A new robust circular Gabor based object matching by using weighted Hausdorff distance

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Abstract

This paper describes a new and efficient circular Gabor filter-based method for object matching by using a version of weighted modified Hausdorff distance. An improved Gabor odd filter-based edge detector is performed to get edge maps. A rotation invariant circular Gabor-based filter, which is different from conventional Gabor filter, is used to extract rotation invariant features. The Hausdorff distance (HD) has been shown an effective measure for determining the degree of resemblance between binary images. A version of weighted modified Hausdorff distance (WMHD) in the circular Gabor feature space is introduced to determine which position can be possible object model location, which we call 'coarse' location, and at the same time we get correspondence pairs of edge pixels for both object model and input test image. Then we introduce the geometric shape information derived from the above correspondence pairs of edge pixels to find the 'fine' location. The experimental results given in this paper show the proposed algorithm is robust to rotation, scale, occlusion, and noise etc.

Keywords: Object matching; Gabor filter; Hausdorff distance; Edge detection; Circular Gabor

1. Introduction

This paper describes a new and efficient circular Gabor filter-based object matching by using a version of weighted modified Hausdorff distance. Object matching in two-dimension images has been a deeply investigated field in object recognition, object tracking, image retrieval, etc. (Belongie et al., 2002; Dupplaw et al., 1999; Zhong and Jain, 2000). Object matching means establishing correspondences between object features in a set of images. The performance of matching depends on the properties of features and the matching measure used.

In lower level processing of our algorithm for object matching, we use an improved odd Gabor based edge detector to obtain edge maps for the object model and the input test image. The odd Gabor filter-based edge detector, which shows advantage over Canny’s (1986) edge detector according to the three detection criterions proposed
by Canny, was first proposed by Mehrotra et al. (1992). But it is computationally expensive since different Gabor templates should be constructed at each pixel and the fast algorithm FFT cannot be used to perform convolution. To solve the problem, the paper proposes an improved odd Gabor filter-based edge detector which only needs two odd Gabor templates to be constructed for the image. Thus, we can use FFT to perform convolution of image with the odd Gabor templates.

The Hausdorff distance (HD) is an efficient and widely used tool for object matching. Huttenlocher et al. (1993) proposed a partial Hausdorff measure for object matching which uses a set of edge points extracted by an edge detector. The partial Hausdorff measure based on the ranked order statistic is efficient to estimate the similarity between two sets of edge points extracted from the object model and the test image in the presence of occlusion. Dubuisson and Jain (1994) have adopted a modified HD to estimate the similarity between two objects. This version of HD decreases the impact of outliers making it more suitable for pattern recognition purposes. Sim and Park (2001) and Olson and Huttenlocher (1997) have incorporated the orientation information with the spatial information of edge points into their applications of HD measure. In addition, Olson (1998) developed a probabilistic formulation in term of likelihood estimation that generalizes a version of Hausdorff matching. But how to realize the rotation and scale invariant object matching is a big problem for the above algorithms. Huttenlocher et al. (1993) proposed to search a discretized rotation and scale space to find the possible location of object model in the test image. Although there exists optimal search algorithm (Huttenlocher and Rucklidge, 1993), it is still almost unpractical since the search space is very huge. So here the paper proposes to use circular Gabor-based features (Zhang and Tan, 2002), which are rotation invariant and insensitive to some level of scale variation, to solve the above problem. Since the Hausdorff distance in circular Gabor feature space reflects no spatial information, so only a ‘coarse location’ is obtained. Thus some more shape information is introduced to find the ‘fine location’ of object model in the input test image.

In reality, it is reasonable to consider that even elements of a set maybe play different roles under some conditions, e.g., the change of background. So to avoid such problem, the paper proposes a weighted modified Hausdorff distance (WMHD), which is different from the conventional HD.

The outline of this paper is as follows: Section 2 provides an improved odd Gabor filter-based edge detector and gives some edge detection results. Section 3 gives an introduction to the modified rotation invariant Gabor filter, which is called circular Gabor filter. Section 4 reviews the definition of Hausdorff distance and proposes a weighted modified Hausdorff distance. In this section a ‘coarse location’ for the object model in the input image is obtained by using the weighted modified Hausdorff distance in circular Gabor feature space. Section 5 introduces a shape distance and incorporates it with the above circular Gabor Hausdorff distance to find the ‘fine location’ of object model in the image. Section 6 gives some object matching experiment results. In Section 7 we give a summary for our work.

