OPPORTUNISTIC WIRELESS RELAY NETWORKS

by

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To my parents who provided me with a continuous and endless support, encouragement and motivation throughout every step of my life and career. All I have and will accomplish are only possible due to your love and sacrifices.

To my beloved country, Egypt, and its people who are fighting to rebuild a great nation.

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PREFACE

This dissertation was produced in accordance with guide lines which permit the inclusion as part of the dissertation the text of an original paper or papers submitted for publication. The dissertation must still conform to all other requirements explained in the Guide for the Preparation of Masters Theses and Doctoral Dissertations at The University of Texas at Dallas. It must include a comprehensive abstract, a full introduction and literature review and a final overall conclusion. Additional material (procedural and design data as well as descriptions of equipment) must be provided in sufficient detail to allow a clear and precise judgment to be made of the importance and originality of the research reported.

It is acceptable for this dissertation to include as chapters authentic copies of papers already published, provided these meet type size, margin and legibility requirements. In such cases, connecting texts which provide logical bridges between different manuscripts are mandatory. Where the student is not the sole author of a manuscript, the student is required to make an explicit statement in the introductory material to that manuscript describing the students contribution to the work and acknowledging the contribution of the other author(s). The signatures of the Supervising Committee which precede all other material in the dissertation attest to the accuracy of this statement.

OPPORTUNISTIC WIRELESS RELAY NETWORKS

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This work studies several relay network geometries whose *opportunistic* diversity-multiplexing tradeoff (DMT) has been unknown. In the past, opportunistic analysis has been applied to multiuser diversity and relay selection based on methods of order statistics. These techniques rely on assumptions that break down in other interesting and useful network topologies, for example the opportunistic user selection in a $n \times n$ network with a relay (among many others). This work presents techniques that expand opportunistic analysis to a wider set of networks. New opportunistic methods are proposed for several network geometries and analyzed in the high signal noise ration (SNR) regime. For each of the relay geometries in the work, we study the opportunistic DMT of several relaying protocols, including amplify-and-forward, decode-and-forward, compress-and-forward, non-orthogonal amplify-forward, and dynamic decode-forward. Among the highlights of the results: in a variety of multi-user single-relay networks, simple user selection strategies are developed and shown to be DMT-optimal. It is shown that compress-forward relaying achieves the DMT upper bound for the opportunistic multiple-access relay channel as well as in the $n \times n$ user network with relay. Other protocols, e.g. dynamic decode-forward, are shown to be near optimal in several cases. Finite-precision feedback is analyzed for the opportunistic multiple-access relay channel, the opportunistic broadcast relay channel, and the opportunistic gateway channel, and is shown to be almost as good as full channel state information. In heterogeneous relay channels, it is shown that mixing relays with simple protocols, like amplify and forward, with relays with more complicated protocols, like dynamic decode and forward or compress and forward, can help achieve gains from both types of relays. For the bi-directional multi-relay channel with a direct link between the two sources, we propose a dynamic-decode and forward opportunistic scheme and prove that it is optimal at high multiplexing gains.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Opportunistic communication is generally defined as a method, often applied in the physical layer of communication networks, that chooses the best out of multiple communication alternatives at each point in time. One of the earliest examples of opportunistic communication was developed for the multiple-access quasi-static fading channels, where the channel gains of individual users follow independent distributions [1]. Under this scenario, it is throughput-optimal to allow the user with the best channel to the base station to transmit, while all other users are silent. Thus, at each point in time, the "best" user is selected to transmit to the base station.

Relay selection is an example of opportunistic communication. An early example of relay selection appeared in [2]. Bletsas et al [3, 4, 5] investigated amplify-and-forward (AF) relay selection, followed by several other works including [6, 7, 8, 9]. Decode-and-forward (DF) relay selection has also received attention [10, 11, 12, 13, 14, 15, 16]. The diversity multiplexing tradeoff (DMT) for relay selection has been investigated in relatively few works including [17] for addressing the multiplexing loss of DF relaying, and [9] for a combination of antenna selection and AF relay selection.

Despite its rapid growth, the literature on opportunistic cooperation has focused on a relatively restricted set of conditions. Broadly speaking, the scope of previous work has been on geometries and protocols where node selection can be reduced to scalar comparisons of statistically independent link gains (or simple scalar functions thereof). For example, DF relay selection compares the relay-destination links of relays that have decoded the source

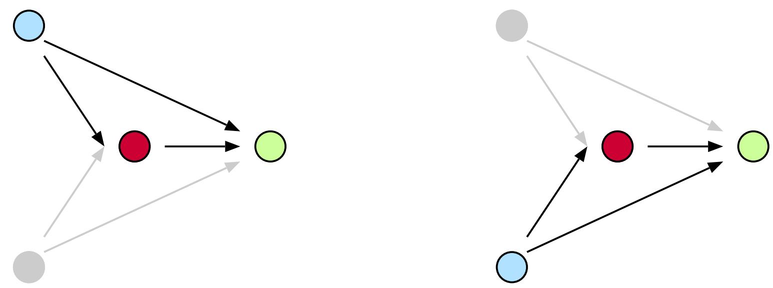

Figure 1.1. The opportunistic modes in the multiple-access relay channel.

message. In the case of AF relaying, the end-to-end SNR (or a proxy, e.g. in [3]) is used to select relays, which is again a scalar comparison among independent random variables.

This leaves open a significant set of problems for whose analysis the existing approaches are insufficient. Among them one may name even some seemingly simple problems, e.g. the DMT of the orthogonal relay on/off problem in the single-relay channel, which has been unsolved until now.

To shed light on the key difficulties, consider the example of the opportunistic multiple-access relay channel (Figure 1.1). Two users transmit messages to the base station with the assistance of a relay. During each transmission interval the system operates in one of two opportunistic modes. In one mode, user 1 transmits and user 2 is silent, while in the other, user 2 transmits and user 1 is silent. The main challenge in the analysis of this system is twofold:

1. The selection is a complex function of multiple link gains, i.e., it is not immediately clear how to select the "better" node in an easy and straight forward way. Not only do all the five link gains participate in this decision, but also the capacity of the component relay networks is generally unknown, and even the achievable rates are only known as expressions that involve nontrivial optimizations. Because the performance analysis must take into account the selection function, the complexity of analysis can quickly

get out of hand with increasing number of nodes.

2. The relay-destination link is shared among the two opportunistic modes, therefore the decision variables for the two modes are not statistically independent. The order statistics of dependent random variables are complicated and often not computable in closed form.

One of the contributions of this work is to address or circumvent the above mentioned difficulties. This work analyzes the diversity and multiplexing gain of a variety of opportunistic relay systems whose asymptotic high-SNR performance has to date been unknown. We study various networks in this work that can have multi-sources, multi-destination or multi-relays under opportunistic access schemes. Among the network geometries that have been studied are the opportunistic multiple-access and broadcast relay channels and several variations of the opportunistic $n \times n$ user network with a relay. In the $n \times n$ network with a relay, if nodes communicate pairwise while crosslink gains cannot be ignored, the links and communication structure resemble an interference channel with a relay, therefore we call it an *opportunistic interference relay channel.*[1] When the crosslink gains can be ignored, we denote it the *opportunistic shared relay channel.* If all transmitters have data for all receivers, we denote the scenario as *opportunistic X-relay channel.* The *gateway channel* represents a scenario where the only path between sources and destination is through a relay. broadcast relay channel, and the opportunistic gateway channel, and is shown to be almost as good as full channel state information. The Heterogeneous relay networks are networks of a single-source and a single-destination where multiple relays with different protocols can co-exist. If both communicating nodes have independent messages to transmits to each other, we denote it the bi-directional multi-relay channel.

[1]The naming is for convenience purposes and only reflects the presence of links not the operation of the network. In opportunistic operation there is no interference among users.

To summarize, the main results of this work are as follows:

- To begin with, the DMT of the opportunistic single-relay on/off problem is calculated under DF and AF. This simple result can be used as a building block for the study of larger networks.

- The diversity-multiplexing tradeoff of the opportunistic interference relay channel is calculated under orthogonal AF and DF, as well as non-orthogonal amplify and forward (NAF), dynamic decode and forward (DDF), and non-orthogonal compress and forward (CF). The nonorthogonal CF is shown to achieve the DMT upper bound.

- For the shared relay channel, an upper bound for the DMT under opportunistic channel access is calculated. Furthermore, it is shown that for the shared relay channel at low multiplexing gain, the DDF outperforms the NAF and CF while at medium multiplexing gains, the CF is the best. At high multiplexing gain, the relay should not be used.

- For the multiple access relay channel, a simple selection scheme based on the source-destination link gains is shown to be optimal for several protocols. Specifically, under the simple selection mechanism, the CF nonorthogonal relaying is shown to achieve the genie-aided DMT upper bound, and the NAF and the DDF also achieve their respective optimal DMTs (in the sense that more general selection schemes do not yield a better DMT).

- For the X-relay channel, an opportunistic scheme is presented that meets the DMT upper bound under the CF protocol. For other relaying protocols, the DMT regions are calculated.

- The results for the opportunistic broadcast relay channel follow from the opportunistic multiple-access relay channel.

- For the gateway channel, the superposition as well as the orthogonal channel access is studied in the absence of transmit CSI, showing that the latter is almost as good as the former. Then, the opportunistic channel access is fully characterized.

- Finite precision feedback is investigated for the multiple access relay channel (and by implication the broadcast relay channel), as well as the gateway channel. The DMT with finite-precision feedback for several other relay channels remains an open problem.

- For the parallel relay channel with n relays of the same type in the presence of a direct source-destination link, the orthogonal opportunistic relaying protocols achieve the same DMT as non-orthogonal opportunistic AF. At low multiplexing gains the DDF relay selection achieves the MISO upper bound. For high multiplexing gains, CF relay selection outperforms both DDF and NAF relay selection. DDF and CF relay selection outperform NAF relay selection at all multiplexing gains.

- For the parallel relay channel with n heterogeneous relays, we observe that the DMT of heterogeneous *orthogonal* opportunistic relaying is similar to the DMT of the corresponding homogeneous AF or DF opportunistic relay systems. For heterogeneous *non-orthogonal* relaying we consider NAF, DDF, and CF. Relay selection among NAF and DDF relays gives maximal performance at high multiplexing gains, while providing diversity somewhere between that of NAF and DDF at low multiplexing gains. Relay selection among CF and NAF relays achieves a diversity that scales with the number of relays at high multiplexing gain as opposed to the fixed diversity for NAF.

- For the bi-directional relay channel with a direct link between the two sources, we propose a DDF opportunistic scheme that is proved to be optimal in a one-relay bi-directional channel for high multiplexing gains and provide a diversity of 1 for low multiplexing gains. For multi-relay bi-directional channel, the DDF bi-directional relay selection outperforms the DF bi-directional relay selection for high multiplexing gain whereas at low multiplexing gains the DF bi-directional relaying outperforms the DDF bi-directional relaying. The DDF bi-directional relaying is optimal at mid multiplexing gains. It is also shown that for the DF bi-direction relaying, selecting two relays where each relay helps one of the sources provides no extra diversity gain compared to selecting one relay to help both sources. If there is no direct link between the sources, the AF bi-directional relaying outperforms the DF and the adaptive DF bi-directional relaying.

The organization is as follows: in Chapter 2; we describe the system model, provide the main tool for calculating the diversity multiplexing tradeoff for an opportunistic system switching between different access modes and solve the problem of a single-relay opportunistic on/off problem. Then, a succession of DMT analysis is presented for a number of network geometries and relaying protocols: in Chapter 3; for various networks of multi-source, multi-destination such as the interference relay channel in Section 3.1, the shared relay channel in Section 3.2, the X-relay channel in Section 3.3 and the gateway channel in Section 3.4, in Chapter 4; for the multiple access relay channel, in Chapter 5; for the heterogeneous relay channel with direct link and in chapter 6; for the bi-directional multi-relay channel. We conclude our work in Chapter 7.

CHAPTER 2

SYSTEM MODEL AND BASIC RESULTS FOR DMT ANALYSIS

2.1 System Model

All the nodes in the network are single-antenna and due to practical limitations, nodes cannot transmit and receive at the same time (half duplex). The channel between any two nodes experiences flat, quasi-static block fading whose coefficients are known perfectly at the receiver. The opportunistic selection mechanism also has access to channel gains, either in full or quantized. The length of the fading states (coherence length) is such that the source message is transmitted and received within one coherence interval. Furthermore, each transmission can accommodate a codeword of sufficient length so that standard coding arguments apply.

The various networks considered in this work may have either multiple sources, multiple destinations, or both. In all scenarios in this work, except for the parallel relay channel, there is one relay. For the the parallel relay channel with direct link, the network has n relays. The channel coefficients between transmitter i and receiver j is denoted with h_{ij}. Channel gains to or from a relay are shown with h_{ir} or h_{rj}. When the network has only one source, a symbolic index s is used for it; similarly if a network has no more than one destination, the index d will be used for it. For example, in a simple relay channel the links are denoted h_{sr}, h_{rd}, h_{sd}. For the parallel relay channel, h_{si} and h_{id} represent the channel gains from the source to relay i and form relay i to the destination, respectively. Channel gains are assumed independent identically distributed circularly symmetric complex Gaussian random variables. The received signals are corrupted by additive white Gaussian noise

(AWGN) which is $n_r \sim \mathcal{CN}(0, N)$ at the relay(s) and $n_j \sim \mathcal{CN}(0, N_j)$ at the destinations. Without loss of generality, in the following we assume all noises have unit variance, i.e., $N = N_j = 1 \ \ \forall j$. The transmitter nodes, the sources and the relay, have short-term individual average power constraints for each transmitted codeword. The transmit-equivalent signal-to-noise ratio (SNR) is denoted by ρ. Due to the normalization of noise variance, the SNR ρ also serves as a proxy for transmit power.

In the original definitions of opportunistic communication, e.g. multi-user diversity, only one transmitter is active during each transmission interval. For the relay networks considered in this work, the definition is slightly generalized in the following manner:

Definition 1 *Opportunistic communication is defined as a strategy where the received signal at each user during each transmission interval is independent of all but one of the transmitted messages. In other words, during each transmission interval, each receiver in the network hears only one message stream unencumbered by other message streams. The target message stream may originate from a source, a relay, or both.*

This definition maintains the spirit of opportunistic communication while allowing various non-orthogonal relaying strategies. It is noteworthy that with this generalized definition, in some networks (e.g. shared relay channel) more than one message may be in transit at the same time.

Definition 2 *An* opportunistic communication mode *is the set of active transmitters, receivers, and respective links in the network during a given transmission interval.*

This work studies the high-SNR behavior of opportunistic relay channels via the diversity-multiplexing tradeoff (DMT), in a manner similar to [18]. Each transmitter i is allocated a family of codes $\mathcal{C}_i(\rho)$ indexed by the SNR, ρ. The rate $R_i(\rho)$ denotes the data rate in bits per second per hertz and is a function of the SNR. The multiplexing gain per user r_i is defined as [18]

$$r_i = \lim_{\rho \to \infty} \frac{R_i(\rho)}{\log \rho}. \tag{2.1}$$

The selection strategy in the opportunistic relay network yields an effective end-to-end channel. The attempted rate into this effective channel is $R_i \approx r_i \log \rho$. The error probability subject to this rate is denoted $P_e(\rho)$ and the diversity gain is defined as follows.

$$d = -\lim_{\rho \to \infty} \frac{\log P_e(\rho)}{\log \rho}, \tag{2.2}$$

For the purposes of this study, since the transmission intervals are sufficiently long, the diversity can be equivalently calculated using the outage probability.

In principle, the high-SNR study of a network can generate a multiplicity of diversities and multiplexing gains. In this work we pursue the symmetric case, i.e., all opportunistic modes the have the same diversity gain d (in a manner similar to [19]) and also are required to support the same multiplexing gain r_i, where $r_i = r/n$ and r is the overall (sum) multiplexing gain.

Finally a few points regarding notation: The probability of an event is denoted with $\mathbb{P}(\cdot)$. We say two functions $f(x)$ and $g(x)$ are exponentially equal if

$$\lim_{x \to \infty} \frac{\log f(x)}{\log g(x)} = 1 \ ,$$

and denote it with $f(x) \doteq g(x)$. The exponential order of a random variable X with respect to SNR ρ is defined as

$$v = -\lim_{\rho \to \infty} \frac{\log X}{\log \rho}, \tag{2.3}$$

and denoted by $X \doteq \rho^{-v}$, $\dot{\leq}$ and $\dot{\geq}$ follow the same definition.

2.2 Basic Results for DMT Analysis

Consider an abstraction of a wireless network, shown in Figure 2.1, consisting of a set of sources, a set of destinations, and a number of data-supporting paths between them. Each of these paths may connect one or more source to one or more destination, and may consist of active wireless links as well as (possibly) relay nodes. Recall that the each collection of active paths and nodes is called an *opportunistic mode.* A concrete example of opportunistic modes was shown in Figure 1.1, where Source 1, Relay, Destination, and corresponding links make one mode, and Source 2, Relay, Destination, and corresponding links form the second mode. For the purposes of this section, the geometry of the links and relays that compose each mode is abstracted away. However, the DMT supported by each of the modes[1] is assumed to be known. Furthermore, it is assumed that only one mode can be active at any given time, i.e., we select one mode during each transmission interval.

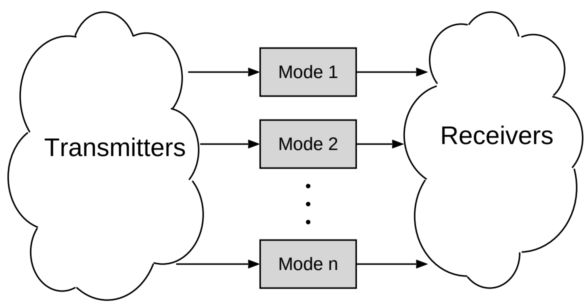

Figure 2.1. General opportunistic wireless scenario model. Each mode consists of active links, potentially including a relay.

We now produce a simple but useful result.

Lemma 1 *Consider a system that opportunistically switches between* n *paths (modes) whose*

[1]The multiplexing gain of each mode can be defined as the prelog of the overall rate carried by that mode, and similarly the diversity defined as the slope of the corresponding aggregate error rate of the data.

conditional DMT is given by $d_i'(r)$. The overall DMT is bounded by:

$$d(r) \leq d_1'(r) + d_2'(r) + \ldots + d_n'(r), \tag{2.4}$$

where $d_i'(r)$ is defined as

$$d_i'(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_i|e_{i-1},\ldots,e_1)}{\log \rho}, \tag{2.5}$$

and $\mathbb{P}(e_i|e_{i-1},\ldots,e_1)$ is the probability of error in access mode i given that all the previous access modes are in error.

Proof: We demonstrate the result for a two-user network, generalization for n users follows directly.

The total probability of error when switching between two subsystems is

$$P_e = \mathbb{P}(e_1, e_2) + \mathbb{P}(U_1, e_1, e_2^c) + \mathbb{P}(U_2, e_1^c, e_2), \tag{2.6}$$

where e_1 and e_2 are the events of error in decoding User 1 and User 2 data, respectively, the complements of error events are denoted with a superscript c, and U_1, U_2 are the events of opportunistically choosing User 1 and User 2, respectively. The event characterized by the probabilities $\mathbb{P}(U_1, e_1, e_2^c)$ and $\mathbb{P}(U_2, e_1^c, e_2)$ represents the error due to wrong selection.

We can upper bound P_e as

$$\begin{aligned} P_e &\geq \mathbb{P}(e_1, e_2) \\ &= \mathbb{P}(e_1)\mathbb{P}(e_2|e_1) \\ &\doteq \rho^{-d_1'(r)}\rho^{-d_2'(r)}, \end{aligned} \tag{2.7}$$

which implies that

$$d(r) \leq d_1'(r) + d_2'(r), \tag{2.8}$$

where $d_i'(r)$ is given by Equation (2.5). This completes the proof. □

Specializing Lemma 1 to the case of independent error probabilities directly yields the following.

Lemma 2 *A DMT upper bound for opportunistically switching between n* independent *wireless subsystems is given by $d(r)$ where*

$$d(r) \leq d_1(r) + d_2(r) + \ldots + d_n(r), \tag{2.9}$$

and $d_i(r)$ is the DMT of the subsystem i.

Lemma 3 *The upper bounds of Lemma 1 and Lemma 2 are tight if the following two conditions are asymptotically satisfied:*

1. *Each selected subsystem uses codebooks that achieve its individual DMT.*
2. *The selection criterion is such that the system is in outage only when all subsystems are in outage, i.e., $\mathbb{P}(U_1, e_1, e_2^c) = \mathbb{P}(U_2, e_1^c, e_2) = 0$.*

Throughout the remainder of the work, we assume that appropriate codebooks are designed and used, therefore the first condition is satisfied. The second condition would be satisfied by selecting access modes according to their instantaneous end-to-end mutual information. For practical reasons, we may consider simpler selection criteria, in which case the tightness of the bounds above is not automatically guaranteed.

2.3 Opportunistic One-Relay Selection

In this section we consider a simple orthogonal relaying scenario with one source, one relay and one destination. During each transmission interval, the source transmits during the first half-interval. In the second half-interval, either the relay transmits, or the relay remains silent and the source continues to transmit (see Figure 2.2). The decision between these two options is made opportunistically based on the channel gains.[2]

The question is: how should the relay on/off decision be made, and what is the resulting high-SNR performance (DMT). The apparent simplicity of the problem can be deceiving, because the random variables representing the performances of our two choices are not independent.

