Design an efficient algorithm for finding an essential edge in $G$.

Problem 5. NP-completeness
Show that, if languages $L_1, L_2$ are in $P$ then $L_1 \cap L_2$ and $L_1L_2$ are also in $P$. 

![Graph Diagram]