2. An improved odd Gabor filter-based edge detection

Edge detection, which has attracted the attention of many researchers, is one of the most important areas in lower level computer vision because the success of higher level processing such as object recognition and scene interpretation relies heavily on good edge detection.

Gabor filters, which have been shown to fit well the receptive fields of the majority of simple cell in the primary visual cortex (Jones and Palmer, 1987; Daugman, 1985, 1988), are modulation products of Gaussian and complex sinusoidal signals. A 2D Gabor filter oriented at angle $\theta$ is given by

$$G(x, y) = \exp \left( -\frac{1}{2\sigma^2} (x^2 + y^2) \right) \cdot \exp[j\omega(x \cos \theta + y \sin \theta)],$$

where $\sigma$ is the standard deviation of the circle Gaussian along $x$ and $y$, and $\omega$ denotes the spatial frequency.
Fig. 1 shows the even real and odd imaginary parts of a 2D Gabor filter. The Gabor filter output is
\[ O(x, y) = G(x, y) * I(x, y), \]
where \( * \) denotes two-dimensional convolution operator and \( I(x, y) \) is the input image. When \( \omega \cdot \sigma \approx 1 \) and \( \theta \) is perpendicular to the edge, i.e.,
\( \theta = \frac{\partial I(x, y)}{\partial y} \), the odd Gabor filter (OGF) i.e., the imaginary part of Gabor filter, has been shown to be an efficient and robust edge detector (Mehrotra et al., 1992) which offers distinct advantages over traditional edge detectors such as Roberts, Sobel, etc. and can be comparable even superior to Canny edge detector (Canny, 1986) generally thought as an optimal edge detector.

According to the algorithm described in (Mehrotra et al., 1992), at each pixel we first need to use some simple edge detectors, e.g., Sobel edge detector, to estimate \( \theta \) which is perpendicular to the edge direction, and then use it to construct the odd Gabor template at that pixel. Thus for an \( m \times n \) image \( I(x, y) \), it is computationally expensive to construct \( m \times n \) Gabor filter templates which are used to convolve with \( I(x, y) \). One can build a look-up table of Gabor filter templates with the discretized \( \theta \), thus we need only to construct 36 Gabor filter templates globally if we discretize \( \theta \) by \( 5^\circ \). Even so, we still cannot to implement the above convolution by using FFT since each pixel corresponds to different odd Gabor filter template; in other words, all odd Gabor filter templates here are ‘local’.

Aiming at solving the above problem, the improved algorithm needs only to construct two odd Gabor filter templates: horizontal (OG\(_h\)) and vertical (OG\(_v\)) odd Gabor filter templates, which correspond to \( \theta = 0 \) and \( \theta = \pi/2 \), respectively, and without need to estimate \( \theta \) at each pixel. Thus we have
\[ \text{OG}_h(x, y) = \exp \left[ -\frac{(x^2 + y^2)}{2\sigma^2} \right] \sin(\omega x), \]
\[ \text{OG}_v(x, y) = \exp \left[ -\frac{(x^2 + y^2)}{2\sigma^2} \right] \sin(\omega y). \]

Since both \( \text{OG}_h \) and \( \text{OG}_v \) are independent of \( \theta \), so they can be thought as ‘global’ and the convolution between the input image \( I(x, y) \) and the two OGFs can be implemented by using FFT. The post processing of the improved algorithm is similar to that in the web address: http://robotics.eecs.berkeley.edu/~sastry/ee20/cacode.html.

Because of the subjectivity of edge detection, it is difficult to compare the performance of two edge detectors on most real world images (Heath et al., 1998). Here we use some synthetic (http://pretty-view.com/edge/nsedge.shtml#to) and real world images to test our algorithm. All experiments in Figs. 2 and 3 are with the same fixed parameter, i.e., \( \sigma = 1.8 \). In addition, no pre-processing was applied to the original images before the edge detector was used to obtain the results below since pre-processing may yield detail information loss if we do not have prior knowledge of the input original images.

Fig. 2 shows some edge detection results of the synthetic images created with a step edge added with Gaussian noise. Fig. 3 shows the edge detection results of the real world images with different SNR. Fig. 4 shows the edge detection results of Fig. 2(c) and (d) with adjusted parameter \( \sigma = 3 \).
In Fig. 2(c) and (d), since the original step edge is badly contaminated by Gaussian noise, so the detection results are unsatisfactory with parameter $r = 1.8$ which are generally used in our algorithm. But in Fig. 4, the edge detection results are visually improved with adjusted parameter $r = 3$.