Theorem 1 *The DMT of a three-node simple relay channel, under either AF or DF, subject to opportunistic relay selection, is given by:*

$$d(r) = (1-r)^{+} + (1-2r)^{+}. \tag{2.10}$$

Proof: The proof is somewhat involved and is therefore relegated to Appendices A and B. An outline of the proof is as follows. The DMT of a point-to-point non-relayed link is $d(r) = (1-r)^{+}$. DF and AF orthogonal relaying [20] have the DMT $d(r) = (1-2r)^{+}$. Using the techniques described in the previously, these two DMTs are combined. The main part of the proof is to establish that the *conditional* DMT of the relay channel subject to the direct link being in outage is $d(r) = (1-2r)^{+}$, similar to its unconditional DMT, therefore the overall result follows from Lemma 1. □

This result will be used in the upcoming composite relay channels.

Remark 1 *For the simple relay channel shown above, opportunistic CF, DDF and NAF are not investigated for the following reason. In both NAF and DDF, it can be shown that the end-to-end mutual information is never increased by removing the relay from the network. Channel state information is already incorporated into the operation of NAF and DDF in such*

[2] Recall that both half-intervals are within the same coherence interval, i.e., the entire operation observes one set of channel realizations.

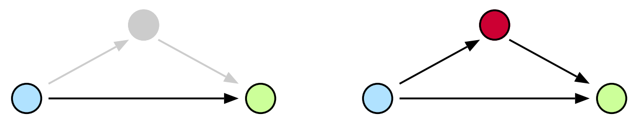

Figure 2.2. The opportunistic modes in the simple orthogonal relay channel.

a way that the usage of the relay automatically adjusts to the quality of the links. Similarly, in non-orthogonal compress-and-forward the mutual information is never increased by removing the relay.

Remark 2 *The opportunistic orthogonal amplify-and-forward attains the same DMT as the non-orthogonal amplify-and forward (NAF). The NAF has decoding complications arising from self-interference while the opportunistic orthogonal AF requires a small exchange of channel state information for opportunistic relaying (1-bit feedback from the destination node to the source and the relay).*

CHAPTER 3

MULTI-SOURCE MULTI-DESTINATION CHANNELS

3.1 Opportunistic Interference Relay Channel

This chapter is dedicated to the study of a $n \times n$ network with a relay in the opportunistic mode. The topology of the links in this network is identical to an interference relay channel, therefore this structure is called an *opportunistic interference relay channel*. The naming is a device of convenience inspired by the topology of the network.

For reference purposes, we briefly outline the background of *non-opportunistic* interference relay channel. The interference channel [21, 22] together with a relay was introduced by Sahin and Erkip [23] (Figure 3.1) who present achievable rates using full duplex relaying and rate splitting. Sridharan et al. [24] present an achievable rate region using a combination of the Han-Kobayashi coding scheme and Costa's dirty paper coding, and calculate the degrees of freedom. Maric et al. [25] study a special case where the relay can observe the signal from only one source and forward the interference to the other destination. Tannious and Nosratinia [17] show that the degrees of freedom for a MIMO interference relay channel with number of antennas at the relay matching or exceeding the number of users, is $k/2$ where k is the number of users.

As mentioned earlier, opportunistic modes are defined such that the data streams do not interfere, i.e., each receiving node is exposed to one data stream at a time. Therefore, the two-user interference relay channel has up to four access modes[1] as shown in Figure 3.2.

[1]In non-orthogonal CF, DDF, and NAF relaying protocols, the non-relayed modes never

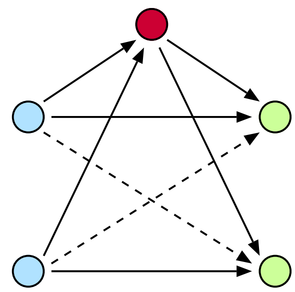

Figure 3.1. Interference relay channel.

The system selects one of the modes based on the instantaneous link gains. In the following we analyze the network under various relaying protocols and calculate the DMT in each case.

We start by developing a simple genie upper bound. Consider a genie that provides the relay with perfect knowledge of the messages of the transmitting sources. Thus access modes (c) and (d) are transformed into a MISO channel with a DMT of $2(1-r)^+$. If the *genie-aided* access mode (c) and (d) are in outage, then access modes (a) and (b) will be in outage as well, therefore they need not be considered. Applying Lemma 1, the DMT of the 2×2 user opportunistic interference relay channel is upper bounded by $4(1-r)^+$. This genie upper bound directly extends to $2n(1-r)^+$ for the $n \times n$ user topology.

3.1.1 Orthogonal Relaying

Orthogonal relaying supports the full set of four access modes in Figure 3.2. Two of the modes do not involve the relay. In the relay-assisted modes, a source transmits during the first half of the transmission interval and the relay transmits in the second half of the transmission interval.

support higher rates than the relayed modes. Therefore in CF, DDF, NAF some of these modes are never selected and can be ignored.

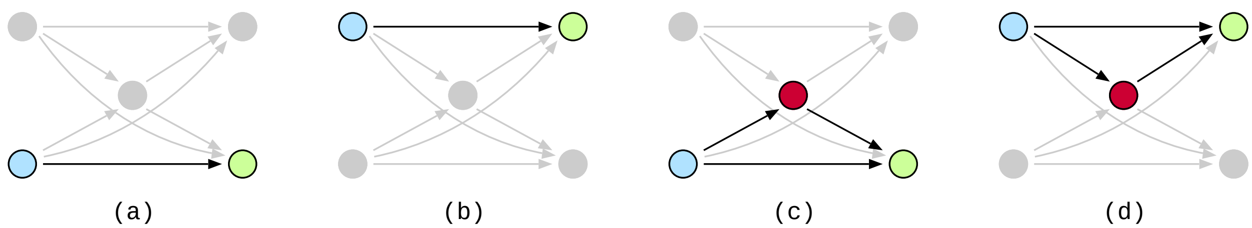

Figure 3.2. The opportunistic access modes for the interference relay channel with orthogonal relaying.

3.1.1.1 Amplify and Forward Orthogonal Relaying

In the relay-assisted modes, the relay amplifies the received signal and forwards it to the destination. We select the mode that minimizes the outage probability. The instantaneous mutual information of the non-relay access modes is given by $I_i = \log(1 + |h_{ii}|^2\rho)$ where $i = 1, 2$. The instantaneous mutual information for the relay-assisted modes under orthogonal AF is given by [20, 26]

$$I_{i+2} = \frac{1}{2}\log(1 + |h_{ii}|^2\rho + f(|h_{ir}|^2\rho, |h_{ri}|^2\rho)), \qquad i = 1, 2, \tag{3.1}$$

where $f(x, y) = \frac{xy}{x+y+1}$. The selection criterion is as follows. We first check the direct links. If none of the direct links can support the rate $r \log \rho$, we check the access modes (c) and (d). Using Lemma 1, the total DMT is given by

$$d(r) = d'_1(r) + d'_2(r) + d'_3(r) + d'_4(r), \tag{3.2}$$

where

$$d'_1(r) = \lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_1)}{\log \rho}, \qquad d'_2(r) = \lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_2)}{\log \rho},$$
$$d'_3(r) = \lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_3|e_1)}{\log \rho}, \qquad d'_4(r) = \lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_4|e_2)}{\log \rho},$$

It is easy to verify that e_1 and e_3 are independent from e_2 and e_4. Using techniques similar to the proof of Theorem 1, the outage probability of the opportunistic orthogonal AF 2×2

interference relay channel at high SNR is given by

$$\mathbb{P}(I < r\log\rho) \approx \left(\frac{e^{-2\rho^{2r-1}} - e^{-\rho^{r-1}} - e^{-2\rho^{2r-1}+\rho^{r-1}} + 1}{1 - e^{-\rho^{r-1}}}\right)^2 (1 - e^{-\rho^{r-1}})^2.$$

The total DMT can be shown to be:

$$d(r) = 2(1-r)^+ + 2(1-2r)^+. \tag{3.3}$$

Generalization to n source-destination pairs follows easily; the corresponding DMT is $d(r) = n(1-r)^+ + n(1-2r)^+$.

3.1.1.2 Decode and Forward Orthogonal Relaying

We use the same selection technique used in the orthogonal AF relaying. The instantaneous mutual information for the relay-assisted modes is given by by [20]

$$I_{i+2} = \frac{1}{2}\log\big(1 + \rho U_i\big), i = 1, 2 \tag{3.4}$$

where

$$U_i = \begin{cases} 2|h_{ii}|^2 & |h_{ir}|^2 < \frac{\rho^{2r}-1}{\rho} \\ |h_{ii}|^2 + |h_{ri}|^2 & |h_{ir}|^2 \geq \frac{\rho^{2r}-1}{\rho} \end{cases} \tag{3.5}$$

With the same type of argument used to calculate the DMT for the opportunistic orthogonal AF interference relay channel and Appendix A, the outage probability of the opportunistic orthogonal 2×2 DF interference relay channel at high SNR is given by

$$\mathbb{P}(I < r\log\rho) \approx \left(1 - e^{-\rho^{2r-1}} + \frac{(1 - e^{-\rho^{r-1}} - \rho^{r-1}e^{-\rho^{2r-1}})e^{-\rho^{2r-1}}}{1 - e^{-\rho^{r-1}}}\right)^2 (1 - e^{-\rho^{r-1}})^2.$$

It can be shown that the DMT in case of orthogonal DF is

$$d(r) = n(1-r)^+ + n(1-2r)^+. \tag{3.6}$$

3.1.2 Non-Orthogonal Relaying

In the non-orthogonal protocols considered in this section, the source transmits throughout the transmission interval, while the relay transmits during part of the transmission interval. The source and relay signals are (partially) superimposed at the destination. Note that this superposition does not violate our working definition of opportunistic communication, which states that received signals at the destinations are independent of all but one of the transmitted messages.

Under the non-orthogonal relaying protocols, the interference relay channel has only two access modes, Figure 3.2 (c) and (d). Access modes (a) and (b) are not considered, because it can be shown that in non-orthogonal relaying, the end-to-end mutual information of the relay-assisted modes is always greater than the corresponding non-relayed modes.

3.1.2.1 Non-Orthogonal Amplify and Forward

For half the transmission interval, the received signal at the destination and at the relay are given by [27]

$$y_{1i} = \sqrt{\rho}\, h_{ii}\, x_{1i} + n_{1i}, \quad y_{1r} = \sqrt{\rho}\; h_{ir}\; x_{1i} + n_{1r},$$

The variables x, y, n have two subscripts indicating the appropriate half-interval and node identity, respectively. For example, y_{1r} is the received signal during the first half-interval at the relay, while x_{1i} is the transmit signal at the first half-interval from source i. At the second half of the transmission interval the relay normalizes the received signal (to satisfy the relay power constraint) and retransmits it. The destination received signal in the second half is given by

$$y_{2i} = \sqrt{\rho}\; h_{ii}\; x_{2i} + \frac{\sqrt{\rho}\, h_{ri}}{\sqrt{\rho |h_{ir}|^2 + 1}}\; y_{1r} + n_{2i},$$

where a similar notation holds, for example x_{2i} is signal transmitted in the second half-interval from source node i. The effective destination noise during this time is $\frac{\sqrt{\rho} h_{ri}}{\sqrt{\rho |h_{ir}|^2+1}}\, n_{1r} +$

n_{2i}. For convenience, in a manner following [27] we divide y_{2i} by a constant factor to normalize the effective noise variance to unity, while not otherwise affecting the SNR. We can write the received signal at destination node i as follows

$$Y_i = \mathbf{H}_i X_i + N, \tag{3.7}$$

where $X_i = [x_{1i} \;\; x_{2i}]^t$ is a vector of the transmit signals the two half-intervals, N is the vector of Gaussian noise in the two half-intervals, which is (subject to the latest normalization) a circularly symmetric complex Gaussian random variable, and $\mathbf{H}_i$ is the effective channel gain matrix:

$$\mathbf{H}_i = \begin{bmatrix} \sqrt{\rho}\, h_{ii} & 0 \\ \frac{\rho\, h_{ri} h_{ir}}{\sqrt{\rho|h_{ir}|^2+1}\sqrt{\frac{\rho|h_{ri}|^2}{\rho|h_{ir}|^2+1}+1}} & \frac{\sqrt{\rho}\, h_{ii}}{\sqrt{\frac{\rho|h_{ri}|^2}{\rho|h_{ir}|^2+1}+1}} \end{bmatrix}. \tag{3.8}$$

The instantaneous mutual information is given by

$$\begin{aligned} I_i &= \frac{1}{2}\log\left|\mathbf{I} + \mathbf{H}_i\mathbf{H}_i^*\right| \\ &= \frac{1}{2}\log\left(1 + |h_{ii}|^2\rho + \frac{|h_{ri}|^2|h_{ir}|^2\rho^2}{|h_{ri}|^2\rho + |h_{ir}|^2\rho + 1} + \frac{|h_{ii}|^2(|h_{ir}|^2\rho+1)\rho}{|h_{ri}|^2\rho + |h_{ir}|^2\rho + 1} + \frac{|h_{ii}|^4(|h_{ir}|^2\rho+1)\rho^2}{|h_{ri}|^2\rho + |h_{ir}|^2\rho + 1}\right). \end{aligned} \tag{3.9}$$

User i^* is selected to maximize the mutual information, which at high SNR can be shown to lead to the following selection rule:

$$i^* = \arg\max_i I_i = \arg\max_i \left\{\frac{|h_{ii}|^4|h_{ir}|^2}{|h_{ri}|^2 + |h_{ir}|^2}\right\}. \tag{3.10}$$

Using our knowledge of the DMT of non-opportunistic NAF [28] which is given by $d(r) = (1-r)^+ + (1-2r)^+$, and applying Lemmas 2, 3 and using the selection criterion i^* from Equation (3.10), the DMT of opportunistic NAF interference relay channel with n source-destination pairs is

$$d(r) = n(1-r)^+ + n(1-2r)^+. \tag{3.11}$$

3.1.2.2 Dynamic Decode and Forward

The relay listens to the source until it has enough information to decode. The relay re-encodes the message using an independent Gaussian codebook and transmits it during the remainder of the transmission interval. The time needed for the relay to decode the message depends on the quality of the source-relay channel. Using [28] and Lemma 1, the DMT of the optimal opportunistic DDF interference relay channel is as follows:

$$d(r) = \begin{cases} 2n(1-r) & 0 \leq r \leq \frac{1}{2}, \\ n\frac{1-r}{r} & \frac{1}{2} < r \leq 1. \end{cases} \tag{3.12}$$

Compared to the other protocols considered for the interference relay channel, the DDF mutual information for each node has a more complex expression. This provides an impetus for the analysis of simpler selection scenarios. It has been observed elsewhere in this work that selection based on source-destination link gains sometimes may perform well, therefore we consider that choice function for the DDF interference relay channel. Following the same technique as [29], the resulting DMT can be shown to be

$$d(r) = \begin{cases} (n+1)(1-r) & 0 \leq r < \frac{n}{n+1} \\ n\frac{1-r}{r} & \frac{n}{n+1} \leq r \leq 1 \end{cases} \tag{3.13}$$

It is observed that for DDF, selection based on direct link gains is clearly suboptimal, especially at low multiplexing gains.

3.1.2.3 Compress and Forward

Following [30], the relay listens to the selected source for a percentage t of the transmission interval. The source and the relay perform block Markov superposition coding, and the destination employs backward decoding [31]. The relay performs Wyner-Ziv compression, exploiting the destination's side information. This ensures that the relay message can be received error free at the receiver. The relay compression ratio must satisfy

$$I(\hat{y}_r; y_r | x_r, y_d) \leq I(x_r; y_d). \tag{3.14}$$

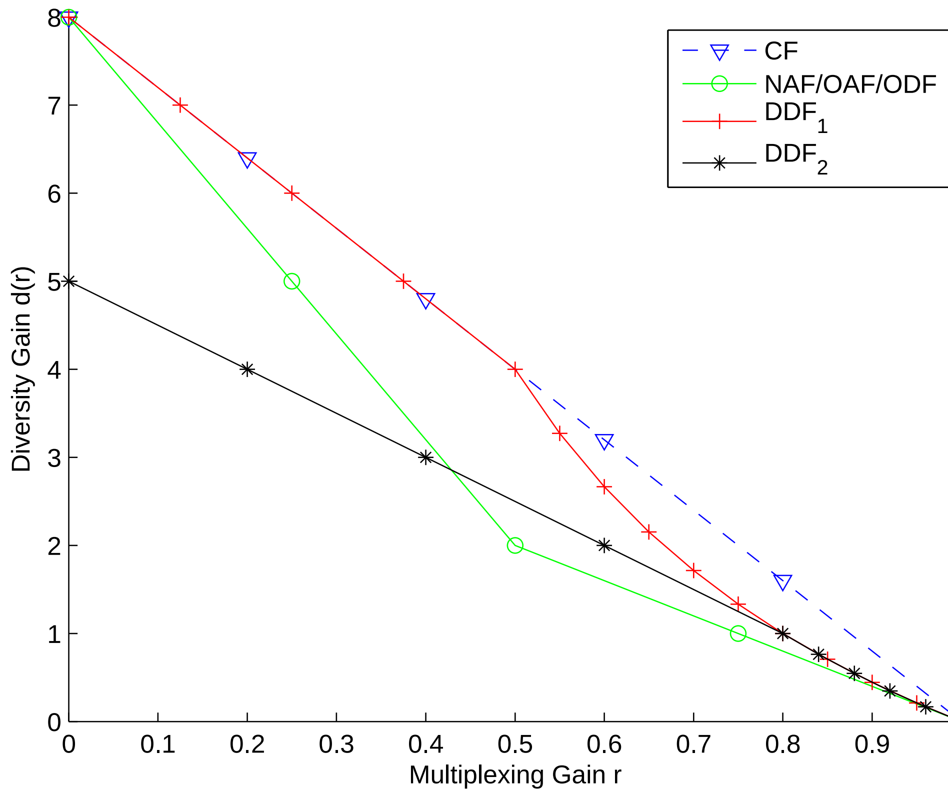

Figure 3.3. Diversity multiplexing trade-off for a 4 source-destination pairs interference relay channel using different opportunistic relaying schemes.

Yuksel and Erkip [30] show that the optimal DMT, $d(r) = 2(1-r)^+$, is achieved when the relay listens for half the transmission interval and transmits during the remainder of time in the interval[2].

For opportunistic compress and forward interference relay channel, the user $i^* = \arg\max_i I_i$ is selected, where I_i is the mutual information for each access mode. At high-

[2]The work in [30] assumes transmit channel state information at the relay to insure that the relay's message reaches the destination error free. Recent work [32] proves that the same DMT can be achieved using quantize-and-forward relaying with only receivers channel state information. Another relaying protocol, dynamic compress-and-forward, is analyzed in [33] without direct link and is shown to achieve the optimal DMT without channel state information at the relay.

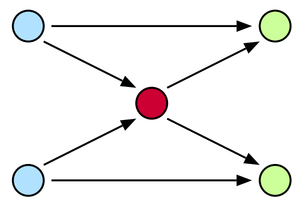

Figure 3.4. Shared relay channel.

SNR, using results from [30], the selected user i^* can be proved to be

$$i^* = \arg\max_i \frac{(|h_{s_i,r}|^2 + |h_{s_i,d_i}|^2)(|h_{r,d_i}|^2 + |h_{s_i,d_i}|^2)|h_{s_i,d_i}|^2}{(|h_{s_i,r}|^2 + |h_{s_i,d_i}|^2) + (|h_{r,d_i}|^2 + |h_{s_i,d_i}|^2)}.$$

Each mode can achieve a DMT $d(r) = 2(1-r)^+$, hence the opportunistic system with n source-destination pairs can achieve the DMT $d(r) = 2n(1-r)^+$.

Figure 3.3 compares the DMT of various relaying schemes for the interference relay channel with four source-destination pairs. The optimal opportunistic DDF relaying is denoted by DDF1 and DDF relaying with the simple selection criterion (based on source-destination link gains) is denoted by DDF2. Compress and forward achieves the optimal DMT but requires full CSI at the relay.

3.2 Opportunistic Shared Relay Channel

The shared relay channel (SRC) (Figure 3.4) was introduced in [34] with the sources using TDMA channel access and orthogonal source and relay transmissions. In [35], based on superposition and dirty paper coding, lower and upper bounds on the capacity of additive white Gaussian noise (AWGN) MIMO shared relay channel are presented.

In the shared relay channel, the direct link between each source and its destination is free from interference from the other source, however, the relay can cause indirect interference if it assists both sources at the same time. Therefore, in the opportunistic mode the relay should either assist one of the users or none of them (Figure 3.5). We assume the access

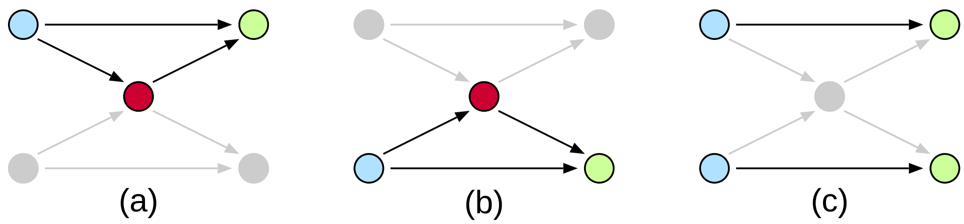

Figure 3.5. Opportunistic access modes for the shared relay channel.

mode that minimizes the outage probability is chosen. In our analysis, access modes support equal rate, thus in the first two access modes, one source transmits at rate $R = r \log \rho$, while in the third access mode both sources transmit, each with a rate $R_i = r/2 \log \rho$.

3.2.1 DMT Upper Bound

First we use Lemma 1 to derive an upper bound. This bound will be tightened subsequently.

