Homework 6

Problem 1. Show that, if languages $L_1, L_2$ are in $P$ then $L_1 \cap L_2$ and $L_1 L_2$ are also in $P$.

Problem 2. (a) Can a bipartite graph with 8 vertices be hamiltonian? Show an example or prove that any such graph is not hamiltonian.
(b) Can a bipartite graph with 11 vertices be hamiltonian? Show an example or prove that any such graph is not hamiltonian.

Problem 3. A problem APPROX-SAT is defined as follows. Given a CNF formula $\phi$ with $k \geq 2$ clauses $C_1, \ldots, C_k$ and $n$ variables $x_1, \ldots, x_n$, is there an assignment of the variables such that exactly $k - 1$ clauses are true.
(a) Prove that APPROX-SAT is in NP.
(b) Prove that 3-SAT $\leq_P$ APPROX-SAT.

Problem 4. Suppose that HAM-CYCLE is in $P$ and a graph $G$ is hamiltonian. Show that a hamiltonian cycle in $G$ can be computed in polynomial time.

Problem 5. A Hamiltonian path in a graph $G$ is a simple path that contains all the vertices of $G$.
HAM-CYCLE problem: Given a graph $G$, is there a Hamiltonian path in $G$?
Prove that HAM-PATH $\leq_P$ HAM-CYCLE.