3. Circular Gabor-based filter

Circular Gabor-based filter is a modified version of traditional Gabor filter. It is rotation invariant i.e., it is isotropic which is the main different point from traditional Gabor filter. It has been successfully used for invariant texture segmentation (Zhang and Tan, 2002). The circular Gabor filter is defined as

$$cG(x, y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x^2 + y^2)}{2\sigma^2}\right] \cdot \exp\left[2\pi i F(\sqrt{x^2 + y^2})\right],$$

where $F$ is the central frequency of the circular Gabor filter. Its spatial responses (even real and odd imaginary parts) are shown in Fig. 5(a) and (b).

The frequency domain representation of the circular Gabor filter is given by

$$F(u, v) = \frac{\sqrt{2\pi}}{2} z \cdot \exp\left[-\frac{(\sqrt{u^2 + v^2} - F)^2}{2z^2}\right],$$

where $z = \frac{1}{\sqrt{\pi}\sigma}$. The frequency representation is shown in Fig. 5(c). In this paper, the parameter selection is similar to (Zhang and Tan, 2002).

4. Matching circular Gabor features using Hausdorff distance

4.1. The Hausdorff distance

Let $A = \{a_1, a_2, \ldots, a_p\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ denote two finite sets. Then the Hausdorff distance is defined as

$$H(A, B) = \max(h(A, B), h(B, A)),$$

where

$$h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\|.$$
tance. In the context of recognition, the Hausdorff distance is used to determine the quality of a match between an object model and an input image. If \( A \) is the set of the object model features and \( B \) is the set of the input image features, the bi-directional Hausdorff distance determines the distance of the worst matching object edge pixel to its closest image edge pixel in the feature space. Of course, due to occlusion, it cannot be assumed that each object pixel appears in the image. The partial Hausdorff distance (Huttenlocher et al., 1993) between these sets is thus often used. It is given by

\[
h_{K} (A, B) = K^{th}_{x \in A} \min_{b \in B} \| a - b \|, \tag{9}
\]

where \( K^{th}_{x \in X} g(x) \) denotes the \( K \)th quantile value of \( g(x) \) over the set \( X \).

For image processing applications it has proven to be useful to apply a slightly different measure, the (directed) modified Hausdorff distance (MHD), which was introduced by Dubuisson and Jain (1994). It is defined as

\[
h_{\text{mod}} (A, B) = \frac{1}{|A|} \sum_{a \in A} \min_{b \in B} \| a - b \|, \tag{10}
\]

where \( |A| \) denotes the number of elements in \( A \). By taking the average of the single point distances, this version decreases the impact of outliers making it more suitable for pattern recognition purposes.

It is reasonable to consider that elements in both \( A \) and \( B \) maybe play different roles in Hausdorff measure, so here we integrate Eq. (9) with Eq. (10) to generate a weighted (directed and backward) modified partial Hausdorff distance (WMHD) which can be obtained by

\[
h_{\text{WMHD}} (A, B) = \frac{1}{|A|} \sum_{i=1}^{K} \left( w_{a} \cdot \min_{b \in B} \| a - b \| \right)_{(i)}, \tag{11}
\]

\[
h_{\text{WMHD}} (B, A) = \frac{1}{|B|} \sum_{i=1}^{K} \left( w_{b} \cdot \min_{a \in A} \| b - a \| \right)_{(i)}, \tag{12}
\]

where \( w_{a}, w_{b} \) are weights, the subscript \( i \) denotes the \( i \)th minimum. Thus (7) should be substituted by

\[
H_{\text{WMHD}} (A, B) = \max(h_{\text{WMHD}} (A, B), h_{\text{WMHD}} (B, A)). \tag{13}
\]