Theorem 2 *An upper bound for the opportunistic shared relay channel with the access modes shown in Figure 3.5 is given by*

$$d(r) \leq \begin{cases} 2n & r < \frac{n}{2n^2+1} \\ (2n+1) - (2n + \frac{1}{n})r & \frac{n}{2n^2+1} \leq r < 1 \\ (1 - \frac{r}{n}) & 1 \leq r \leq n. \end{cases} \tag{3.15}$$

Proof: The proof uses Lemma 1, adding the DMT of the three access modes and the fact that the maximum diversity order for the shared relay channel cannot exceed $2n$. Details of the proof are given in Appendix C. $\square$

A tighter upper bound can be found by assuming a genie that provides the relay with the source information. In Figure 3.5, we call modes (a) and (b) *relay-assisted access modes* while denoting mode (c) a *non-relayed access mode*. Thus, in the presence of a genie, the relay-assisted access modes are essentially equivalent to MISO links. The non-relay access mode is obviously not affected by the genie.

Theorem 3 *A DMT upper bound for the genie aided opportunistic shared relay channel is given by*

$$d(r) \leq \left(1 - \frac{r}{n}\right)^{+} + (2n-1)(1-r)^{+} \tag{3.16}$$

$$= \begin{cases} 2n - (2n - 1 + \frac{1}{n})r & 0 \leq r \leq 1 \\ (1 - \frac{r}{n}) & 1 < r \leq n \end{cases} \tag{3.17}$$

Proof: The proof uses Lemma 1 taking into account the dependency between the different access modes. Details of the proof are given in Appendix D. □

We notice that for high multiplexing gain, $r > 1$, the first and second access modes do not contribute to the diversity gain where the third mode is always active. For low multiplexing gain, $r \leq 1$, the three access modes are contributing to the total diversity gain of the system and switching between the three access modes should be considered.

For clarity of exposition, we assume two source-destination pairs in the remainder of the analysis. However, the analysis is expandable to any number of node pairs in a manner that is straightforward.

3.2.2 Achievable DMT

If we allow ourselves to be guided by the upper bound above, it is reasonable to use the non-relay access mode for high multiplexing gains ($r > 1$). This makes intuitive sense, since relayed access modes cannot support high multiplexing gains. For multiplexing gains less than 1, switching between the three access mode should be considered.

In the following we sometimes consider a simplified selection by partitioning the decision space: in one partition choosing only among relayed access modes (easier due to their independence) and in another partition using only the non-relayed mode. We shall see that sometimes this easier switching scheme suffices and one can thus avoid the cost of the comparison among all three modes.

The following DMT are subject to the two conditions mentioned in Lemma 3.

3.2.2.1 Non-Orthogonal Amplify and Forward

First consider NAF relaying while using only the two relayed opportunistic modes (ignoring for the moment the non-relay mode). Using Lemma 1 and the results in [28] one can show:

$$d(r) = 2(1-r)^{+} + 2(1-2r)^{+}. \tag{3.18}$$

The non-relayed mode achieves the following DMT

$$d(r) = (1-\frac{r}{2})^{+}. \tag{3.19}$$

To begin with, consider a hybrid scheme that chooses between the two relay-assisted access modes when $r < \frac{2}{3}$, and uses the non-relayed mode when $r \geq \frac{2}{3}$. It is easy to obtain the DMT of this strategy, since the only opportunistic action is between independent modes. This strategy leads to the following DMT

$$\begin{aligned} d(r) &= \max\left\{2(1-r)^{+} + 2(1-2r)^{+}, \left(1-\frac{r}{2}\right)^{+}\right\} \\ &= \left\{ \begin{array}{ll} 4-6r & 0 \leq r \leq 0.5, \\ 2(1-r) & 0.5 \leq r \leq \frac{2}{3} \\ (1-\frac{r}{2}) & \frac{2}{3} < r \leq 2. \end{array} \right. \end{aligned} \tag{3.20}$$

Naturally, there is no guarantee that the above strategy is optimal. For the best results, once must compare directly the three opportunistic modes, but then the DMT requires nontrivial calculations, as characterized by the following result. We note that the comparison of mutual informations for the amplify-and-forward strategy in the high-SNR regime has been characterized earlier.

Theorem 4 *The overall DMT for the opportunistic shared relay channel under NAF relay-*

ing protocol is given by

$$d(r) = 2(1-2r)^+ + (1-\frac{r}{2})^+ + (1-r)^+$$
$$= \begin{cases} 4 - \frac{11}{2}r & 0 \le r \le 0.5 \\ 2 - \frac{3}{2}r & 0.5 < r \le 1 \\ 1 - \frac{r}{2} & 1 < r \le 2. \end{cases} \tag{3.21}$$

Proof: The proof uses Lemma 1 and results from MIMO point to point communication [18] and NAF relaying [28] taking into account the dependency between the different access modes. Details are given in Appendix E. $\square$

3.2.2.2 Dynamic Decode and Forward

We saw that a simple hybrid method using the relay-assisted modes at low r and the non-relay mode at high r achieves a reasonable (although suboptimal) DMT for NAF, so we begin by investigating a similar strategy for the DDF. Using Lemma 1 and [28], opportunistic communication using only the relay-assisted modes has at best the following DMT

$$d(r) = \begin{cases} 4(1-r) & 0 \le r \le 0.5 \\ 2\frac{1-r}{r} & 0.5 < r \le 1. \end{cases} \tag{3.22}$$

The DMT of the non-relayed scheme is $(1-\frac{r}{2})^+$. We now choose at each r the better of the non-relayed DMT or opportunistic relay-assisted DMT of Equation (3.22). This hybrid (partitioned) strategy will yield:

$$d(r) = \begin{cases} 4(1-r) & 0 \le r \le 0.5 \\ 2\frac{1-r}{r} & 0.5 < r \le 3-\sqrt{5} \\ 1-\frac{r}{2} & 3-\sqrt{5} < r \le 2. \end{cases} \tag{3.23}$$

Although this strategy is simple and easy to analyze, it can be shown that it is sub-optimal for the DDF. For some values of $r < 1$, in DDF it is advantageous to make all three access modes available to the selection algorithm. This result is developed in the following theorem.

Theorem 5 *The overall DMT for the opportunistic shared relay channel under DDF relaying protocol is given by*

$$d(r) = \begin{cases} \left(1 - \frac{r}{1-r}\left(1 - \frac{r}{2}\right)\right) + 2(1-r) + \left(1 - \frac{r}{2}\right), & 0 \le r \le 0.5 \\ 2\frac{(1-r)}{r}, & 0.5 < r \le 2 - \sqrt{2} \\ \frac{(1-r)}{r} + \left(1 - \frac{r}{2}\right), & 2 - \sqrt{2} < r \le 1, \\ \left(1 - \frac{r}{2}\right), & 1 < r \le 2. \end{cases} \tag{3.24}$$

Proof: The proof uses Lemma 1 and results for DDF relaying [28], while taking into account the dependency between the three access modes. Details are given in Appendix G. □

3.2.2.3 Compress and forward

Using the CF scheme in [30], optimally selecting among the two relay-assisted modes can achieve $d(r) = 4(1-r)^+$. In a manner similar to NAF and DDF cases above, we can consider relay-assisted opportunistic DMT and non-relay mode DMT, and take the maximum of the two expressions at each value of r. This hybrid strategy yields the following DMT.

$$d(r) = \begin{cases} 4(1-r), & 0 \le r \le \frac{6}{7} \\ (1 - \frac{r}{2}), & \frac{6}{7} < r \le 2 \end{cases} \tag{3.25}$$

One can show that optimization between all three access modes at each r cannot yield a better DMT under CF relaying, therefore the result above cannot be improved upon. The proof is given in Appendix H.

Remark 3 *The trivial hybrid scheme of using the relay-assisted modes at low multiplexing gains and the direct links at high multiplexing links is not always suboptimal. It is shown that for NAF and DDF, better performance is achieved by considering the three access modes at low multiplexing gains. For CF relaying, the non-relayed access mode is not helping at low multiplexing gains, hence, the hybrid scheme is optimal.*

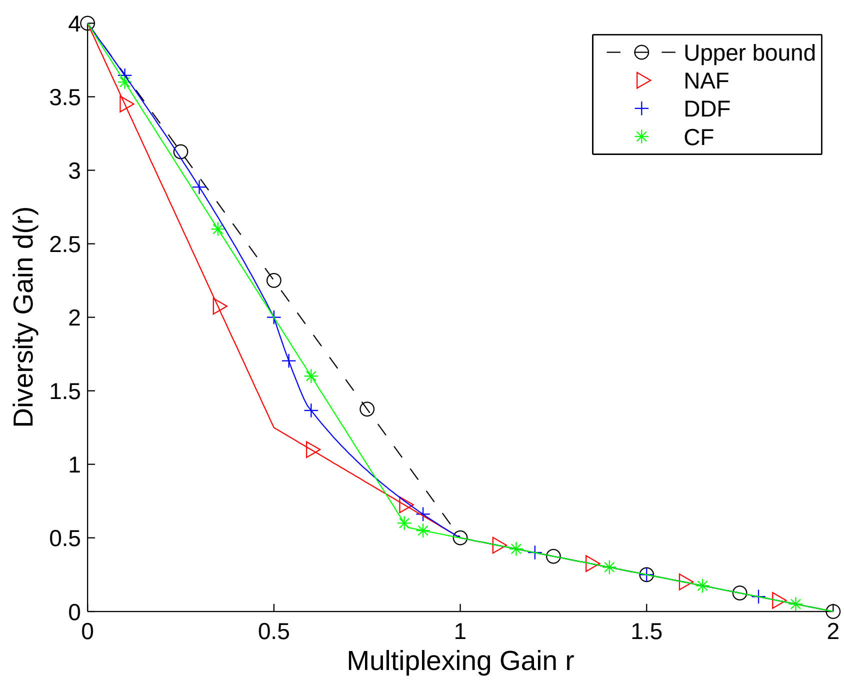

Figure 3.6. Diversity multiplexing trade-off for a 2-pair shared relay channel, demonstrating the performance of various protocols.

Remark 4 *Using the same technique used to prove the DMT of the orthogonal opportunistic simple relay channel, Appendix A and B, and Lemma 1, one can show that the DMT of the opportunistic shared relay channel under either orthogonal AF or orthogonal DF is given by*

$$d(r) = 2(1-2r)^{+} + (1-r/2)^{+}, \tag{3.26}$$

where the access modes are defined as before and the relay always transmits orthogonal to the sources.

To summarize the results, as seen in Figure 3.6, for the opportunistic shared relay channel, a brief comparison between three relaying protocols NAF, DDF, and CF is as follows. At low multiplexing gain the DDF outperforms NAF and CF. At medium multiplexing gains, the relay does not have enough time to fully forward the decoded message to the destination and the CF in this case outperforms the DDF. At multiplexing gains above 1, it does not matter which relaying protocol is used since the DMT-optimal strategy uses direct (non-relayed) mode.

3.3 Opportunistic X-Relay Channel

The X-relay channel is defined as a $n \times n$ node wireless network with a relay, where each of the n sources has messages for each of the n destinations (see Figure 3.7). The sources are not allowed to cooperate with each other, but the relay cooperates with all sources.

There are only a few results available on the X channel, among them, it has been shown [36] that the X-channel with no relay has exactly $\frac{4}{3}$ degrees of freedom when the channels vary with time and frequency. The X-relay channel introduces a relay to the X channel for improved performance.

The opportunistic X-relay channel has four access modes as shown in Figure 3.8. These modes avoid interference across different message streams and satisfy our working

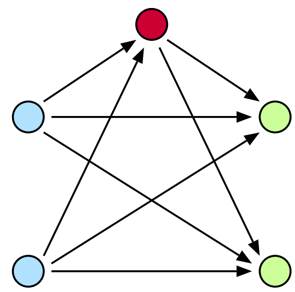

Figure 3.7. The X-relay channel.

definition of opportunistic modes in relay networks.

3.3.1 DMT Upper Bound

To find an upper bound for the DMT of opportunistic X-relay channel, we assume a genie transfers the data from the sources to the relay and also allows the sources to know each other's messages. For the upper bound we also allow the destinations to fully cooperate, noting that it can only improve the performance. Figure 3.9 shows the genie-aided opportunistic modes, where the two-sided arrows indicate the free exchange of information by the genie. From this figure, it is easy to see that the genie-aided X-relay channel is equivalent to a MIMO system with 3 transmit antennas and 2 receive antennas.

The performance of the *opportunistic* X-relay channel is therefore upper bounded by a 3×2 MIMO system with *antenna selection*, choosing for each codeword two transmitting and one receiving antennas. It is noteworthy that the 3×2 antenna selection allows one configuration that does not have a counterpart in the opportunistic modes in the X-relay channel, therefore due to the extra flexibility the MIMO system with antenna selection upper bounds the performance of the genie-aided opportunistic X-relay channel.

Using the result from Equation (4.1), a 3×2 MIMO system with two antennas selected from the transmitter side and one antenna selected from the receiver side has a DMT that is upper bounded by $d(r) = 6(1-r)^+$. This in turn is an upper bound to the performance

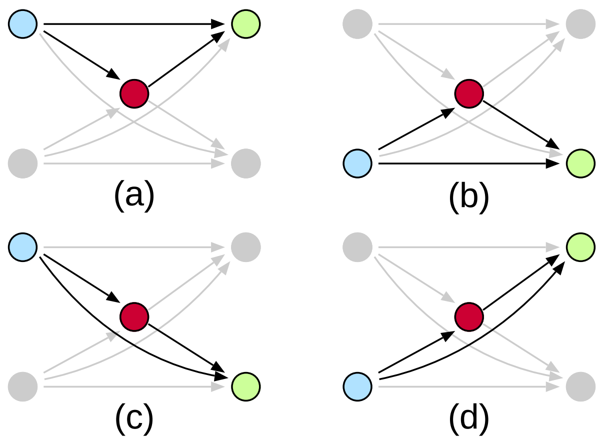

Figure 3.8. Opportunistic modes of the X-relay channel

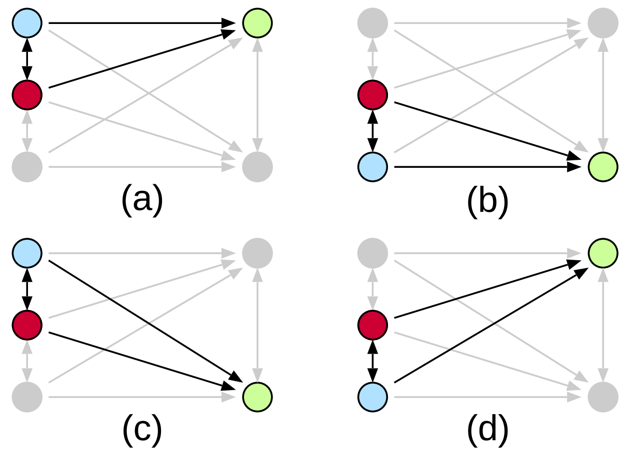

Figure 3.9. Opportunistic modes of genie-aided X-relay channel.

of the opportunistic X-relay channel.

3.3.2 Achievable DMT

For deriving achievable rates, we consider the following simplified opportunistic scheme. First, we choose between the two access modes (a) and (b) in Figure 3.8. If both these two modes are in outage, we consider *only* the direct link of the two access modes (c) and (d), i.e., the relay is not allowed to cooperate in modes (c) and (d). Note that this is only a simplification for the purposes of achievable-DMT analysis, the idea being that if the relay is useful in neither of the access modes (a) and (b), it is unlikely to be useful at all. The approximation involving the conditional removal of the relay from (c) and (d) allows the access modes to become independent and simplifies the analysis. The resulting achievable rate is tight against the upper bound for compress-forward, as seen in the sequel, but not demonstrably so for other protocols.

Access modes (a) and (b) do not share any common links, therefore their statistics are independent. Each of them is an ordinary relay channel which can achieve $d(r) = 2(1-r)^+$ via the CF protocol [30]. The (c) and (d) access modes, which were reduced to a single link, each achieves the DMT $d(r) = (1-r)^+$. Furthermore, the source-destination links in (c) and (d) are disjoint from the links in (a) and (b), therefore the statistics are independent and we can use Lemma 1 to find the overall DMT $d(r) = 6(1-r)^+$. Note that this achievable DMT meets the upper bound, therefore the DMT of the X-relay channel under CF is exactly $d(r) = 6(1-r)^+$.

Achievability results for relaying protocols other than CF can be obtained along the same lines. We begin with NAF. Recall that the DMT of a simple relay network (source, relay, destination) under NAF is $d(r) = (1-r)^+ + (1-2r)^+$. Combining the four access modes (a), (b), (c), (d) mentioned earlier for the X-relay channel together with the NAF protocol results in:

$$\begin{aligned} d_{XNAF}(r) &= 2(1-r)^+ + 2\Big[(1-r)^+ + (1-2r)^+\Big] \\ &= 4(1-r)^+ + 2(1-2r)^+ \end{aligned} \tag{3.27}$$

A similar result exists for the DDF where the DMT is given by

$$d_{XDDF}(r) = \begin{cases} 6(1-r) & 0 \leq r < \frac{1}{2} \\ 2\frac{1-r}{r} + 2(1-r) & \frac{1}{2} \leq r \leq 1 \end{cases} \tag{3.28}$$

Applying the same analysis to orthogonal AF and DF yields a diversity $d(r) = 2(1-r)^+ + 4(1-2r)^+$, but there is more to be said for orthogonal transmission. In orthogonal transmission it may be beneficial at high multiplexing gains to shut down the relay, therefore a complete analysis requires two more opportunistic modes that are derived by shutting down the relay from modes (a) and (b). Using this extended set of six access modes, the DMT of the opportunistic X-relay channel with orthogonal AF or orthogonal DF is

$$d(r) = 4(1-r)^+ + 2(1-2r)^+ \tag{3.29}$$

which matches the DMT of NAF.

Thus far, to find achievable DMTs for the X relay channel we used simplified selection rules and access modes. In the case of CF, this simplified achievable DMT is in fact optimal since it matches the genie upper bound. Other protocols do not meet the genie-aided bound, therefore the question of the optimality of simplified selection for other protocols is more involved. Nevertheless, for NAF and DDF also, no DMT gains can be obtained by more sophisticated selection rules and access modes, as outlined below.

To find the overall optimal DMT without the simplifications, we need to solve a linear optimization problem similar to (4.27) where

$$d_o(r) = \inf_{(v_1^{(ij)}, v_2^{(rj)}, u^{(jr)}) \in O,\ i,j \in \{1,2\}} \sum_{j=1}^{2} \Big(\sum_{i=1}^{2} v_1^{(ij)} + v_2^{(rj)} + u^{(jr)} \Big), \tag{3.30}$$

where $v_1^{(ij)}, v_2^{(rj)}$ and $u^{(jr)}$ represent the exponential order of $1/|h_{ij}|^2$, $1/|h_{rj}|^2$ and $1/|h_{jr}|^2$, respectively. The outage event O is characterized by $\mathcal{O}_1^+ \cap \mathcal{O}_2^+ \cap \mathcal{O}_3^+ \cap \mathcal{O}_4^+$, i.e., the system is in outage if all access modes are in outage. The outage event is given by Equation (4.29) for NAF and Equation (4.31) for DDF. In a straight forward manner, the optimization above gives the same DMTs found by the simplified selection criterion, therefore the calculated DMTs cannot be improved upon and are optimal. A comparison between all these relaying schemes is shown in Figre 3.10.

3.4 The Gateway Channel

The gateway channel [37] is a multi-node network with M source-destination pairs that communicate with the help of a relay (see Figure 3.11). Each source communicates only with its corresponding destination. A two-hop communication scheme is used, where at the first hop the sources transmit to the relay and at the second hop the relay transmits to the destinations. No direct link exists between the sources and destinations, therefore if the relay is in outage the destination will surely be in outage. Under these conditions, the most natural mode of operation is decode-and-forward, although amplify-and-forward may also be considered due to practical limitations. In this work we concentrate on the DF gateway channel.

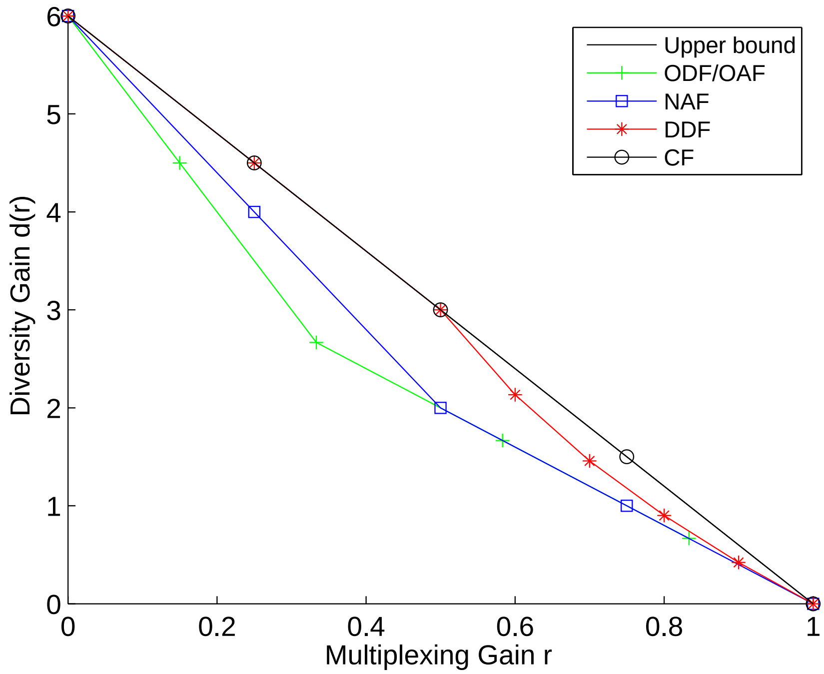

Figure 3.10. The DMT of the opportunistic X-relay channel under various relaying protocols

We do not require data buffering at the relay. With an infinite buffer at the relay, the gateway channel decomposes into a concatenation of a MAC and a broadcast channel. An infinite buffer would thus simplify the analysis but also increase the overall latency and relay complexity. One of the interesting outcomes of the forthcoming analysis is that that data buffering in the asymptotic high-SNR regime does not provide a performance advantage (in the sense of DMT).

