Pulse Shaping and PAM

- When the message signal is digital, it must be converted into an analog signal in order to be transmitted.
  - “pulse shaping” filter changes the symbol into a suitable analog pulse

- Each symbol $w(kT)$ initiates an analog pulse that is scaled by the value of the signal

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Pulse Shaping

- Compose the analog pulse train entering the pulse shaping filter as
  \[ w_a(t) = \sum_k w(kT)\delta(t - kT) \]
  - which is $w(kT)$ for $t = kT$ and 0 for $t \neq kT$
- Pulse shaping filter output
  \[ x(t) = w_a(t) * p(t) \Rightarrow X(f) = W_a(f)P(f) \]
Pulse Shaping

- 4-PAM symbol sequence triggering baud-spaced rectangular pulse

Intersymbol Interference

- If the analog pulse is wider than the time between adjacent symbols, the outputs from adjacent symbols may overlap
  - A problem called intersymbol interference (ISI)
- What kind of pulses minimize the ISI?
- Choose a shape that is one at time $kT$ and zero at $mT$ for all $m \neq k$
- Then, the analog waveform contains only the value from the desired input symbol and no interference from other nearby input symbols.
- These are called Nyquist Pulses
Nyquist Pulses

- The impulse response \( p(t) \) is a Nyquist pulse for a \( T \)-spaced symbol sequence if there exists a \( \tau \) such that

\[
p(t)|_{t=kT+\tau} = \begin{cases} 
  c, & k = 0 \\
  0, & k \neq 0
\end{cases}
\]

- Rectangular pulse:

\[
p_R = \begin{cases} 
  1, & 0 \leq t < T \\
  0, & \text{otherwise}
\end{cases}
\]

Nyquist Pulses

- Sinc Pulse

\[
p_S(t) = \frac{\sin \pi f_0 t}{\pi f_0 t}
\]

where \( f_0 = 1/T \). Sinc is Nyquist pulse because \( p_S(0) = 1 \) and \( p_S(kT) = \sin(\pi k)/\pi k = 0 \).

- Sinc envelope decays at \( 1/t \).

- Raised-cosine pulse:

\[
p_{RC}(t) = 2f_0 \left( \frac{\sin(2\pi f_0 t)}{2\pi f_0 t} \right) \left[ \frac{\cos(2\pi f_\Delta t)}{1 - (4f_\Delta t)^2} \right]
\]

- with roll-off factor \( \beta = f_\Delta / f_0 \).
- \( T = 1/2f_0 \) because \( p_{RC} \) has a sinc factor
- \( \sin(\pi k)/\pi k \) which is zero for all nonzero integers \( k \).
- Raised-cosine envelope decays at \( 1/|\beta| \).
- As \( \beta \to 0 \), raised-cosine \( \to \) sinc.
Frequency Domain

- Fourier transform

\[ P_{RC}(f) = \begin{cases} 
1, & |f| < f_1 \\
\frac{1 + \cos(\pi \beta)}{2}, & f_1 < |f| < B \\
0, & |f| > B 
\end{cases} \]

- where
  - \( B \) is the absolute bandwidth,
  - \( f_0 \) is the 6db bandwidth,
  - \( f_\Delta = B - f_0 \),
  - \( f_1 = f_0 - f_\Delta \), and
  - \( \alpha = \pi (|f| - f_1) / 2f_\Delta \)

Spectrum

- Spectral comparison of rectangular and raised-cosine pulses
  - Note the band-limitation of raised-cosine shaping
**Eye Diagram**

- *Eye diagram is a popular robustness evaluation tool.*
- For 4-PAM, single-baud-wide Hamming blip with additive broadband channel noise, retriggering oscilloscope after every 2 baud intervals produces

![Eye Diagram](image1)

**Eye Diagrams**

- Eye diagrams with raised-cosine pulse shaping with 2-PAM and 4-PAM systems

![Eye Diagrams](image2)
Eye Diagram Examples

- Multiple-baud wide pulses

DSP Implementation

- The input is zero padded.
- For oversampling rate of $M$, there are $M-1$ zeros between two consecutive nonzero points.