### 4.2. Matching circular Gabor features

Let us assume that the sizes of the input image \( I(x, y) \) and \( M(x, y) \) are \( m1 \times n1 \) and \( m2 \times n2 \), respectively. The edge maps for the input image and the object model obtained by Section 2 are \( I_{E}(x, y) \) and \( M_{E}(x, y) \). During object matching, we use a circular object model template not the whole object model, that is to say we only consider those edge pixels within the circle area. In the following description we refer the above object model \( M(x, y) \) as the circular object model. When the object model \( M(x, y) \) is translated in the input image \( I(x, y) \), we denote the circle area coincident with the translated object model \( M(x, y) \) within the input image \( I(x, y) \) at the current translation as \( M_{t}(x, y) \) and the edge map relates to \( M_{t}(x, y) \) as \( M_{E_{t}}(x, y) \). For \( K \) in Eqs. (11) and (12), we have \( K = f_{1} \cdot M_{n} \), where \( f_{1} \) is a fraction which is set between zero and 1 and \( M_{n} \) is the number of edge pixels within \( M(x, y) \). We denote \( M^{G} \) as a set of \( M^{n} \) 1-D circular Gabor feature vectors extracted from edge pixel locations within the object model \( M(x, y) \) and \( M_{t}^{G} \) as a set of \( M_{t}^{n} \) 1-D circular Gabor feature vectors extracted from edge pixel locations within \( M_{t}(x, y) \), where \( M_{t}^{n} \) is the number of edge pixels in \( M_{t}(x, y) \). Note that we cannot derive \( K \) one to one correspondence pairs between \( M_{E}(x, y) \) and \( M_{E_{t}}(x, y) \) only in circular Gabor feature space by using Hausdorff measure. So in order to get it, here we add adjustment component to each vector of \( M^{G} \) and \( M_{t}^{G} \). Thus we have:

\[
M^{G} \oplus M_{A} \rightarrow M^{G},
\]

\[
M_{t}^{G} \oplus M_{A} \rightarrow M_{t}^{G},
\]

where

\[
M_{A} = \left\{ \frac{|m - C_{M}|}{\text{mean}(|m - C_{M}|)} \mid m \in M_{E} \right\}
\]
and
\[
M_{A_\ell} = \left\{ \frac{||m - C'_M||}{\text{mean}(||m - C'_M||)} | m \in M_{E_\ell} \right\},
\]

where \(C'_M, C_M\) are the centers of \(M(x,y)\) and \(M_\ell(x,y)\), and \(\oplus\) denotes to change \(l\)-dimension vectors in the \(M^{G}\) and \(M'_G\) into \((l + 1)\)-dimension by ‘adding’ corresponding \(M_{a}(x,y)\) and \(M_{d}(x,y)\), respectively.

The weighted modified Hausdorff distance \(\text{WMHD}\) (see Eq. (13) and \(A = M^{G_f}, B = M^{G_g}\)) is performed at each translation location. Thus we obtain an \(m \times n\) circular Gabor Hausdorff distance matrix \(HD_c\). And at the same time, we obtain two maps \(Q_d\) and \(Q_b\). Each of them consists of \(K\)-correspondence pairs between edge pixels in \(M(x,y)\) and those in \(M_\ell(x,y)\) and we have:
\[
Q_d : M_{E_\ell}^{K_d} \leftrightarrow M_{E_{\ell}}^{K_d},
\]
\[
Q_b : M_{E_\ell}^{K_b} \leftrightarrow M_{E_{\ell}}^{K_b},
\]

where \(M_{E_\ell}^{K_d} \leftrightarrow M_{E_{\ell}}^{K_d}\) and \(M_{E_\ell}^{K_b} \leftrightarrow M_{E_{\ell}}^{K_b}\) denote \(K\)-correspondence pairs between edge pixels in \(M_{E}(x,y)\) and in \(M_{E_\ell}(x,y)\) under \(h_{\text{WMHD}}(M^{G}, M^{G}_f)\) and \(h_{\text{WMHD}}(M^{G}_f, M^{G})\), respectively. If \(M^{h}_{\ell}\), the number of edge pixels in \(M_{\ell}\), is less than \(K\), we can assign a big enough value to the circular Gabor Hausdorff distance at that location which means the object model \(M(x,y)\) would not appear at that location in the input image \(I(x,y)\). So we need not to compute the circular Gabor Hausdorff distance at each translation location.

There are many weight functions for weights in Eqs. (11) and (12). Here we adopt a binomial weight function, which is given by
\[
w_a = \frac{0.4r_a^2}{[(\max(m2, n2) + 1)/2]^2} + 1,
\]

where \(r_a\) is the distance from edge pixel \(a\) to the center of \(M(x,y)\). The weight function adopted here can decrease the influence from the change of background. Now the location \((x_c, y_c)\) where \(HD_{c}\) has the lowest score is the ‘coarse location’ in the input image \(I(x,y)\) for the object model \(M(x,y)\). The ‘coarse location’ \((x_c, y_c)\) can be obtained by
\[
(x_c, y_c) = \arg\min_{x,y} [\text{HD}_{c}(x,y)].
\]

5. Fine location using geometric shape information

Since the Hausdorff measure in circular Gabor space reflects no much spatial information, so \((x_c, y_c)\) obtained in Section 4.2 is only a ‘coarse location’. Some more shape information should be considered to get the ‘fine location’.