We start with the non-opportunistic gateway channel, and then move to the opportunistic scheme.

3.4.1 No Transmit CSI

We first consider the case where all nodes have receive-side CSI, but the nodes, and in particular the relay, do not have transmit-side CSI. Under these conditions, we cannot choose

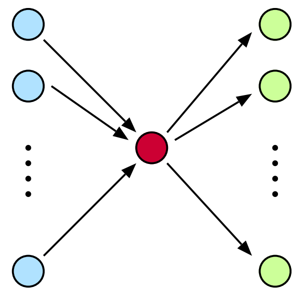

Figure 3.11. The gateway channel.

source-destination pairs according to their SNR. Then the choice of transmission strategies on the MAC and broadcast side of the network are as follows.

On the broadcast side, the channel gains are random and unknown to the relay. In light of symmetric rate requirements, the transmit strategy must be symmetric with respect to the destinations. Under this symmetry, the best achievable rate is according to orthogonal transmission [38] and superposition coding does not give better results.

For the multiple-access side, under symmetric rate requirement, both orthogonal and superposition channel access are viable. It has been shown that superposition access gives slightly better performance at medium SNR, while at high and low SNR the two methods have asymptotically the same capacity under symmetric rates [38, pp. 243-245].

In the absence of transmit-side CSI, and with symmetric rate requirements, the network does indeed decompose into a cascade of a multiple-access and a broadcast subnetworks, and the overall outage probability is given by:

$$\begin{aligned} P_{\mathcal{O}} &= 1 - (1 - P_{MAC})(1 - P_{BC}) \\ &= P_{MAC} + P_{BC} - P_{MAC}P_{BC}. \end{aligned} \tag{3.31}$$

Where P_{MAC} (respectively P_{BC}) denotes the outage of the MAC (respectively broadcast channel), defined as the probability that one or more of the users in the MAC (respectively broadcast channel) cannot support rate R. In a slowly fading environment, for a power allocation vector $P_s = (P_1, \ldots, P_M)$ and a fading state $H = (h_{1r}, \ldots, h_{Mr})$, the following rates are achievable for the MAC under superposition coding

$$\mathcal{C}_{MAC}(H,P) = \left\{ \bar{R} : \sum_{i \in S} R_i \leq \frac{1}{2} \log \left(1 + \frac{1}{N} \sum_{i \in S} |h_{ir}|^2 P_i \right) \right\}, \tag{3.32}$$

where $\bar{R}$ is the rate vector and $S \subseteq \{1, \ldots, M\}$. The outage is:

$$\begin{aligned} P_{MAC} &= \mathbb{P}\big(\bar{R} \notin \mathcal{C}_{MAC}\big) \\ &= \mathbb{P}\left(|S|R > \frac{1}{2} \log \left(1 + \rho \sum_{i \in S} |h_{ir}|^2 \right) \right) \\ &= \mathbb{P}\left(\sum_{i \in S} |h_{ir}|^2 < \frac{2^{2|S|R} - 1}{\rho} \right), \end{aligned} \tag{3.33}$$

where $|S|$ denotes the cardinality of the set S. From Equation (3.33), with the assumption of independent identically distributed Rayleigh fading coefficients, the outage probability in the MAC can be shown to be

$$\begin{aligned} P_{MAC} &= 1 - \prod_{i=1}^{M} \mathbb{P}\left(|h_{ir}|^2 \geq \frac{\rho^{2r} - 1}{\rho} \right) \\ &\times \prod_{k=2}^{M} \left(\frac{\mathbb{P}\left(\sum_{j=1}^{k} |h_{jr}|^2 \geq \frac{2^{2kR}-1}{\rho} \right)}{\mathbb{P}\left(\sum_{j=1}^{k} |h_{jr}|^2 \geq \left(\frac{2^{2(k-1)R}-1}{\rho} \right) \frac{k}{k-1} \right)} \right)^{\binom{M}{k}} \\ &= 1 - \left(e^{\left(-\lambda \frac{\rho^{2r}-1}{\rho} \right)} \right)^{M} \\ &\times \prod_{k=2}^{M} \left(\frac{1 - \gamma\left(k, \lambda \frac{2^{2kR}-1}{\rho} \right)}{1 - \gamma\left(k, \frac{k}{k-1} \lambda \frac{2^{2(k-1)R}-1}{\rho} \right)} \right)^{\binom{M}{k}}, \end{aligned} \tag{3.34}$$

where $\lambda = 1$ is the exponential parameter of the distribution of $|h_{ir}|^2$ and $|h_{ri}|^2$, the function $\gamma(i,x) \triangleq \frac{1}{\Gamma(i)} \int_0^x t^{i-1} e^{-t} dt = 1 - e^{-x} \sum_{k=0}^{i-1} \frac{x^k}{k!}$ is the (lower) incomplete gamma function, and $\Gamma(\cdot)$ is the Gamma function. The above outage was calculated using superposition coding.

With a time-sharing MAC, the outage probability is:

$$\begin{aligned} P_{MAC} &= \mathbb{P}\Big(\Big\{R_i > \frac{1}{2M}\log\big(1+\rho|h_{ir}|^2\big),\ i=1,\ldots,M\Big\}\Big) \\ &= 1-\exp\bigg(-M\lambda\frac{2^{2MR}-1}{\rho}\bigg) \end{aligned} \tag{3.35}$$

On the broadcast side, the following rates are achievable in the fading state $h_{rd} = (h_{r1},\ldots,h_{rM})$ by time sharing

$$\mathcal{C}_{BC}(h_{rd}) = \Big\{\bar{R} : R_i \le \frac{1}{2M}\log\big(1+\rho|h_{ri}|^2\big)\Big\}, \tag{3.36}$$

Thus the outage is

$$\begin{aligned} P_{BC} &= \mathbb{P}\big(\bar{R} \notin \mathcal{C}_{BC}\big) \\ &= \mathbb{P}\Big(\Big\{R_i > \frac{1}{2M}\log\big(1+\rho|h_{ri}|^2\big),\ i=1,\ldots,M\Big\}\Big) \\ &= \mathbb{P}\Big(\Big\{|h_{ri}|^2 \le \frac{2^{2MR}-1}{\rho},\ i=1,\ldots,M\Big\}\Big) \\ &= 1-\bigg(e^{\left(-\lambda\frac{2^{2MR}-1}{\rho}\right)}\bigg)^M. \end{aligned} \tag{3.37}$$

Without transmit CSI, the DMT is the minimum of the DMT of the MAC and the broadcast channel. For the MAC channel, it has been shown [19] that for multiplexing gains $r \le \frac{M}{M+1}$, the diversity $d = 1 - r/M$ is achievable, while for higher rates $\frac{M}{M+1} < r \le 1$, the diversity of $d = M(1-r)$ is obtained.

For the broadcast channel, since time sharing achieves the maximum sum-rate bound, the broadcast DMT is similar to the single-user DMT. The DMT of the network is bounded by the DMT of the broadcast part of the network. Thus, including the half-duplex consideration, the best achievable DMT is

$$d(r) = (1-2r)^+. \tag{3.38}$$

The same DMT can be obtained with orthogonal channel access; superposition coding has no effect on the DMT.

3.4.2 Opportunistic Channel Access

In this scenario, the relay is assumed to have channel state information (either perfect or incomplete) about its incoming and outgoing links. Using this information, during each transmission interval the relay selects the best overall source-destination pair and gives it access to the channel. Form Lemma 1, it is easy to see that the DMT of an opportunistic gateway channel is upper bounded by $d(r) \leq n(1-2r)^+$. We start by assuming perfect CSI at the relay.

3.4.2.1 Full CSI at the Relay

We start by defining

$$\gamma_i \triangleq \min(|h_{ir}|^2, |h_{ri}|^2) \ .$$

In the decode-and-forward protocol, end-to-end data transmission is feasible if and only if both source-relay and relay-destination links can support the desired rate, therefore γ_i is the effective channel gain that governs the rate supported by a DF protocol for any node pair i. In the opportunistic mode, we would like to support the maximum instantaneous rate, therefore user i^* will be selected such that:

$$i^* = \arg\max_i \gamma_i, \tag{3.39}$$

We now investigate the statistics of γ_{i^*}. Since the channel fading coefficients h_{ri} and h_{ir} are complex Gaussian random variables, the channel gains $|h_{ri}|^2$ and $|h_{ir}|^2$ obey exponential distributions with exponential parameters $\frac{1}{E[|h_{ri}|^2]}$ and $\frac{1}{E[|h_{ir}|^2]}$, respectively. It is known that the minimum of M exponential random variables with parameters λ_k is an exponential random variable with parameter $\sum_{k=1}^{M} \lambda_k$, therefore the pdf of γ_i is an exponential distribution with parameter $\lambda = 2$. Therefore, the cdf of the maximum SNR for all the source-relay-destinations links γ_{i^*} is

$$F_{\gamma_{i^*}}(x) = \left(1 - e^{-2x}\right)^M \tag{3.40}$$

The network is considered in outage when none of the source-destination pairs can support the desired transmission rate R. The outage condition is therefore:

$$\begin{aligned} P_{\mathcal{O}} &= \mathbb{P}\Big(R > \frac{1}{2}\log\left(1+\rho\gamma_{i^*}\right)\Big) \\ &= \mathbb{P}\Big(\gamma_{i^*} < \frac{\rho^{2r}-1}{\rho}\Big) \\ &= \Big(1-\exp\Big(-2\frac{\rho^{2r}-1}{\rho}\Big)\Big)^M . \end{aligned} \tag{3.41}$$

The block sizes in our analysis are large enough so that the error events are dominated by outage events, therefore the probability of error can be approximated by the outage probability. Using the Taylor approximation $1-\exp(-x) \approx x$, we get:

$$\begin{aligned} P_e &\doteq \Big(\frac{\rho^{2r}-1}{\rho}\Big)^M \\ &\doteq \rho^{-M(1-2r)} . \end{aligned} \tag{3.42}$$

where the Taylor approximation is valid for $2r < 1$. Hence, the opportunistic gateway channel achieves the following DMT

$$d(r) = M(1-2r)^+. \tag{3.43}$$

Remark 5 *If the path selection criterion uses one set of channel gains, i.e. either* $\{h_{ir}\}$ *alone or* $\{h_{ri}\}$ *alone, no diversity gain would result. For example, selecting on the MAC side of the network would give* $\gamma_i = \min(|h_{i^*r}|^2, |h_{ri^*}|^2)$ *where* $i^* = \arg\max |h_{ir}|^2$*. Since the channel gains on the two sides are independent,* $|h_{ri^*}|^2$ *is still exponential and dominates the diversity order.*

Remark 6 *The outage calculations assume that upon selection each source must be connected to its corresponding destination within one transmission interval, implying that no long-term storage and buffering is taking place at the relay. In addition to simplifying the relay, this is also helpful in terms of reducing the end-to-end delay due to opportunistic communication.*

Remark 7 *An infinite buffer at the relay may increase the throughput, but it does not improve the DMT. If the relay can hold onto the data, the incoming packets could wait indefinitely until the path to their destination is dominant. Under this condition, the opportunistic MAC and opportunistic broadcast operations can be performed independently, each giving rise to a diversity* $d = M(1-2r)^+$*, thus the overall diversity would also be* $d = M(1-2r)^+$*. However, this is no more than the diversity obtained without the buffer.*

To summarize, a buffer would not improve the DMT, however, it would allow us to achieve the optimal DMT via local decision making (using MAC information on the MAC side, and broadcast channel information on the broadcast side). Without buffering, the relay must make decisions jointly in order to achieve optimal DMT.

3.4.2.2 Limited Feedback

We now assume the relay does not have perfect CSI but rather has access to one bit of information per node from each destination and is further able to send one bit of information per node to each of the sources. We wish to explore the DMT of this network under the one-bit feedback strategy.

Each destination node knows its incoming channel gain via the usual channel estimation techniques. Each destination compares its incoming channel gain to a threshold α, reporting the result via the one-bit feedback to the relay. The k destination nodes that report "1"(and their respective channels) are characterized as eligible for data transmission in that interval. From among these k eligible destinations, the relay chooses the one whose corresponding source-relay channel is the best.

The network is considered in outage if there is no source-relay-destination link that can support the target rate R. We design the threshold of the second hop of the network such that each destination reports "1" if the corresponding relay-destination link can support this rate R, i.e., $\alpha = \frac{\rho^{2r}-1}{\rho}$. The outage event occurs if no destination reports positively, or if some destinations are eligible, but none of the corresponding source-relay links can support the rate R. If according to this methodology the relay detects more than one end-to-end path that can support the rate R, the relay selects one of them randomly.

We define A_m as the event of m destinations reporting "1", and $\mathbb{P}(e|A_m)$ as the probability of error given that m destinations report "1". This is the probability that none of the m eligible relay-destination channels have a corresponding source-relay link that can support the rate R. The probability of outage in this case is

$$P_{\mathcal{O}} = \mathbb{P}(A_0) + \sum_{m=1}^{M} \mathbb{P}(A_m)\mathbb{P}(e|A_m). \tag{3.44}$$

The probability of m destinations reporting "1" and $M-i$ destinations reporting "0" is

$$\begin{aligned} \mathbb{P}(A_m) &= \binom{M}{m} F_\gamma(\alpha)^m \big(1 - F_\gamma(\alpha)\big)^{M-m} \\ &= \binom{M}{m} \big(e^{-\lambda\alpha}\big)^m \big(1 - e^{-\lambda\alpha}\big)^{M-m}, \end{aligned} \tag{3.45}$$

where $F_\gamma(x)$ is the cdf of the channel gains $\gamma = |h|^2$, which is exponentially distributed with parameter $\lambda = 1$. The probability of error given that m destinations report "1" is

$$\begin{aligned} \mathbb{P}(e|A_m) &= \mathbb{P}\big(\max_{j\in S} |h_{jr}|^2 \leq \alpha\big) \\ &= \big(1 - F_\gamma(\alpha)\big)^m = \big(1 - e^{-\lambda\alpha}\big)^m, \end{aligned} \tag{3.46}$$

where $S \subset \{1, \dots, M\}$, $|S| = m$, and we use the fact that source-relay and relay-destination channel gains have the same distribution F_γ. Assuming non-identical exponential distributions introduces more variables into analysis but the end results will be identical. Substituting (3.45), (3.46) in (3.44), the outage probability becomes

$$\begin{aligned} P_{\mathcal{O}} &= \big(1 - e^{-\lambda\alpha}\big)^M + \sum_{m=1}^{M} \binom{M}{m} \big(e^{-\lambda\alpha}\big)^m \big(1 - e^{-\lambda\alpha}\big)^{M-m} \big(1 - e^{-\lambda\alpha}\big)^m \\ &= \sum_{m=0}^{M} \binom{M}{m} \big(e^{-\lambda_g\alpha}\big)^m \big(1 - e^{-\lambda\alpha}\big)^{M-m} \big(1 - e^{-\lambda\alpha}\big)^m. \end{aligned} \tag{3.47}$$

To calculate the DMT, from (3.44), the outage probability is

$$\begin{aligned}
P_{\mathcal{O}} &= \mathbb{P}\left(\frac{1}{2}\log\left(1+\max_i |h_{ri}|^2\rho\right) \le r\log\rho\right) + \sum_{m=1}^{M}\binom{M}{m}\mathbb{P}\left(\frac{1}{2}\log\left(1+|h_{rd}|^2\right) \le r\log\rho\right)^{M-m} \\
&\qquad \times \mathbb{P}\left(\frac{1}{2}\log\left(1+|h_{rd}|^2\right) \ge r\log\rho\right)^{m}\mathbb{P}\left(\frac{1}{2}\log\left(1+\max_{j\in S,|S|=m}|h_{jr}|^2\right) \le r\log\rho\right) \\
&\doteq \mathbb{P}\left(\max_i |h_{ri}|^2 \le \rho^{2r-1}\right) + \sum_{m=1}^{M}\mathbb{P}\big(|h_{rd}|^2 \le \rho^{2r-1}\big)^{M-m}\mathbb{P}\big(|h_{rd}|^2 \ge \rho^{2r-1}\big)^{m} \\
&\qquad\quad \times \mathbb{P}\left(\max_{j\in S,|S|=m}|h_{jr}|^2 \le \rho^{2r-1}\right) \\
&\doteq \sum_{m=1}^{M}\left(e^{-\lambda\rho^{2r-1}}\right)^{m}\left(1-e^{-\lambda\rho^{2r-1}}\right)^{M-m}\left(1-e^{-\lambda\rho^{2r-1}}\right)^{m} \\
&\doteq \rho^{M(2r-1)}.
\end{aligned} \tag{3.48}$$

So we have:

$$d(r) = M(1-2r)^{+}. \tag{3.49}$$

Thus, even 1-bit feedback is enough to achieve optimal DMT.

CHAPTER 4

OPPORTUNISTIC MULTIPLE ACCESS AND BROADCAST RELAY CHANNELS

The multiple access relay channel (MARC) [39] consists of the standard multiple access channel together with one relay (see Figure 4.1). No results for the DMT of the opportunistic MARC have been available until now, but its *non-opportunistic* DMT under superposition coding with single-antenna nodes is analyzed in [40, 41, 42, 30]. The following results are known for the non-opportunistic MARC: It is known that the dynamic decode and forward is DMT optimal for low multiplexing gain [40]. The compress and forward protocol achieves a significant portion of the half duplex DMT upper bound for high multiplexing gain [30] but suffers from diversity loss in the low multiplexing regime. The multiple-access relay amplify and forward (MAF) is proposed in [42], it dominates the CF and outperform the DDF protocol in high multiplexing regime.

The broadcast relay channel (BRC) was introduced independently in [43] and [44]. Assuming single-antenna nodes, the *opportunistic* BRC is identical to the *opportunistic* MARC save for certain practicalities in the exchange of channel state information, which does not

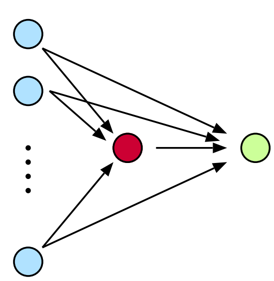

Figure 4.1. The multiple access relay channel.

make a difference at the abstraction level of the models used in this work. Therefore for the demonstration purposes we focus on MARC; the results carry over to the BRC directly.

4.1 DMT Upper Bound

In order to calculate a DMT upper bound for the opportunistic MARC, we assume a genie gives the relay an error-free version of the messages originating from all the sources. We also assume full cooperation on the transmit side. Under these conditions, the source that maximizes the instantaneous end-to-end mutual information is selected. Each of the n sources has an independent link to the destination and they all share the same relay-destination link. The opportunistic modes are demonstrated in Figure 4.2. The genie-aided MARC is equivalent to a MISO system with $n+1$ transmit antennas and one receive antenna.

The performance of the *opportunistic* genie-aided MARC is therefore upper bounded by a $(n+1)\times 1$ MISO system with antenna selection that chooses for each codeword transmission two transmit antennas. The $(n+1)\times 1$ antenna selection allows configurations that do not have a counterpart in the opportunistic modes in the MARC channel, therefore due to the extra flexibility the MISO system with antenna selection upper bounds the performance of the genie-aided opportunistic MARC channel.

The DMT of a $M \times N$ MIMO link with $L_t < M$ selected transmit antennas and $L_r < N$ selected receive antennas is upper bounded by a piecewise linear function obtained by connecting the following $K+2$ points [45]

$$\left\{\Big(n, (M_r - n)(M_t - n)\Big)\right\}_{n=0}^{K}, \big(\min(L_r, L_t), 0\big), \tag{4.1}$$

where

$$K = \arg\min_{k\in\mathbb{Z}} \frac{(M_r - k)(M_t - k)}{\min(L_r, L_t) - k}, \quad \text{subject to } 0 \le k \le \min(L_r, L_t) - 1$$

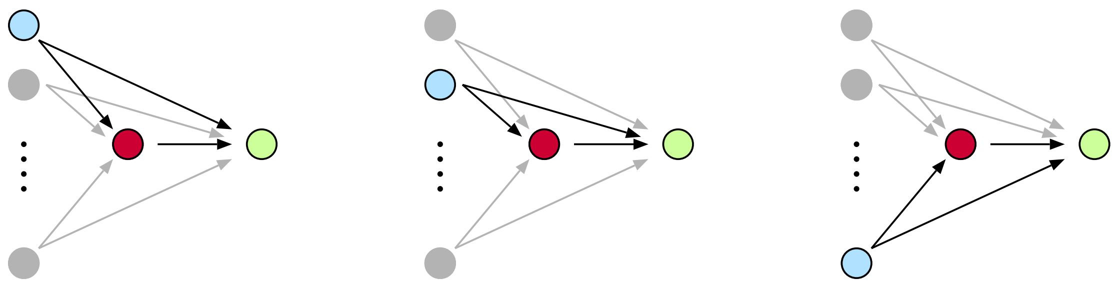

Figure 4.2. Opportunistic access modes for the genie-aided multiple access relay channel.

Using this result, a $(n+1)\times 1$ MISO system with two selected transmit antennas has a DMT that is upper bounded by

$$d(r) = (n+1)(1-r)^{+}. \tag{4.2}$$

This represents our genie-aided upper bound for opportunistic MARC.

4.2 Achievable DMT

In this section, we propose a node selection rule and calculate the corresponding achievability results for a number of relaying protocols in opportunistic MARC and BRC. As mentioned earlier, one of the difficulties in the computation of DMT in opportunistic scenarios is the dependencies among the statistics of the node selections, which itself is a result of selection rules. To circumvent these difficulties, we propose a selection rule that relies only on the source-destination links in the MARC. Because this method does not observe the shared link in the system, the resulting node statistics are independent and many of the computational difficulties disappear.

We shall see that this simplified selection works surprisingly well in the high-SNR regime. It will be shown that for some relaying protocols this selection algorithm yields achievable DMT that is tight against the upper bound.

The proposed schemes for the MARC can be also be used for the BRC, therefore for demonstration purposes we limit ourselves to MARC. The only difference is that for the BRC the CSI must be fed back to the source to make the scheduling decision.

4.2.1 Orthogonal Amplify and Forward

The maximum instantaneous mutual information between the inputs and the output is

$$I_{AF} = \frac{1}{2}\log\left(1 + \rho|h_{i^*d}|^2 + f(\rho|h_{i^*r}|^2, \rho|h_{rd}|^2)\right), \tag{4.3}$$

where $i^* = \arg\max_i |h_{id}|$. The outage probability is given by

$$\begin{aligned} P_{AF} &= \mathbb{P}\big(I_{AF} < r\log\rho\big) \\ &= \mathbb{P}\left(|h_{i^*d}|^2 + \frac{1}{\rho}f(\rho|h_{i^*r}|^2, \rho|h_{rd}|^2) < \frac{\rho^{2r}-1}{\rho}\right). \end{aligned} \tag{4.4}$$

Since channel coefficients h_{ij} are complex Gaussian, $|h_{ij}|^2$ obey exponential distributions. We therefore use the following result to characterize (4.4) in the high-SNR regime.

Lemma 4 *Assume random variables u_i, v and w follow exponential distributions with parameters λ_u, λ_v and λ_w, respectively, and ϵ is a constant and $f(x, y) = \frac{xy}{x+y+1}$.*

$$\lim_{\rho\to\infty} \frac{1}{\left(\frac{\rho^{2r}-1}{\rho}\right)} \mathbb{P}\left(u_i < \frac{\rho^{2r}-1}{\rho}\right) = \lambda_u \tag{4.5}$$

$$\lim_{\rho\to\infty} \frac{1}{\left(\frac{\rho^{2r}-1}{\rho}\right)^n} \mathbb{P}\left(\max_i u_i < \frac{\rho^{2r}-1}{\rho}\right) = \lambda_u^n \tag{4.6}$$

$$\lim_{\rho\to\infty} \frac{1}{\left(\frac{\rho^{2r}-1}{\rho}\right)^{n+1}} \mathbb{P}\left(\max_i u_i + v < \frac{\rho^{2r}-1}{\rho}\right) = \frac{\lambda_v\lambda_u^n}{n+1}, \tag{4.7}$$

$$\lim_{\rho\to\infty} \frac{1}{\left(\frac{\rho^{2r}-1}{\rho}\right)^{n+1}} \mathbb{P}\left(\max_i u_i + f(\frac{v}{\epsilon}, \frac{w}{\epsilon}) < \frac{\rho^{2r}-1}{\rho}\right) = \frac{\lambda_u^n(\lambda_v+\lambda_w)}{2}, \tag{4.8}$$

Proof: Expression (4.5) is proved in [20]. The proof of the other expressions is similar (with slight modifications). $\square$

From (4.4) and (4.8), the probability of outage at high SNR is

$$P_{AF} \doteq \frac{1}{2}\lambda_{i^*d}^{n}\big(\lambda_{i^*r} + \lambda_{rd}\big)\left(\frac{\rho^{2r}-1}{\rho}\right)^{n+1} \tag{4.9}$$

where $\lambda_{i^*r}, \lambda_{rd}, \lambda_{i^*d}$ are the exponential parameters of the channel gains for the links corresponding to the selected opportunistic mode. It follows that the DMT of the opportunistic n-user MARC with orthogonal amplify-and-forward, under a selection rule based on the source-destination channel gain, is given by

$$d(r) = (n+1)(1-2r)^{+}. \tag{4.10}$$

4.2.2 Orthogonal Decode and Forward

With the orthogonal DF protocol, outage happens if either of the following two scenarios happen: (1) the relay cannot decode and the direct source-destination channel is in outage, or (2) the relay can decode but the source-destination and relay-destination links together are not strong enough to support the required rate. In other words:

$$\begin{aligned} P_{DF} &= \mathbb{P}\left(|h_{i^*r}|^2 \geq \frac{\rho^{2r}-1}{\rho}\right)\mathbb{P}\left(|h_{i^*d}|^2 + |h_{rd}|^2 < \frac{\rho^{2r}-1}{\rho}\right) \\ &\quad + \mathbb{P}\left(|h_{i^*r}|^2 < \frac{\rho^{2r}-1}{\rho}\right)\mathbb{P}\left(|h_{i^*d}|^2 < \frac{\rho^{2r}-1}{\rho}\right) \\ &\doteq \mathbb{P}\left(|h_{i^*r}|^2 \geq \rho^{2r-1}\right)\mathbb{P}\left(|h_{i^*d}|^2 + |h_{rd}|^2 < \rho^{2r-1}\right) \\ &\quad + \mathbb{P}\left(|h_{i^*r}|^2 < \rho^{2r-1}\right)\mathbb{P}\left(|h_{i^*d}|^2 < \rho^{2r-1}\right). \end{aligned} \tag{4.11}$$

Using Lemma 4 (specifically equations (4.5), (4.6), (4.7)) the outage probability can be approximated thus:

$$P_{DF} \doteq \left(\frac{\lambda_{i^*d}^{n}\lambda_{rd}}{n+1} + \lambda_{i^*d}^{n}\lambda_{rd}\right)\rho^{(n+1)(2r-1)} \tag{4.12}$$

It follows directly that the n-user opportunistic MARC, subject to selection based on source-destination channel gains and operating with orthogonal DF, has the following DMT

$$d(r) = (n+1)(1-2r)^{+}. \tag{4.13}$$

Remark 8 *We know that an orthogonal relay may not be helpful in high multiplexing gains, but the above orthogonal MARC dedicates time to the relay, therefore it may be improved. To do that, we add to the system n* unassisted *modes, where the relay does not play a role. For an opportunistic MARC that can choose between $2n$ opportunistic modes, one can show that the maximum achieved DMT is $d(r) = n(1-r)^+ + (1-r/2)^+$. A simple selection rule achieves this DMT: take the best source-destination link. If it is viable without the relay, use it without relay, otherwise use it with the relay.*

4.2.3 Non-Orthogonal Amplify and Forward

In this protocol, the source with the maximum source-destination channel coefficient is selected. Recall that the index of this source is denoted i^*. This source continues transmitting throughout the transmission interval.

The DMT of the MARC with n sources and opportunistic channel access *based on the source-destination channel gain* using NAF relaying is

$$d(r) = n(1-r) + (1-2r)^+. \tag{4.14}$$

This result indicates that at multiplexing gains $r > 0.5$ the relay does not play any role; the only available diversity at $r > 0.5$ is that of multiuser diversity generated by selection among n sources.

To prove the result, we make use of the calculation method in [18, 28]. An outline of the proof is as follows. We assume that v_1 is the exponential order of the random variable $\frac{1}{|h_{i^*d}|^2}$, i.e.

$$v_1 = -\frac{\log(|h_{i^*d}|^2)}{\log \rho}. \tag{4.15}$$

The probability density function of the exponential order is

$$p_v = n\ln(\rho)\rho^{-v}e^{-\rho^{-v}}(1-e^{-\rho^{-v}})^{n-1}, \tag{4.16}$$

which, asymptotically,

$$p_v \doteq \begin{cases} 0, & v < 0 \\ \rho^{-nv}, & v \geq 0. \end{cases} \tag{4.17}$$

The probability of outage can be characterized by $P_O \doteq \rho^{-d_o}$ where

$$d_o = \inf_{(v_1,v_2,u)\in O^+} nv_1 + v_2 + u, \tag{4.18}$$

where v_2 and u are the exponential order of $1/|h_{i^*r}|^2$ and $1/|h_{rd}|^2$, respectively. The set O characterizes the outage event and O^+ is $O \bigcap R^{3+}$. Optimization problems of this form have been solved in [28] and also in the context of opportunistic relay networks we have demonstrated a solution in Appendix E for the shared relay channel, therefore we omit a similar solution here in the interest of brevity.

4.2.4 Dynamic Decode and Forward

The DMT of the opportunistic DDF MARC, where the selection is based on the source-destination channel gain, is given by

$$d(r) = \begin{cases} (n+1)(1-r), & \frac{n}{n+1} \geq r \geq 0 \\ n\frac{(1-r)}{r}, & 1 \geq r \geq \frac{n}{n+1}. \end{cases} \tag{4.19}$$

The proof follows [18, 28] together with the basic Lemmas of this work and the NAF MARC proof. The DDF achieves the optimal trade-off (the genie-aided DMT) for $\frac{n}{n+1} \geq r \geq 0$. For multiplexing gains $r > \frac{n}{n+1}$ the relay does not have enough time to perfectly help the selected source. However, as n grows, the DMT approaches the upper bound (genie-aided).

4.2.5 Compress and Forward

The node selected by the opportunistic algorithm has index i^*. The system will be in outage if the transmission rate $r \log \rho$ is less than the instantaneous mutual information $I(x_{i^*}; \hat{y_r}, y_d | x_r)$, where $\hat{y_r}$ represents the compressed signal at the relay, y_r and y_d are the received signals at the relay and the destination, respectively, and x_{i^*} and x_r are the source and relay transmitted signals, respectively. Using selection scheme based on the direct link only and applying the same techniques as in [30], it follows that the CF protocol achieves the following DMT

$$d(r) = \min\big(d_{BC}(r), d_{MAC}(r)\big), \tag{4.20}$$

where d_{BC}, d_{MAC} correspond to the outage of broadcast and MAC cutsets, as follows:

$$\begin{aligned} d_{BC}(r) &\triangleq -\lim_{\rho\to\infty} \frac{\min\limits_{p(x_{i^*},x_r)} \mathbb{P}(I(x_{i^*};y_r y_d|x_r) < r\log\rho)}{\log\rho} \\ &= -\lim_{\rho\to\infty} \frac{\mathbb{P}\big(\log|I+\rho\, H_{BC}H_{BC}^{\dagger}| < r\log\rho\big)}{\log\rho} \end{aligned} \tag{4.21}$$

$$\begin{aligned} d_{MAC}(r) &\triangleq -\lim_{\rho\to\infty} \frac{\min\limits_{p(x_{i^*},x_r)} \mathbb{P}(I(x_{i^*}x_r;y_d) < r\log\rho)}{\log\rho} \\ &= -\lim_{\rho\to\infty} \frac{\mathbb{P}\big(\log|I+2\rho\, H_{MAC}H_{MAC}^{\dagger}\,| < r\log\rho\big)}{\log\rho}, \end{aligned} \tag{4.22}$$

The transmit signals x_{i^*} and x_r are from random codebooks that are drawn according to complex Gaussian distributions with zero mean and variance $\sqrt{\rho}$. We define $H_{BC} \triangleq \begin{bmatrix} h_{i^*r} \\ h_{i^*d} \end{bmatrix}$, $H_{MAC} \triangleq [h_{i^*d}\ h_{rd}]$ and $(\ )^{\dagger}$ denotes the Hermitian operator. The derivation of Equations (4.21), (4.22) uses the fact that a constant scaling in the transmit power does not change the DMT [18].

Using the techniques in [18, 28] and following the NAF MARC DMT proof, it is possible to calculate the following:

$$d_{BC}(r) = \begin{cases} (n+1) - \frac{r}{t}, & r \le t < \frac{1}{n+1} \\ n\frac{(1-r)}{(1-t)}, & t < \min(r, \frac{1}{n+1}) \\ (n+1)(1-r), & t \ge \frac{1}{n+1} \end{cases} \tag{4.23}$$

$$d_{MAC}(r) = \begin{cases} (n+1) - \frac{r}{1-t}, & 1-r \ge t > \frac{n}{n+1} \\ n\frac{(1-r)}{t}, & t > \max\{1-r, \frac{n}{n+1}\} \\ (n+1)(1-r), & t \le \frac{n}{n+1} \end{cases} \tag{4.24}$$

Details of the derivation are similar to, e.g., Theorem 5.

From Equations (4.20)-(4.24), it follows that the genie aided DMT upper bound can be achieved for any value of $\frac{1}{n+1} \leq t \leq \frac{n}{n+1}$. The maximum achieved DMT is given by

$$d(r) = (n+1)(1-r)^{+} \tag{4.25}$$

4.3 Optimality of the Achievable DMTs

Although the previous DMTs were calculated using simplified selection schemes that only observed the source-destination direct link, one can show that for each of the relaying protocols, no improvement in DMT is possible by more sophisticated selection schemes.

This fact is self-evident for the CF relaying result, since it meets the genie-aided upper bound. The NAF and DDF do not meet the genie-aided bound, therefore it is not obvious that they perform optimally under the simplified selection scheme. We now proceed to investigate this question for DDF and NAF.

The DMT of the multiple access relay channel with opportunistic user selection is given by

$$d(r) = \lim_{\rho \to \infty} \frac{\log \mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n)}{\log \rho}, \tag{4.26}$$

where $\mathcal{O}_i$ represents the outage event for the access mode characterized by source i transmitting to the destination with the help of the relay.

In a manner similar to [28] and Equation (E.6), the probability of outage $\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n)$ can be expressed as follows

$$\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n) \doteq \rho^{-d_o(r)},$$

where

$$d_o(r) = \inf_{(v_1^{(1)}, u^{(1)}, \ldots, v_1^{(n)}, u^{(n)}, v_2) \in O} v_2 + \sum_{j=1}^{n} \left(v_1^{(j)} + u^{(j)} \right) . \tag{4.27}$$

The random variables $v_1^{(j)}, u^{(j)}$ and v_2 represent the exponential order of $1/|h_{jd}|^2,\ 1/|h_{jr}|^2$ and $1/|h_{rd}|^2$, respectively. Each of these random variables has a probability density function that is asymptotically equal to

$$p(x) \doteq \begin{cases} 0 & x < 0 \\ \rho^{-x} & x \geq 0. \end{cases} \tag{4.28}$$

The set O represents the outage event for the opportunistic network. We know $O = \mathcal{O}_1^+ \cap \ldots \cap \mathcal{O}_n^+$, i.e, the opportunistic system is considered in outage when no access mode is viable.

For NAF the outage region is defined by [28]

$$\mathcal{O}_j^+ = \Big\{ \big(v_1^{(j)}, v_2, u^{(j)}\big) \in R^{3+} \Big| (l-2m)(1-v_1^{(j)})^+ + m\max\{2(1-v_1^{(j)}), \\ 1-(v_2+u^{(j)})\})^+ < rl \Big\}, \tag{4.29}$$

where m is rank of the relay amplification matrix and l is the block length. The solution to Equations (4.27) and (4.29) is facilitated by the knowledge that $d_o(r)$ is maximized when $m = l/2$, leading to:

$$d_{NAF}(r) = n(1-r) + (1-2r), \tag{4.30}$$

This is the best diversity obtained for NAF, which is similar to the simplified selection based on the source-destination link. Therefore the optimality of the simplified selection rule is established for NAF.

For DDF the outage region is defined by [28]

$$\mathcal{O}_j^+ = \Big\{ \big(v_1^{(j)}, v_2, u^{(j)}\big) \in R^{3+} \Big| t^{(j)}(1-v_1^{(j)})^+ + (1-t^{(j)})(1-\min(v_1^{(j)}, v_2))^+ < r \Big\}, \tag{4.31}$$

where $t^{(j)}$ is the listening-time ratio of the half-duplex relay when source j is transmitting, with $r \leq t^{(j)} \leq 1$. In the following we outline the solution of Equations (4.27) and (4.31) for a two-user MARC. The generalization to n users is straight forward.

Our strategy for solving the optimization problem is to partition the optimization space into eight regions, solve the optimization problem over each region as a function of $t^{(1)}$ and $t^{(2)}$, maximize over $t^{(1)}$ and $t^{(2)}$ and then find the minimum of the eight solutions. The eight regions correspond to the Cartesian product of whether each of the three positive variables $v_1^{(1)}, v_1^{(2)}, v_2$ is greater than or less than 1. Following the calculations, which are straight forward, the DMT for DDF is

$$d_{DDF}(r) = \begin{cases} (n+1)(1-r) & \frac{n}{n+1} \geq r \geq 0 \\ n\frac{1-r}{r} & 1 \geq r > \frac{n}{n+1}, \end{cases} \tag{4.32}$$

which matches the DMT of simplified selection based on the source-destination links. Therefore the optimality of simplified selection for the DDF is established.

Figure 4.3 shows a comparison between the various studied relaying schemes for the opportunistic N-user multiple-access relay channel. The insert shows the point at which the DDF is no longer achieving the channel upper bound, $r = \frac{n}{n+1}$. The CF is shown to achieve the upper bound for all multiplexing gain. The NAF outperforms the orthogonal AF and the orthogonal DF for all multiplexing gains.

We can follow essentially the same steps for the broadcast relay channel and obtain the same DMTs for both the NAF and DDF. The optimization problem in the broadcast case is slightly different: the shared link in BRC is the source-relay channel while it is the relay-destination channel in the MARC. Nevertheless, very similar strategies follow through for the BRC with only small adjustments.

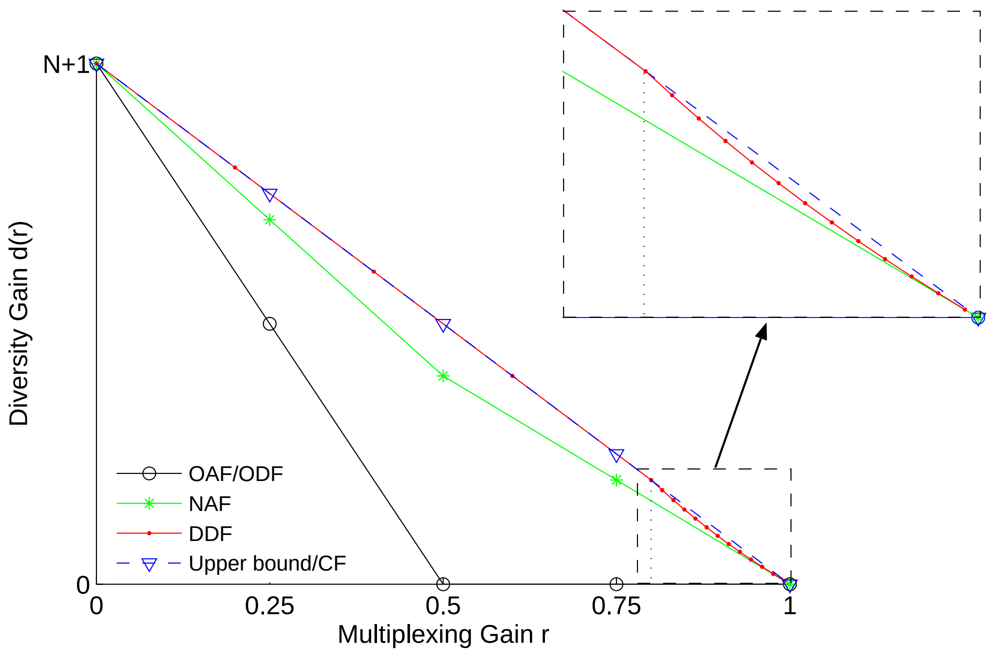

Figure 4.3. DMT for a N-user opportunistic multiple-access relay channel. The insert shows the high-multiplexing gain region.

CHAPTER 5

HETEROGENEOUS RELAY CHANNEL

In a multi-relay scenario, relay selection can harvest diversity with relatively modest requirements compared with alternatives such as distributed space-time codes or distributed beamforming. An early example of cooperative selection appeared in [2]. Bletsas et al [3] investigated amplify-and-forward (AF) relay selection, followed by many other works including [6, 7, 8]. Decode-and-forward (DF) relay selection has also received attention, e.g. [11] and many others [10, 11, 13]. The diversity multiplexing tradeoff (DMT) for relay selection has been investigated in [46] for addressing the multiplexing loss of DF relaying, and [9] for a combination of antenna selection and AF relay selection.

This chapter studies the performance of relay selection in heterogeneous relay networks where relays with different protocols co-exist. For example, a heterogeneous network may contain both DF relays and AF relays, or in the non-orthogonal transmission mode, the network may include dynamic decode-forward (DDF) relays and non-orthogonal amplify-forward (NAF) relays. We study a system of one source, one destination and n relays, all half-duplex single-antenna nodes, see Figure 5.1. There is a viable link between the source and destination.

In the process of developing heterogeneous relay selection results, a long-standing restriction on relay selection networks is relaxed and removed: In relay selection the source-destination link introduces dependencies among decision variables[1] and because of that previous works on relay selection often assumed the direct link to be non-existent, i.e., they

[1]With rare exceptions, e.g. [46]

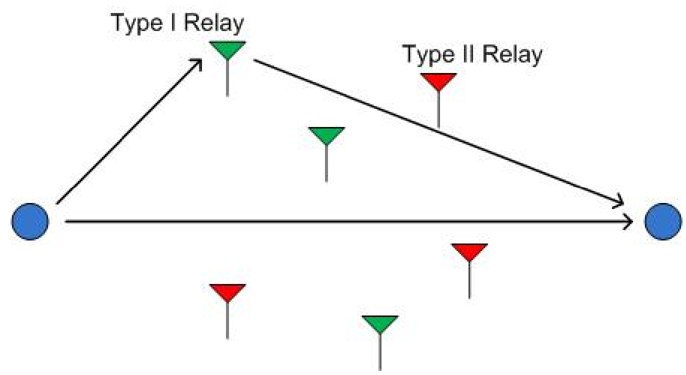

Figure 5.1. Multi-relay channel with direct link, relays can be of different types.

assumed a two-hop geometry. However, ignoring the direct link is in general wasteful, and in particular interferes with the operation of compress-forward (CF), one of the more powerful relaying protocols.[2] So there is ample motivation to produce analysis that can account for the direct link.