In the input image \(I(x,y)\) we define an \(n \times n\) window centered at \((x_c, y_c)\) as \(I_w\), here \(n = 7\). For each pixel in \(I_w\), we compute a shape distance by using the map \(Q\) at that pixel location, which is given by
\[
Q = \begin{cases} 
Q_d & \text{if } h_{\text{WMHD}}(M^{G}, M^{G}_f) \\
Q_b & \text{if } h_{\text{WMHD}}(M^{G}_f, M^{G})
\end{cases}.
\]

The shape distance \(D_s(p,q)\) at each pixel location within the \(I_w\) is given by
\[
D_s^W(p,q) = \frac{1}{K} \sum_{m \in E_\ell} \frac{||m - C_M||}{\text{mean}(||m' - C_M||)} - \frac{||m - C_M||}{\text{mean}(||m' - C_M||)}.
\]

The denominators in Eq. (17) mean scale normalization. The normalized shape distance matrix \(D_s(p,q)\) is given by
\[
D_s^W(p,q) = \frac{[D_s^W(p,q) - \min(D_s^W(\cdot, \cdot))]}{[\max(D_s^W(\cdot, \cdot)) - \min(D_s^W(\cdot, \cdot))]}.
\]

We denote the part corresponding to \(I_w\) in circular Gabor Hausdorff distance matrix \(HD_{cG}\) as \(HD_{cG}^W\). Thus within the \(I_w\), the \(HD_{cG}^W\) is normalized and given by
\[
HD_{cG}^W(p,q) = \frac{[HD_{cG}^W(p,q) - \min(\text{HD}_{cG}^W(\cdot, \cdot))]}{[\max(\text{HD}_{cG}^W(\cdot, \cdot)) - \min(\text{HD}_{cG}^W(\cdot, \cdot))]}.
\]

Now we can incorporate Eqs. (18) and (19) to obtain the final similarity measure ‘SM’:
\[
SM = \beta(1) \cdot HD_{cG}^W + \beta(2) \cdot D_s^W,
\]
where \( \beta(1) \) and \( \beta(2) \) are weights assigned to \( \text{HD}_G \) and \( \text{D}_W \). In our experiments, they are assigned 1 and 0.2, respectively. Thus the final ‘fine location’ \((x_f, y_f)\) of object model \( M(x, y) \) in the input image \( I(x, y) \) can be obtained by finding the location where \( \text{SM} \) has the lowest score:

\[
(x_f, y_f) = \arg\min_{x,y} \{ \text{SM}(x, y) \}.
\]

\[ (21) \]

6. Experimental results

The following figures show some matching results of different cases. In each figure, the upside image is the object model and the downside one is the corresponding input test image. The white circle is just used to show the located position. In Fig. 6, all models are taken from the input test images, which are all contaminated by Gaussian noise with standard deviation \( \sigma = 25 \). Fig. 7 illustrates the robustness of proposed algorithm to occlusion. Note that the object models in Fig. 7(c) and (d) are all taken from what are different from the test images. Figs. 8–10 give some results of rotation and scale change cases. Since we did not make special scale change consideration, Fig. 9 just showed the insensitive of our algorithm to slight scale change.

For the above reason the white circles on the input

Fig. 6. Matching results for Gaussian noise cases (\( \sigma = 25 \)).

Fig. 7. Matching results for occlusion cases.

Fig. 8. Matching results for rotation cases.
test images appeared in Fig. 9 just gave the located positions without scaling over the located object. Generally scale change between 0.8 and 1.3 can be met. For space consideration, the input test images here have been trimmed.

Note that the object matching in our experience is only for single object. Of course it can be extended to multi-objects cases. Thus we need a threshold and find multi-local minimum not simply globally minimizing Eq. (15). Generally speaking, the selection of threshold is task dependent. Some researches on fast multi-object matching based on multi-local optimization are going on.

7. Conclusion

In this paper, a new and efficient circular Gabor filter-based method for object matching using the weighted modified Hausdorff distance is introduced. The experiment results show the proposed algorithm is robust to rotation and scale change, occlusion, noise etc. Since the Gabor filter is a local operator, so the proposed algorithm is somewhat sensitive to distinct illumination change. Further research will focus on the improvement of the proposed algorithm by finding a fast search strategy and the development of parallel algorithm to reduce the computation consume.

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