A heterogeneous network (Figure 5.1) where one relay is selected for each transmission interval is studied. In particular, the following combinations are analyzed: mixture of DF relays and AF relays, a mixture of NAF relays and DDF relays, and a mixture of NAF relays and CF relays. The diversity-multiplexing tradeoff of heterogeneous relay selection is analyzed in each of these cases.

A brief overview of our results is as follows: in a network of n relays of the same type in the presence of a direct source-destination link, the orthogonal relaying protocols achieve the same DMT as non-orthogonal amplify-forward. At low multiplexing gains the dynamic decode-forward achieves the MISO upper bound. For high multiplexing gains, CF relay selection outperforms both DDF and NAF. DDF and CF relay selection outperform NAF at all multiplexing gains.

[2]With no direct link and one selected relay, the destination is in outage if the relay cannot decode (due to cutset bound). So compress-forward is not particularly useful in a two-hop network.

We observe that the DMT of heterogeneous *orthogonal* relaying is similar to the DMT of the corresponding homogeneous AF or DF relay systems. For heterogeneous *non-orthogonal* relaying we consider NAF, DDF, and CF. Relay selection among NAF and DDF relays gives maximal performance at high multiplexing gains, while providing diversity somewhere between that of NAF and DDF at low multiplexing gains. Relay selection among CF and NAF relays achieves a diversity that scales with the number of relays at high multiplexing gain as opposed to the fixed diversity for NAF.

An interesting direction that on first sight may seem related to this work is hybrid relaying, where a relay adaptively changes its relaying protocol (e.g. AF/DF) [47, 48, 49, 50]. This class of work describes a new type of relay that obeys a complex decoding law. But this is distinct from a system containing multiple relays of different kind, which we call a *heterogeneous system*. True heterogeneous schemes have appeared very rarely in the literature: Jeong et al. [51] studied a system with multiple AF and DF relays without a direct link. Lusina et al. [52] presented a variation of the slotted Amplify-Forward (SAF) scheme of Yang and Belfiore [53] with two relays, where one of the relays is AF while the other (unlike [53]) is DF.

A relay is allowed to opportunistically access the channel and assist the source by transmitting to the destination. This generates various ways to access the channel (access modes), as seen in Figure 5.2. The source and destination can communicate without a relay, as shown in the first access mode, or a relay can forward a version of the signal received from the source. Since the source-destination link is shared among the relays, the selection process is much more complicated than the two-hop variety. More importantly, the shared link introduces dependency between the decision variables and this complicates the analysis. Lemma 3 provide a technique to analyze such a system by providing a method to combine the conditional DMTs of each access mode to produce the overall DMT. Naturally this is useful only if the conditional DMTs are tractable. For n relay, $n+1$ access mode are defined

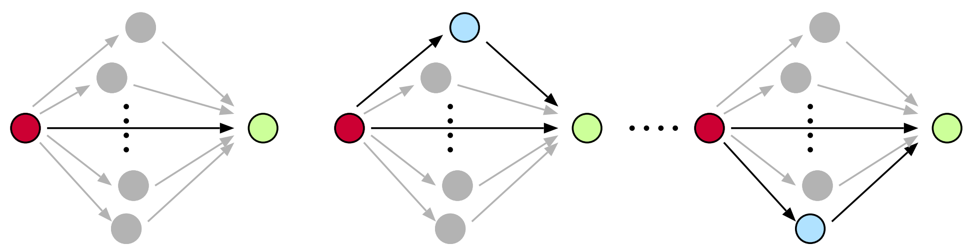

Figure 5.2. The opportunistic modes for the multi-relay channel with non-orthogonal access scheme.

and the DMT of the system is given by

$$d(r) = d_1(r) + d'_2(r) + \ldots + d'_n(r), \tag{5.1}$$

where $d'_i(r)$ is conditional DMT for access mode i and defined as Equation (2.5).

5.1 Orthogonal Relaying

A system is considered with n orthogonal relays that are AF, DF, or a combination. The selection criterion is as follows. If the direct link alone can support the transmission rate, no relay will be selected. Otherwise, the transmission will be assisted by a single selected relay, in which case the source transmits for half the transmission interval and the relay transmits for the second half of the transmission interval.

This creates $n+1$ access modes, where n is the total number of relays in the system (Figure 5.2). The first access mode, *the non-relayed access mode*, is relay-free. The remaining *relay-assisted access modes* each involves a relay in the usual two-interval orthogonal relay framework. Some of these access modes may be AF and some of them may be DF. Each will have an end-to-end mutual information calculated according to its own protocol.

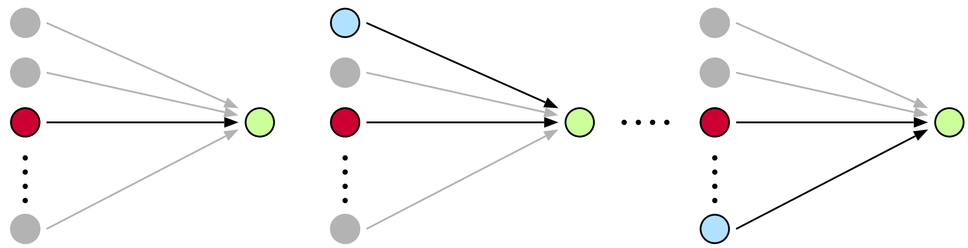

Figure 5.3. The genie upper bound opportunistic modes for the multi-relay channel

5.1.1 Upper Bound

Assume a genie provides the relays with a noise-free, error-free copy of the source message. Thus the source-relay channel is removed from the picture (effectively replaced with a perfect wire), and each selected relay together with the source will constitute a 2×1 MISO channel (Figure 5.3). Recall that there are n different ways to select a relay, each corresponding to a relay-assisted mode, all of them sharing one link: the source destination link. The non-relayed mode is not affected by the genie.

Theorem 6 *An upper bound for the opportunistic parallel relay channel with n orthogonal relays is*

$$d(r) = (n+1)(1-r)^+. \tag{5.2}$$

Proof: Using Lemma 1 and Lemma 3 tightness conditions, the genie upper bound DMT is

$$d(r) = d_0(r) + d_1'(r) + \ldots + d_n'(r), \tag{5.3}$$

where $d_0(r)$ is the DMT of the non-relay mode (denoted Mode 0). This mode consists of a simple direct link whose DMT is $d_0(r) = (1-r)^+$ and $d_i'(r), i = 1, \ldots, n$, are the conditional DMTs of the relay-assisted modes that are given by Equation (2.5).

For the first relay-assisted access mode (denoted Mode 1), the DMT is given by

$$d_1'(r) = -\lim_{\rho \to \infty} \frac{\log \mathbb{P}(\mathcal{O}_1 | \mathcal{O}_0)}{\log \rho}, \tag{5.4}$$

where $\mathcal{O}_i$ represents the outage event for Mode i. The outage probability of Mode 1 given that Mode 0 is in outage

$$\begin{aligned}\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) &= \mathbb{P}\big(\{\log(1+(|h_{sd}|^2+|h_{1d}|^2)\rho) < r\log\rho\}\big|\{\log(1+|h_{sd}|^2\rho) < r\log\rho\}\big)\\ &= \mathbb{P}\Big(\Big\{|h_{sd}|^2+|h_{1d}|^2 < \frac{\rho^r-1}{\rho}\Big\}\Big|\Big\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\Big\}\Big),\end{aligned}$$

where Mode 1 is equivalent to a 2×1 MISO channel and the conditioning means that one of its links is not capable of supporting the whole transmission rate. Using results from the Appendix I and defining $\alpha = g_1(r,\rho) = \frac{\rho^r-1}{\rho}$, the conditional outage probability can be calculated as follows

$$\begin{aligned}\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) &= \int_0^{g_1(r,\rho)} f_{Z|B}(z)dz = \int_0^{g_1(r,\rho)} \frac{ze^{-z}}{1-e^{g_1(r,\rho)}}dz\\ &\doteq \frac{1-e^{-\rho^{r-1}}-\rho^{r-1}e^{-\rho^{r-1}}}{1-e^{-\rho^{r-1}}} \doteq \rho^{r-1}.\end{aligned} \tag{5.5}$$

Hence, $d_1'(r) = (1-r)^+$. For Modes $i>1$, one can show that

$$\begin{aligned}\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}) &\doteq \mathbb{P}\Big(|h_{sd}|^2+|h_{id}|^2 < \rho^{r-1}\Big|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}\Big)\\ &\dot{\le}\mathbb{P}\Big(|h_{id}|^2 < \rho^{r-1}\Big|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}\Big) = \mathbb{P}\Big(|h_{id}|^2 < \rho^{r-1}\Big).\end{aligned} \tag{5.6}$$

At the same time, $\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1})$ is lower bounded by $\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0)$, which can be calculated using the same technique used for $\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0)$, i.e.,

$$\begin{aligned}\mathbb{P}\Big(|h_{id}|^2 < \rho^{r-1}\Big)&\dot{\ge}\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}) \ge \mathbb{P}(\mathcal{O}_i|\mathcal{O}_0)\\ \rho^{r-1}&\dot{\ge}\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1})\dot{\ge}\rho^{r-1}.\end{aligned} \tag{5.7}$$

Hence, $d_i'(r) = (1-r)^+$ for $1 < i \le n$. Substituting the conditional DMTs into Equation (5.1) completes the proof. $\square$

Note that the above selection procedures are designed to be DMT-optimal and to simplify the analysis. Selecting the mode $i^* = \arg\max_{i\in\{0,\ldots,n\}} I_i$ gives the same DMT.

5.1.2 Opportunistic Amplify and Forward Orthogonal Relaying

If the direct link is in outage, the selected relay amplifies its received signal during the first half-interval and forwards it to the destination in the second half-interval. The instantaneous mutual information of the non-relay mode is given by $I_0 = \log(1+|h_{sd}|^2\rho)$. The end-to-end instantaneous mutual information for the relay-assisted modes under orthogonal AF is given by [20, 26]

$$I_i = \frac{1}{2}\log(1+|h_{sd}|^2\rho + f(|h_{ir}|^2\rho, |h_{ri}|^2\rho)), \quad i = 1,\dots,n,$$

where $f(x,y) = \frac{xy}{x+y+1}$. At high SNR, $f(|h_{ir}|^2\rho, |h_{ri}|^2\rho)$ can be approximated to $\frac{|h_{ir}|^2|h_{ri}|^2\rho}{|h_{ir}|^2+|h_{ri}|}$ and relay i^* is selected such that $i^* = \arg\max_i \frac{|h_{ir}|^2|h_{ri}|^2}{|h_{ir}|^2+|h_{ri}|^2}$.

Theorem 7 *The DMT of the orthogonal opportunistic AF parallel relay channel with direct link between the source and the destination is given by*

$$d(r) = (1-r)^+ + n(1-2r)^+. \tag{5.8}$$

Proof: The overall DMT in terms of individual conditional DMTs is given by Equation (5.1). To calculate the right hand side, we start with the non-relay mode (Mode 0) whose DMT is $d_0(r) = (1-r)^+$. At high SNR, the conditional outage of Mode 1 is

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) \doteq \mathbb{P}\left(\left\{|h_{sd}|^2 + \frac{|h_{s1}|^2|h_{1d}|^2}{|h_{s1}|^2+|h_{1d}|^2} < \frac{\rho^{2r}-1}{\rho}\right\}\Big|\left\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\right\}\right). \tag{5.9}$$

In order to calculate the conditional outage probability distribution, we first calculate the conditional density function of $Z = |h_{sd}|^2 + V$ where $V = \frac{|h_{s1}|^2|h_{1d}|^2}{|h_{s1}|^2+|h_{1d}|^2}$. The term $\frac{|h_{s1}|^2|h_{1d}|^2}{|h_{s1}|^2+|h_{is}|^2}$ represents half the harmonic mean of two independent exponential random variables. Using a result of [54], the harmonic mean of two exponential random variables with parameters λ can be approximated by an exponential random variable with parameter $\lambda + \lambda = 2\lambda$.

Using results from the Appendix I and defining $\alpha = g_1(r,\rho) = \frac{\rho^r - 1}{\rho}$ and $g_2(r,\rho) = \frac{\rho^{2r}-1}{\rho}$, the conditional outage probability is

$$\begin{aligned}\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = &\int_0^{g_2(r,\rho)} f_{Z|B}(z)dz \doteq 2\int_0^{g_1(r,\rho)} \frac{e^{-2z}(e^z - 1)}{1-e^{-g_1(r,\rho)}}dz + 2\int_{g_1(r,\rho)}^{g_2(r,\rho)} \frac{e^{-2z}\left(e^{g_1(r,\rho)} - 1\right)}{1-e^{-g_1(r,\rho)}}dz \\ =&\frac{e^{-2\rho^{2r-1}} - e^{-\rho^{r-1}} - e^{-2\rho^{2r-1}+\rho^{r-1}} + 1}{1-e^{-\rho^{r-1}}} \doteq \rho^{2r-1}. \end{aligned} \tag{5.10}$$

From Equations (5.10) and (5.4), $d_1'(r) = (1-2r)^+$.

For Mode 2, the conditional outage can be shown to be

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \geq \mathbb{P}(\mathcal{O}_2|\mathcal{O}_0) \doteq \rho^{2r-1}, \tag{5.11}$$

where (5.11) follows the same proof as $\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0)$. Also one can show

$$\begin{aligned}\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \doteq &\mathbb{P}\left(\left\{|h_{sd}|^2 + f\left(|h_{s2}|^2, |h_{2d}|^2\right) < \rho^{2r-1}\right\}\middle| \mathcal{O}_1,\mathcal{O}_0\right) &(5.12)\\ \dot{\leq}&\mathbb{P}\left(\left\{f\left(|h_{s2}|^2, |h_{2d}|^2\right) < \rho^{2r-1}\right\}\middle| \mathcal{O}_1,\mathcal{O}_0\right) &(5.13)\\ \dot{\leq}&\mathbb{P}\left(f\left(|h_{s2}|^2, |h_{2d}|^2\right) < \rho^{2r-1}\right) \doteq \rho^{2r-1}. \end{aligned}$$

Hence, $\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \doteq \rho^{2r-1}$ and $d_2'(r) = (1-2r)^+$. Following similar steps, one can show that $d_i'(r) = (1-2r)^+$ for $i = 3,\ldots,n$. Substituting conditional DMTs into Equation (5.1) completes the proof. $\square$

5.1.3 Opportunistic Decode and Forward Orthogonal Relaying

If the direct link is not capable of supporting the transmission rate, the transmission interval is divided into two intervals. In the first transmission interval, the source transmits. If some relays decode the source message, then a relay will forward the decoded message to the destination in the second half of the transmission interval and the source remains silent. If neither the non-relayed mode nor any relayed mode is capable of supporting the desired rate, the system is in outage.

Theorem 8 *The DMT of the orthogonal opportunistic DF parallel relay channel with direct link between the source and the destination is given by*

$$d(r) = (1-r)^+ + n(1-2r)^+. \tag{5.14}$$

Proof: The total DMT as a function of conditional DMTs is given by Equation (5.1) and $d_0(r) = (1-r)^+$. The conditional outage probability of the first relay-assisted mode is given by

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = \mathbb{P}\Big(\Big\{\frac{1}{2}\log(1+U_1\rho) < r\log\rho\Big\}\Big|\Big\{\log(1+|h_{sd}|^2\rho) < r\log\rho\Big\}\Big), \tag{5.15}$$

where the random variable U_1 is defined as

$$U_i = \begin{cases} |h_{sd}|^2 & |h_{si}|^2 < \frac{\rho^{2r}-1}{\rho} \\ |h_{sd}|^2 + |h_{id}|^2 & |h_{si}|^2 \geq \frac{\rho^{2r}-1}{\rho}. \end{cases} \tag{5.16}$$

$$\begin{aligned}\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) =& \mathbb{P}\Big(\Big\{|h_{sd}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\Big|\Big\{|h_{sd}|^2 < \frac{\rho^{r}-1}{\rho}\Big\}\Big)\mathbb{P}\Big(|h_{s1}|^2 < \frac{\rho^{2r}-1}{\rho}\Big) \\ &+ \mathbb{P}\Big(\Big\{|h_{sd}|^2 + |h_{1d}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\Big|\Big\{|h_{sd}|^2 < \frac{\rho^{r}-1}{\rho}\Big\}\Big)\mathbb{P}\Big(|h_{s1}|^2 \geq \frac{\rho^{2r}-1}{\rho}\Big).\end{aligned} \tag{5.17}$$

One can show that $\frac{\rho^{2r}-1}{\rho} \dot{>} \frac{\rho^{r}-1}{\rho}$, therefore

$$\mathbb{P}\Big(\Big\{|h_{sd}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\Big|\Big\{|h_{sd}|^2 < \frac{\rho^{r}-1}{\rho}\Big\}\Big) \doteq 1. \tag{5.18}$$

Using results from the Appendix I and defining $g_1(r,\rho) \triangleq \frac{\rho^{r}-1}{\rho}$ and $g_2(r,\rho) \triangleq \frac{\rho^{2r}-1}{\rho}$

$$\begin{aligned}&\mathbb{P}\Big(\Big\{|h_{sd}|^2 + |h_{1d}|^2 < g_2(r,\rho)\Big\}\Big|\Big\{|h_{sd}|^2 < g_1(r,\rho)\Big\}\Big) \\ &\qquad = \int_0^{g_1(r,\rho)} \frac{ze^{-z}}{1-e^{-g_1(r,\rho)}}dz + \int_{g_1(r,\rho)}^{g_2(r,\rho)} \frac{g_1(r,\rho)e^{-z}}{1-e^{-g_1(r,\rho)}}dz \\ &\qquad \doteq \frac{1-e^{-\rho^{r-1}} - \rho^{r-1}e^{-\rho^{2r-1}}}{1-e^{-\rho^{r-1}}} \doteq \rho^{2r-1}.\end{aligned} \tag{5.19}$$

Substituting (5.18) and (5.19) into (5.17), the conditional probability of outage is given by

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) \doteq \rho^{2r-1} + \rho^{2r-1}(1-\rho^{2r-1}) \doteq \rho^{2r-1}. \tag{5.20}$$

Hence, the conditional DMT for the first relay-assisted access mode is $d'_1(r) = (1-2r)^+$. Using the same argument as the orthogonal AF, it follows that $\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0, \ldots, \mathcal{O}_{i-1}) \doteq \rho^{2r-1}$ and hence $d'_i(r) = (1-2r)^+$ for $i = 2, \ldots, n$. Substituting conditional DMTs into Equation (5.1) completes the proof. □

5.1.4 Heterogeneous Network with AF and DF Orthogonal Relays

In this scenario both AF and DF relays are present in the network. At high SNR, the selected relay i^* is such that $i^* = \arg\max_i\{\gamma^{DF}_{i\in D}, \gamma^{AF}_{i\in A}\}$, where D and A represent the DF and AF relays set, respectively, and γ_i is the SNR at the destination when Relay i is active.

Theorem 9 *The DMT of the orthogonal opportunistic relay selection with N AF relay and M DF relays channel and a direct link between the source and the destination is given by*

$$d(r) = (1-r)^+ + (N+M)(1-2r)^+. \tag{5.21}$$

Proof: The total DMT as a function of conditional DMTs is given by Equation (5.1). As mentioned earlier for the non-relay mode (Mode 0) DMT is $d_0(r) = (1-r)^+$.

We know there is at least one AF and one DF relay in this network. For the purposes of exposition, we assign Mode 1 to an AF relay and Mode 2 to a DF relay. For Mode 1, At high SNR, the analysis is identical to Mode 1 analysis in Section 5.1.2. From Equation (5.10), the conditional outage $\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) \doteq \rho^{2r-1}$, hence for Mode 1, the conditional DMT is $d'_1(r) = (1-2r)^+$.

For Mode 2, we first bound the conditional outage from below:

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \geq \mathbb{P}(\mathcal{O}_2|\mathcal{O}_0). \tag{5.22}$$

The calculation of $\mathbb{P}(\mathcal{O}_2|\mathcal{O}_0)$ is similar to Equation (5.20) and can be shown to be

$$\begin{aligned}\mathbb{P}(\mathcal{O}_2|\mathcal{O}_0) =& \mathbb{P}\Big(\Big\{\frac{1}{2}\log(1+U_2\rho) < r\log\rho\Big\}\Big|\Big\{\log(1+|h_{sd}|^2\rho) < r\log\rho\Big\}\Big)\\ \doteq& \rho^{2r-1}.\end{aligned} \tag{5.23}$$

Now, we bound $\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0)$ from above, using the same technique used in Equation (5.6)

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \dot{\leq} \mathbb{P}\Big(U_2 < \rho^{2r-1}\Big) \doteq \rho^{2r-1}. \tag{5.24}$$

Thus, using a sandwich argument, we have $\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \doteq \rho^{2r-1}$, indicating the conditional DMT of Mode 2 is $d_2'(r) = (1-2r)^+$. Following the same steps, one can show that $d_i'(r) = (1-2r)^+$ for $i = 3,\ldots,N+M$. Substituting the conditional DMTs into Equation (5.1) completes the proof. $\square$

5.2 Non-Orthogonal Relaying

In this scenario the source transmits throughout the transmission interval. The relays listens for a portion of the transmission interval and then, depending on the channel conditions, one of the relays may transmit simultaneously with the source for a portion of the transmission interval. The destination receives the source signal, potentially superimposed with the relay signal. Since in non-orthogonal modes the end-to-end mutual information always improves with a relay, there is no need to consider a non-relay access mode for the purposes of DMT calculation. Thus in the analysis of non-orthogonal protocols, the system has n access modes as opposed to $n+1$ in the case of orthogonal relaying.

5.2.1 Upper Bound

For the calculation of the upper bound we assume a genie provides the relays with a noise-free and error-free version of the source message. We also assume full cooperation between the source and the relays. The relay that maximizes the instantaneous end-to-end mutual information is selected to transmit simultaneously with the source. Each of the n relays has an independent link to the destination and they all share the same source-destination link. Subject to the genie information, the distinction between relay types (AF, DF, CF) goes away and therefore the upper bound is equally valid for any set of n non-orthogonal relays.

Theorem 10 *An upper bound for the opportunistic parallel relay channel with* n *non-orthogonal relays is*

$$d(r) = (n+1)(1-r)^{+}. \tag{5.25}$$

Proof: The genie-aided opportunistic parallel relay channel is equivalent to a MISO system with $n+1$ transmit antennas and one receive antenna. The performance of the *opportunistic* genie-aided relay selection is therefore upper bounded by a $(n+1) \times 1$ MISO system with antenna selection that chooses for each codeword transmission two transmit antennas. The $(n+1) \times 1$ antenna selection allows configurations that do not have a counterpart in the opportunistic modes in our channel, therefore in addition to the genie we have allowed a second relaxation of conditions for the computation of the upper bound in this theorem.

The DMT of a $M \times N$ MIMO link with $L_t < M$ selected transmit antennas and $L_r < N$ selected receive antennas is upper bounded by a piecewise linear function obtained by connecting the following $K+2$ points [45]

$$\Big\{\Big(n, (M_r - n)(M_t - n)\Big)\Big\}_{n=0}^{K}, \big(\min(L_r, L_t), 0\big), \tag{5.26}$$

where

$$K = \arg\min_{k\in\mathbb{Z}} \frac{(M_r-k)(M_t-k)}{\min(L_r,L_t)-k},$$
$$\text{subject to } \; 0 \le k \le \min(L_r,L_t)-1$$

Using this result, a $(n+1)\times 1$ MISO system with two selected transmit antennas has a DMT that is upper bounded by

$$d(r) = (n+1)(1-r)^{+}. \tag{5.27}$$

This represents our genie-aided upper bound. $\square$

5.2.2 Non-Orthogonal Amplify and Forward

For half the transmission interval, the received signal at the destination and at Relay i are given by

$$y_{1i} = \sqrt{\rho}\, h_{ii}\, x_{1i} + n_{1i}, \quad y_{1r} = \sqrt{\rho}\; h_{ir}\; x_{1i} + n_{1r}.$$

The variables x, y, n represent source transmitted signal, received signal and the noise, respectively. The variable have two subscripts indicating the appropriate half-interval and node identity, respectively. At the second half of the transmission interval the relay normalizes the received signal and retransmits it. The destination received signal in the second half is

$$y_{2i} = \sqrt{\rho}\; h_{ii}\; x_{2i} + \frac{\sqrt{\rho}\, h_{ri}}{\sqrt{\rho|h_{ir}|^2+1}}\; y_{1r} + n_{2i}.$$

At the destination $Y = \mathbf{H}_i X + N$ where $X_i = [x_{1i} \;\; x_{2i}]^t$ is the transmitted signals vector, N is the noise vector, and $\mathbf{H}_i$ is the effective channel gain matrix [27].

$$\mathbf{H}_i = \begin{bmatrix} \sqrt{\rho}\, h_{ii} & 0 \\ \frac{\rho\, h_{ri} h_{ir}}{\sqrt{\rho|h_{ir}|^2+1}\sqrt{\frac{\rho|h_{ri}|^2}{\rho|h_{ir}|^2+1}+1}} & \frac{\sqrt{\rho}\, h_{ii}}{\sqrt{\frac{\rho|h_{ri}|^2}{\rho|h_{ir}|^2+1}+1}} \end{bmatrix}. \tag{5.28}$$

The instantaneous mutual information is given by

$$
\begin{aligned}
I_i &= \frac{1}{2}\log\left|\mathbf{I}+\mathbf{H}_i\mathbf{H}_i^*\right| \\
&= \frac{1}{2}\log\left(1+|h_{ii}|^2\rho+\frac{|h_{ri}|^2|h_{ir}|^2\rho^2}{|h_{ri}|^2\rho+|h_{ir}|^2\rho+1}+\frac{|h_{ii}|^2(|h_{ir}|^2\rho+1)\rho}{|h_{ri}|^2\rho+|h_{ir}|^2\rho+1}+\frac{|h_{ii}|^4(|h_{ir}|^2\rho+1)\rho^2}{|h_{ri}|^2\rho+|h_{ir}|^2\rho+1}\right).
\end{aligned}
$$

Relay i^* is selected to maximize the mutual information, which at high SNR is:

$$i^* = \arg\max_i I_i = \arg\max_i \left\{\frac{|h_{ir}|^2}{|h_{ri}|^2}\right\}. \tag{5.29}$$

Theorem 11 *The maximum DMT for n parallel relay channel with opportunistic NAF relay selection is*

$$d(r) = (1-r)^+ + n(1-2r)^+. \tag{5.30}$$

Proof: The DMT of a system that switches between n dependent access modes is given by

$$d(r) = -\lim_{\rho\to\infty}\frac{\log\mathbb{P}(\mathcal{O}_1,\ldots,\mathcal{O}_n)}{\log\rho}. \tag{5.31}$$

In a manner similar to [28], the probability of outage $\mathbb{P}(\mathcal{O}_1,\ldots,\mathcal{O}_n)$ can be expressed as follows

$$\mathbb{P}(\mathcal{O}_1,\ldots,\mathcal{O}_n) \doteq \rho^{-d_o(r)}, \tag{5.32}$$

$$d_o(r) = \inf_{(v_1,u^{(1)},v_2^{(1)},\ldots,u_1^{(n)},v_2^{(n)})\in O} v_1 + \sum_{i=1}^{n}\left(u^{(i)}+v_2^{(i)}\right). \tag{5.33}$$

The random variables $v_1, u^{(i)}$ and $v_2^{(i)}$ represent the exponential order of $1/|h_{sd}|^2$, $1/|h_{si}|^2$ and $1/|h_{id}|^2$, respectively. Each of these random variables has a probability density function $p(x)$ that is asymptotically equal to ρ^{-x} for $x \geq 0$ and 0 otherwise [28]. The set O represents the outage event for the network. The network is considered in outage when no access mode is viable, i.e., $O = \mathcal{O}_1^+ \cap \ldots \cap \mathcal{O}_n^+$. For NAF, the outage region for Mode i is defined by [28]

$$\mathcal{O}_i^+ = \left\{\left(v_1, v_2^{(i)}, u^{(i)}\right) \in R^{3+}\middle|(l-2m^{(i)})(1-v_1)^+ + m^{(i)}\max\{2(1-v_1), 1-(v_2^{(i)}+u^{(i)})\})^+ < rl\right\}, \tag{5.34}$$

where $m^{(i)}$ is rank of the amplification matrix and l is the block length. The solution to the optimization problem in Equations (5.33) and (5.34) is facilitated by the knowledge that $d_o(r)$ is maximized when $m^{(i)}$ is maximum, i.e, $m^{(i)} = l/2$, thus leading to the result. □

5.2.3 Dynamic Decode and Forward

In this scenario the selected relay listens to the source until it has enough information to decode the message. From that point on, the relay uses the remainder of the transmission interval to send the decoded information to the destination. The selection criterion is, once again, according to the maximum end-to-end mutual information.

Theorem 12 *The maximum DMT for the n parallel relay channel with opportunistic DDF relay selection is*

$$d_{DDF}(r) = \begin{cases} (n+1)(1-r) & \frac{1}{n+1} \geq r \geq 0 \\ (n+1) - n\frac{r}{1-r} & \frac{1}{2} \geq r > \frac{1}{n+1} \\ \frac{1-r}{r} & r \geq \frac{1}{2}. \end{cases} \tag{5.35}$$

Proof: Following Equations (5.31), (5.32), and (5.33), for the relay channel characterized by Relay i, the outage region under DDF can be shown to be [28]

$$\mathcal{O}_i^+ = \left\{ \left(v_1, v_2^{(i)}, u^{(i)}\right) \in R^{3+} \middle| t_i(1-v_1)^+ + (1-t_i)(1-\min(v_1, v_2^{(i)}))^+ < r \right\}, \tag{5.36}$$

where t_i is the listening-time ratio of the relay i, with $r \leq t_i \leq 1$. In the following we outline the solution of Equations (5.33) and (5.36) for a two-relay channel. The generalization to n users follows the same lines.

Our strategy for solving the optimization problem is to partition the optimization space into eight regions, solve the optimization problem over each region as a function of t_1 and t_2, maximize over t_1 and t_2 and then find the minimum of the eight solutions. The eight regions correspond to the Cartesian product of whether each of the three positive variables $v_1^{(1)}, v_1^{(2)}, v_2$ is greater than or less than 1. Following the calculations, which are omitted due to their length, the DMT for DDF is established. □

5.2.4 Compress and Forward

In this scenario, following [30], the selected relay uses Wyner-Ziv compression and the destination uses the received signal from the source as side information to decode the signal. The compression rate ensures that the compressed signal is decoded at the destination error-free. Yuksel and Erkip [30] show that the one-relay compress-forward achieves the DMT $d(r) = 2(1-r)^+$, which coincides with the MISO upper bound.

The relay $i^* = \arg\max_i I_i$ is selected which at high-SNR, using results from [30], is:

$$i^* = \arg\max_i \frac{(|h_{si}|^2 + |h_{sd}|^2)(|h_{id}|^2 + |h_{sd}|^2)}{(|h_{si}|^2 + |h_{sd}|^2) + (|h_{id}|^2 + |h_{sd}|^2)}.$$

Following [30], the DMT for the n parallel relay channel with opportunistic CF relay selection can be shown to be

$$\begin{aligned} d(r) &= \max_t \min(d_{MAC}(r,t), d_{BC}(r,t)), \qquad (5.37)\\ d_{MAC} &= -\lim_{\rho\to\infty} \frac{\log \min_{p(x_s,x_r|q)} \mathbb{P}\big(I_{MAC} < r\log\rho\big)}{\log\rho},\\ d_{BC} &= -\lim_{\rho\to\infty} \frac{\log \min_{p(x_s,x_r|q)} \mathbb{P}\big(I_{BC} < r\log\rho\big)}{\log\rho}, \end{aligned}$$

where t is the the time ratios vector, $t = [t_1, \ldots, t_n]$, q represents the state of the relay (listening vs. transmitting), $p(x_s, x_r|\, q)$ is the probability density according to which the random codebooks are generated for the source and the relay, and I_{BC} and I_{MAC} represent the total mutual information across the source and the destination cutsets, respectively. We have:

$$I_{MAC} \leq (1 - t_{i^*}) \log(1 + |h_{sd}|^2\rho) + t_{i^*} \log(1 + (|h_{sd}|^2 + |h_{i^*d}|^2)\rho).$$

We can show that $\mathbb{P}\big(I_{MAC} < r\log\rho\big) \doteq \rho^{-d_{MAC}(r)}$ where

$$d_{MAC}(r) = \inf_{(v_1, v_2^{(1)}, \ldots, v_2^{(n)}) \in O} v_1 + \sum_{i=1}^{n} v_2^{(i)}, \qquad (5.38)$$

and the outage event O_i^+ is defined as

$$O_i^+ = \Big\{(v_1, v_2^{(i)}) \in R^{2+} \Big| (1-t_i)(1-v_1)^+ + t_i\big(1-\min(v_1, v_2^{(i)})\big)^+ \leq r\Big\}. \tag{5.39}$$

Solving the optimization problem in (5.38), the DMT for the destination cutset is:

$$d_{MAC}(r, t_1, \ldots, t_n) = \begin{cases} (n+1)(1-r) & \sum_{i=1}^{n} \frac{t_i}{1-t_i} < 1 \\ 1 + \sum_{t_i \in \Gamma}(1 - \frac{r}{1-t_i}) & \sum_{t_i \in \Gamma} \frac{t_i}{1-t_i} > 1 \ , \ \Gamma \neq \varnothing \\ \frac{1-r}{t_i} + \sum_{t_j < t_i} \frac{1-r}{1-t_j}\big(1 - \frac{t_j}{t_i}\big) & t_i > 1-r \quad , \frac{1-2t_i}{1-t_i} < \sum_{t_j < t_i} \frac{t_j}{1-t_j} < 1 \end{cases}$$

where $\Gamma = \{t_i : t_i < 1-r\}$, and the third line in the equation above represents multiple expressions, one for each t_i that satisfies the corresponding condition. The details of the optimization are omitted for complying with the length requirement of this manuscript.

Similarly, the source cutset DMT can be obtained by replacing t_i with $1 - t_i$. Maximizing over t, the minimum of the two DMT cutsets gives the DMT of the system.

Thus, the DMT for arbitrary number of relays is a composite expression which does not reduce to a more compact form as long as the number of relays remains a variable. However, for any fixed number of relays the results are more streamlined. For example, for selection among three CF relays:

$$d_{CF}(r) = \begin{cases} 4 - 6r & r < \frac{1}{3} \\ 3(1-r) & \frac{1}{3} \leq r \leq 1 \end{cases} \tag{5.40}$$

5.2.5 Mixture of NAF and DDF Relays

Now consider a scenario consisting of N NAF relays and M DDF relays. If the selected relay is an NAF relay, the relay will listen for half the transmission interval and transmits simultaneously with the source in the second half of the transmission interval. If the selected relay is a DDF relay, the relay listens to the source until it has enough information to decode the message, then transmits during the remainder of the transmission interval.

Theorem 13 *The DMT for an opportunistic multiple relay channel with N NAF relays and M DDF relays is given by*

$$d(r) = \begin{cases} (M+1)(1-r) + N(1-2r) & \frac{1}{M+1} \geq r \geq 0 \\ (M+1) - \frac{Mr}{1-r} + N(1-2r) & \frac{1}{2} \geq r \geq \frac{1}{M+1} \\ \frac{1-r}{r} & 1 \geq r \geq \frac{1}{2}. \end{cases} \tag{5.41}$$

Proof: The DMT of a system that switches between $N+M$ dependent access modes is given by

$$d(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_{N+M})}{\log \rho}. \tag{5.42}$$

In a manner similar to [28], the probability of outage $\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n)$ can be expressed as follows

$$\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n) \doteq \rho^{-d_o(r)}, \tag{5.43}$$

where

$$d_o(r) = \inf_{(v_1, u^{(1)}, v_2^{(1)}, \ldots, u_1^{(N+M)}, v_2^{(N+M)}) \in O} v_1 + \sum_{i=1}^{N+M} \left(u^{(i)} + v_2^{(i)}\right), \tag{5.44}$$

where $O = \mathcal{O}_1^+ \cap \ldots \cap \mathcal{O}_{N+M}^+$.

For the purposes of exposition, the NAF access modes are indexed as Mode i where $i = 1, \ldots, N$. The outage region can be shown to be [28]

$$\mathcal{O}_i^+ = \left\{ \left(v_1, v_2^{(i)}, u^{(i)}\right) \in R^{3+} \middle| (l - 2m^{(i)})(1-v_1)^+ + m^{(i)} \max\{2(1-v_1), 1 - (v_2^{(i)} + u^{(i)})\})^+ < rl \right\}, \tag{5.45}$$

where $m^{(i)}$ is rank of the amplification matrix and l is the block length.

The DDF access modes are indexed as Mode i where $i = N+1, \ldots, N+M$. The outage region can be shown to be [28]

$$\mathcal{O}_i^+ = \left\{ \left(v_1, v_2^{(i)}, u^{(i)}\right) \in R^{3+} \middle| t_i(1-v_1)^+ + (1-t_i)(1 - \min(v_1, v_2^{(i)}))^+ < r \right\}, \tag{5.46}$$

where t_i is the listening-time ratio of the relay i, with $r \leq t_i \leq 1$.

The DMT is found by solving the optimization problem in Equation (5.44) over the union of the regions defined in Equations (5.45) and (5.46). The details of the optimization are omitted for brevity. □

5.2.6 Mixture of NAF and CF Relays

In this scenario we have N NAF relays and M CF relays. The NAF relays work as mentioned in the previous subsection. For CF relays, we again follow [30] where the CF relays use Wyner-Ziv compression and the destination uses the received signal from the source as side information to decode the signal. As mentioned earlier, a single-relay CF channel achieves the DMT $d(r) = 2(1-r)^+$.

The relay $i^* = \arg\max_i I_i$ is selected. At high-SNR, using results from [30] and [27], the selected user i^* is

$$i^* = \arg\max_i \left\{ \left\{ \frac{|h_{si}|^2}{|h_{id}|^2} \right\}_{i=1,\ldots,N}, \left\{ \frac{(|h_{si}|^2 + |h_{sd}|^2)(|h_{id}|^2 + |h_{sd}|^2)}{(|h_{si}|^2 + |h_{sd}|^2) + (|h_{id}|^2 + |h_{sd}|^2)} \right\}_{i=N+1,\ldots,N+M} \right\},$$

where without loss of generality we have indexed the relays so that the first N relays are NAF and the following M relays are CF.

We provide a sketch of the technique used to arrive at the DMT. Following [30], the DMT for the multiple relay channel with opportunistic relay selection can be shown to be

$$\begin{aligned} d(r) &= \max_t \min(d_{CF}(r,t), d_{BC}(r,t)), \qquad (5.47)\\ d_{MAC} &= -\lim_{\rho\to\infty} \frac{\log \mathbb{P}\big(I_{CF-MAC} < r\log\rho, I_{NAF} < r\log\rho\big)}{\log\rho}, \\ d_{BC} &= -\lim_{\rho\to\infty} \frac{\log \mathbb{P}\big(I_{CF-BC} < r\log\rho, I_{NAF} < r\log\rho\big)}{\log\rho}, \end{aligned}$$

where t is the the time ratios vector, $t = [t_{N+1}, \ldots, t_{N+M}]$, for the CF relays, and I_{BC} and I_{MAC} represent the total mutual information across the source and the destination cutsets considering one CF relay active, respectively. It can be shown that

$$I_{CFMAC} \leq (1-t_{i^*})\log(1+|h_{sd}|^2\rho)+t_{i^*}\log(1+(|h_{sd}|^2+|h_{i^*d}|^2)\rho).$$

The probability of outage $\mathbb{P}\big(I_{MAC} < r\log\rho, I_{NAF} < r\log\rho\big)$ can be expressed as

$$\mathbb{P}\big(I_{CFMAC} < r\log\rho, I_{NAF} < r\log\rho\big) \doteq \rho^{-d(r)}, \tag{5.48}$$

where

$$d(r) = \inf_{(v_1, v_2^{(1)},\ldots,v_2^{(N+M)},u^{(1)},\ldots,u^{(N)})\in O} v_1 + \sum_{i=1}^{N} u^{(i)} + \sum_{i=1}^{N+M} v_2^{(i)}, \tag{5.49}$$

For $i = 1,\ldots,N$, the NAF relay-assisted modes, the outage event O_i^+ is defined as given by Equation (5.45). For $i = \{N+1,\ldots,N+M\}$, the CF relay-assisted modes, the outage event O_i^+ is defined as

$$O_i^+ = \Big\{(v_1, v_2^{(i)}) \in R^{2+} \Big| (1-t_i)(1-v_1)^+ + t_i\big(1-\min(v_1, v_2^{(i)})\big)^+ \leq r\Big\}. \tag{5.50}$$

We provide the solution for the optimization problem for the case of 2 CF relays and N NAF. the generalization to the M CF relays follows along the same lines.

$$d(r) = \begin{cases} (3-4r)+N(1-2r) & \frac{3}{7} \geq r \geq 0 \\ \frac{9}{4}(1-r)+N(1-2r) & \frac{1}{2} \geq r \geq \frac{3}{7} \\ \frac{9}{4}(1-r) & 1 \geq r \geq \frac{1}{2}. \end{cases} \tag{5.51}$$

Figure 5.4 compares the opportunistic DMT of a system consists of one source, one destination and two relays with a direct link between the source and the destination. Figure 5.5 shows the DMT for the case of three relays, where various combination of three available relays are analyzed. Figure 5.6 considers a configuration with four relays, where the relays can be all ODF, all OAF, all NAF, all DDF, a mix of 2 OAF and 2 ODF, a mix of 2 NAF and 2 DDF, or a mix of 2 NAF and 2 CF. These figures serve to highlight the relative advantage and disadvantage of hybrid relay configurations at various transmission rates (multiplexing gains).

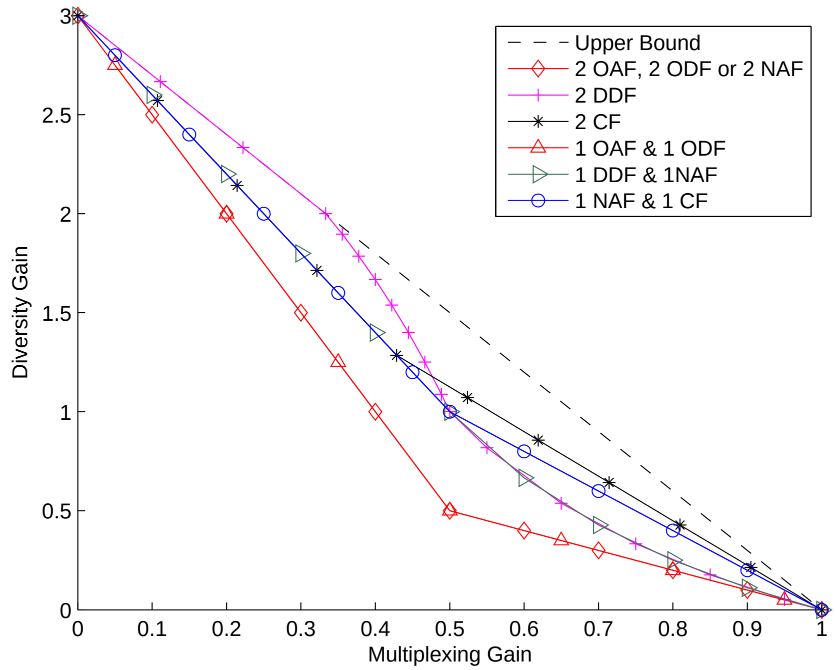

Figure 5.4. The DMT of hybrid relay selection from among two relays

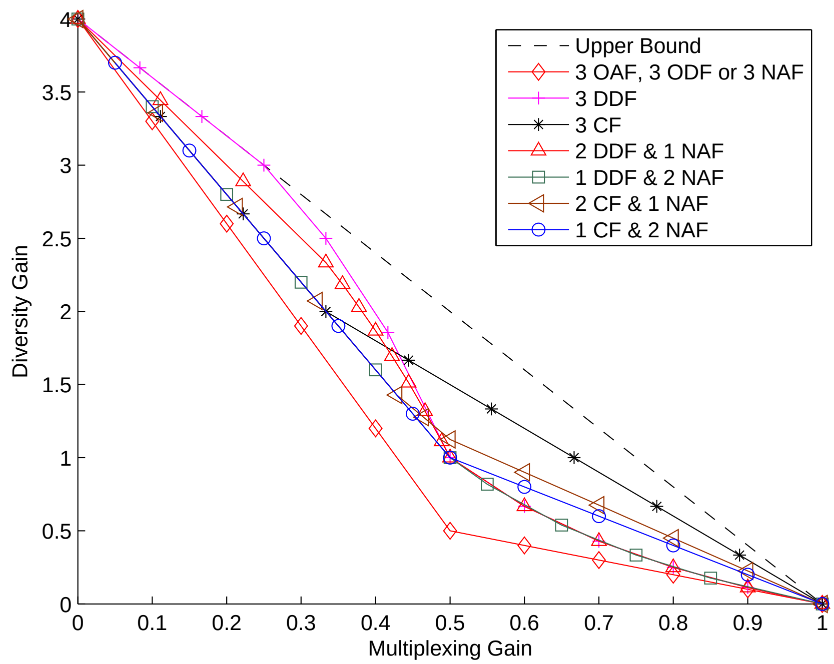

Figure 5.5. The DMT of hybrid relay selection from among three relays

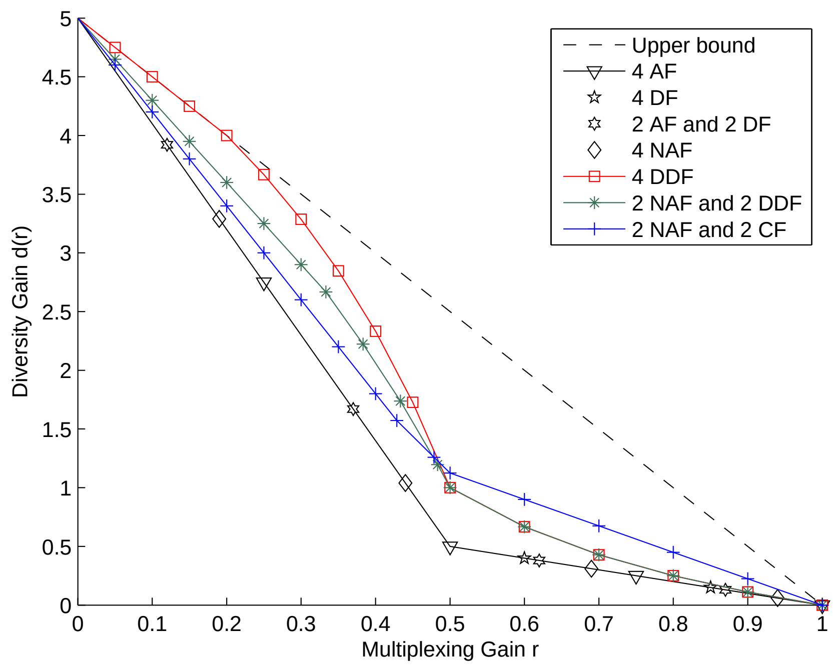

Figure 5.6. The DMT of hybrid relay selection from among four relays

CHAPTER 6

BI-DIRECTIONAL MULTI-RELAY CHANNEL

A bi-directional multi-relay channel consists of two source nodes trying to exchange messages through wireless links. Multiple relays exist to help both nodes delivering their messages to their intended receivers. The relays can listen to the messages from both sources and forward the received signal to any of these two nodes or both of them. There is a viable link between the two nodes, see Figure 6.1.

The channel coefficients for the direct link between the two sources is given by $h_{s_1s_2}$ and $h_{s_2s_1}$ for the reverse channel. The channel between the first source, s_1, and Relay i, r_i, is represented by $h_{s_1r_i}$ and $h_{r_is_1}$ for its reverse channel, respectively. The channel between the second source, s_2, and Relay i, r_i, is represented by $h_{s_2r_i}$ and $h_{r_is_2}$ for its reverse channel, respectively. As mentioned before, for simplicity, we assume that both nodes have the same multiplexing gain $r_1 = r_2 = r$ and the total multiplexing gain in the system should be $r_t = 2r$.

There are various ways the sources can exchange their messages in a bi-directional

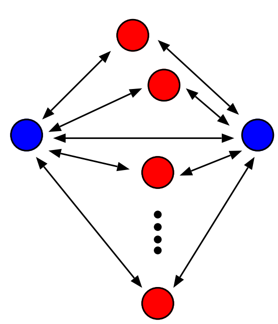

Figure 6.1. Bi-directional multi-relay channel with direct link.

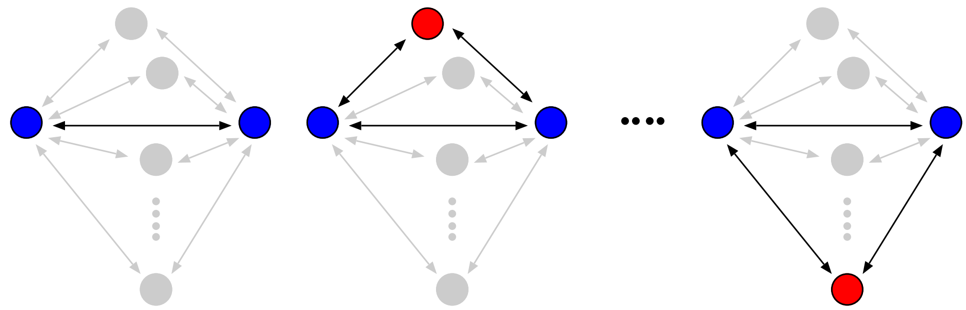

Figure 6.2. Bi-directional multi-relay channel access modes, one relay helping.

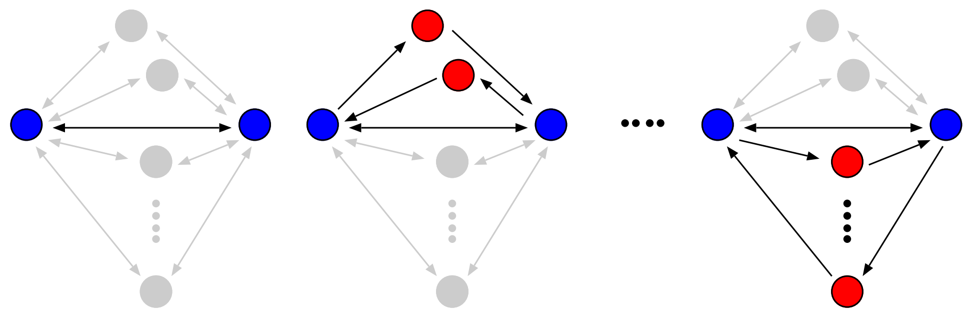

Figure 6.3. Bi-directional multi-relay channel access modes, two relays helping.

multi-relay channel with relay selection. If there is a viable link between the sources, the sources can communicate with each other directly without help from the relays. Another way is through the relays. A one relay can be selected to help both sources or two relays can be selected where each relay is responsible for helping a specific source node. In the network under study, the direct communication between the two source where no relay is helping represents an access modes and is refereed to as Mode 0. More access modes can be defined by allowing a relay or more to take part in helping the source nodes, see Figure 6.2 for the access modes for a one-relay selection scheme and Figure 6.3 for the access modes for a two-relay selection scheme.

The probability of error in each of the previously defined access modes, Mode i, conditioned on the previous modes in error, $\mathbb{P}(e_i|e_{i-1},\ldots,e_0)$ is used to calculate the conditional

DMT for that mode. The total DMT for the opportunistic system is given by

$$d(r) = d_0(r) + d'_1(r) + \cdots + d'_n(r) \tag{6.1}$$

where $d'_i(r)$ is defined as

$$d'_i(r) = -\lim_{\rho \to \infty} \frac{\log P(e_i|e_{i-1}, \ldots, e_1)}{\log \rho}, \tag{6.2}$$

6.1 With a Direct Link Between the Sources

The selection process is as follow. Access mode 0, direct transmission with no relay, is checked. If this mode is in outage, a relay-assisted access mode where a relay or two relays are helping the sources is checked. The process of checking all the defined relay-assisted access modes continues until all have been checked. The system is declared in outage if all access modes are in outage. This selection process is still equivalent to the regular relay selection technique, selecting the "best" relay, but this formulation helps making the analysis easier.

Depending on the channel conditions, the sources use Mode 0 or one of the relay-assisted access modes to exchange their messages. If it is Mode 0, the transmission interval is split into two phases where in Phase 1, s_1 transmits and s_2 listens and in Phase 2, s_2 transmits and s_1 listens. Due to the symmetry of the channel and the assumption of equal sources transmit rate $r \log \rho$, the two phases are equal in time and each occupies half the transmission interval. If a relay-assisted access mode is used, the transmission interval is split into three Phases. The three phases are as follows. In Phase 1, s_1 transmits while s_2 and the active relay(s) listen. In Phase 2, s_2 transmits while s_1 and the active relay(s) listen. In Phase 3, the active relay(s) transmit while the s_1 and s_2 listen.

6.1.1 Upper Bound

An easy upper bound can be calculated by assuming a genie that provides the relays with either s_1 or s_2 message. This makes the network equivalent to a point to point bi-directional channel where one of the sources has $N+1$ antennas and the other source has only one antenna. The relay selection process is equivalent to selecting two antennas out of the $N+1$ available antennas at one of the sources. One of the selected antenna should be the direct link antenna though. One of the sources is equivalent to a MISO system with $n+1$ transmit antennas and one receive antenna. The performance of that source is upper bounded by the performance of a $(n+1)\times 1$ MISO system with antenna selection that chooses two antennas for each codeword transmission. This is an upper bound since the antenna selection process results in some configurations that do not have a counterpart in the relay system under study. On the other hand, the other source under the same assumption is equivalent to a SIMO system with one transmit antenna and $n+1$ receive antennas. The performance of that source is upper bounded by the performance of a $1\times(n+1)$ SIMO system with antenna selection that chooses two antennas for each codeword reception.

The DMT of a $M \times N$ MIMO link with $L_t < M$ selected transmit antennas and $L_r < N$ selected receive antennas is upper bounded by a piecewise linear function obtained by connecting the following $K+2$ points [45]

$$\Big\{\Big(j, (M_r - j)(M_t - j)\Big)\Big\}_{j=0}^{K}, \big(\min(L_r, L_t), 0\big), \tag{6.3}$$

where

$$K = \arg\min_{k\in\mathbb{Z}} \frac{(M_r-k)(M_t-k)}{\min(L_r,L_t)-k}, \\ \text{subject to } \; 0 \le k \le \min(L_r, L_t) - 1$$

Using this result, the $(n+1)\times 1$ MISO system with two selected transmit antennas as well as the $1\times(n+1)$ SIMO system with two selected receive antennas have a DMT that equals to $d(r) = (n+1)(1-r)^+$. Hence the upper bound for the bi-directional multi-relay channel with opportunistic selection is

$$d(r) = (n+1)(1-r)^+. \tag{6.4}$$

6.1.2 Decode and Forward Relaying

We define two ways of relaying the communicating nodes messages under decode and forward relaying. The first relaying method is based on a one relay helping both sources, see Figure 6.2. The second relaying method is based on two relays helping the sources where each one is dedicated to helping one of the sources, see Figure 6.3. Selecting two relays to help both sources allows to select the best relay for each source while selecting one relay requires optimization of this selection decision for both sources.

6.1.2.1 One Relay Selection

We define $n+1$ acces modes. Mode 0 is the two phases direct communication access mode. The n relay-assisted access modes are defined such that at each mode a relay is helping the sources, see Figure 6.2. If the sources messages can not be exchanged through the direct link, Mode 0, a relay is used to forward the messages to the communicating nodes. This is refereed to as Mode i where $i \in \{1, \ldots, n\}$ and Relay i is helping both sources. In any of these relay-assisted modes, the exchange of information is done through a 3-phases relaying scheme, see Figure 6.4. During the first and second phases, one of the sources nodes transmits and the other source node and the associated relay for this access mode listen. The relay associated with this access mode tries to decode both messages transmitted in the first and second phases and forwards the superposition of both messages during the third phase to s_1 and s_2. In the third phase, the relay transmits and the sources nodes listen. Each of the communicating nodes can subtract its own message away from the received signal and decodes the other source message interference free. If the relay could not decode both

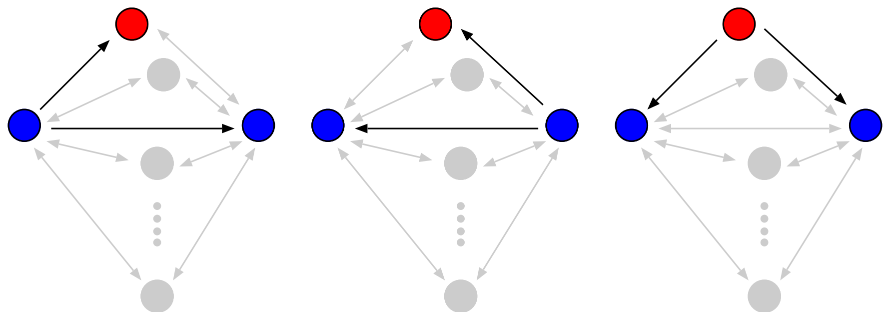

Figure 6.4. An access mode in a 3-phase bi-directional relaying scheme, one relay helping.

messages after Phase 1 and Phase 2 , it transmits nothing in the third phase. At the end of the transmission interval, If any of the transmitting node, s_1 or s_2, could not decode the other source message, that access mode is declared in outage.

Note that since the sources are assumed to use the same transmission rate, a common codebook can be used at both sources and the relay can use a simple network coding technique, for example encoding the module sum of the decoded messages. However, this requires each source at the end of Phase 3 to decode the sum separtly without using the information received in Phase 1 and 2. Superposition, however, does not require separte decoding where the source can substract its message and use the information received in the previous phases to decode. In addition, superposition is more general and can be used if the results needed to be extended to non equal transmission rates.

Theorem 14 *The DMT of an opportunistic bi-directional multi-relay channel with one-relay selection, a direct link between the two sources and DF relaying is given by*

$$d(r) = (1 - 2r) + N(1 - 3r) \tag{6.5}$$

Proof: Mode 0 is a direct transmission access mode; the first source transmits for half the transmission interval and the second source listens. In the second half of the transmission interval, the second source transmits and the first source listens. One can show that the DMT of Mode 0 is $d_0(r) = 1 - 2r$.

Mode i, where $i > 0$, is a 3-phase relay-assisted access mode. The fractional timing of the three phases are assumed equal. In Mode 1, the first source transmits for 1/3 of the transmission interval . The second third of the transmission interval the second source transmits. The last third of the transmission interval, Relay 1 forwards the superposition of the two messages to both communicating nodes. Mode 1 is in outage if s_1 or s_2 are not capable of decoding the message sent to it from the other source by the end of the transmission interval. To calculate the total DMT of the system, we begin by calculating the conditional DMT of each of the access modes. Due to symmetry, the outage event is calculated for s_1 and the calculation of the outage for s_2 should be identical. If $\mathcal{S}_1$ and $\mathcal{S}_2$ represent the outage event for s_1 and s_2 in Mode 1, respectively, one can show that

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = \mathbb{P}(\mathcal{S}_1|\mathcal{O}_0) + \mathbb{P}(\mathcal{S}_2|\mathcal{O}_0) \doteq \mathbb{P}(\mathcal{S}_1|\mathcal{O}_0). \tag{6.6}$$

The probability of outage in Mode 1 given that Mode 0 is in outage is given by

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = \mathbb{P}\bigg(\Big\{\frac{1}{3}\log(1+U_1\rho) < r\log\rho\Big\}\Big|\;\Big\{\frac{1}{2}\log(1+|h_{s_1s_2}|^2\rho) < r\log\rho\Big\}\bigg), \tag{6.7}$$

where for Relay i, the random variable U_i is defined as

$$U_i = \begin{cases} |h_{s_1s_2}|^2 & |h_{s_1r_i}|^2 < \frac{\rho^{3r}-1}{\rho} \\ |h_{s_1s_2}|^2 + |h_{r_is_2}|^2 & |h_{s_1r_i}|^2 \geq \frac{\rho^{3r}-1}{\rho}. \end{cases} \tag{6.8}$$

$$\begin{aligned}\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = &\mathbb{P}\bigg(\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\bigg)\mathbb{P}\bigg(|h_{s_1r_1}|^2 < \frac{\rho^{3r}-1}{\rho}\bigg) \\ &+ \mathbb{P}\bigg(\Big\{|h_{s_1s_2}|^2 + |h_{r_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\bigg)\mathbb{P}\bigg(|h_{s_1r_1}|^2 \geq \frac{\rho^{3r}-1}{\rho}\bigg).\end{aligned} \tag{6.9}$$

One can show that $\frac{\rho^{3r}-1}{\rho} \dot{>} \frac{\rho^{2r}-1}{\rho}$, therefore

$$\mathbb{P}\bigg(\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\bigg) \doteq 1. \tag{6.10}$$

Using results from the Appendix I and defining $g_1(r,\rho) \triangleq \frac{\rho^{2r}-1}{\rho}$ and $g_2(r,\rho) \triangleq \frac{\rho^{3r}-1}{\rho}$

$$\begin{aligned}
\mathbb{P}\Big(\Big\{|h_{s_1s_2}|^2 + |h_{r_1s_2}|^2 < g_2(r,\rho)\Big\}\Big|\Big\{|h_{s_1s_2}|^2 < g_1(r,\rho)\Big\}\Big)& \\
&= \int_0^{g_1(r,\rho)} \frac{ze^{-z}}{1-e^{-g_1(r,\rho)}}dz + \int_{g_1(r,\rho)}^{g_2(r,\rho)} \frac{g_1(r,\rho)e^{-z}}{1-e^{-g_1(r,\rho)}}dz \\
&\doteq \frac{1-e^{-\rho^{2r-1}} - \rho^{2r-1}e^{-\rho^{3r-1}}}{1-e^{-\rho^{2r-1}}} \doteq \rho^{3r-1}. \qquad (6.11)
\end{aligned}$$

Substituting (6.10) and (6.11) into (6.9), the conditional probability of outage is given by

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) \doteq \rho^{3r-1} + \rho^{3r-1}(1-\rho^{3r-1}) \doteq \rho^{3r-1}. \qquad (6.12)$$

Hence, the conditional DMT for the first relay-assisted access mode is $d_1(r) = (1-3r)^+$. For Mode 2, the conditional outage can be shown to be

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \geq \mathbb{P}(\mathcal{O}_2|\mathcal{O}_0) \doteq \rho^{3r-1}, \qquad (6.13)$$

where (6.13) follows the same proof as $\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0)$. Also one can show

$$\begin{aligned}
\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \doteq& \mathbb{P}\Big(\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\mathcal{O}_1,\mathcal{O}_0\Big)\mathbb{P}\Big(|h_{s_1r_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big) \\
&+ \mathbb{P}\Big(\Big\{|h_{s_1s_2}|^2 + |h_{r_2s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\mathcal{O}_1,\mathcal{O}_0\Big)\mathbb{P}\Big(|h_{s_1r_2}|^2 \geq \frac{\rho^{3r}-1}{\rho}\Big) \\
&\dot{\leq} \mathbb{P}\Big(|h_{s_1r_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big) + \mathbb{P}\Big(\Big\{|h_{r_2s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\mathcal{O}_1,\mathcal{O}_0\Big)\mathbb{P}\Big(|h_{s_1r_2}|^2 \geq \frac{\rho^{3r}-1}{\rho}\Big) \\
&\dot{\leq} \mathbb{P}\Big(|h_{s_1r_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big) + \mathbb{P}\Big(\Big\{|h_{r_2s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big)\mathbb{P}\Big(|h_{s_1r_2}|^2 \geq \frac{\rho^{3r}-1}{\rho}\Big) \\
&\doteq \rho^{3r-1} + \rho^{3r-1}(1-\rho^{3r-1}) \doteq \rho^{3r-1}. \qquad (6.14)
\end{aligned}$$

From (6.13) and (6.14), the conditional outage of the second access mode is bounded from above and below such that

$$\rho^{3r-1} \dot{\leq} \mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \dot{\leq} \rho^{3r-1} \qquad (6.15)$$

Hence, $\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1,\mathcal{O}_0) \doteq \rho^{3r-1}$ and $d_2(r) = (1-3r)^+$. Following similar steps, one can show that $d_i(r) = (1-2r)^+$ for $i = 3,\ldots,n$. Substituting conditional DMTs into Equation (6.1), the DMT is $d(r) = (1-2r) + n(1-3r)$. $\square$

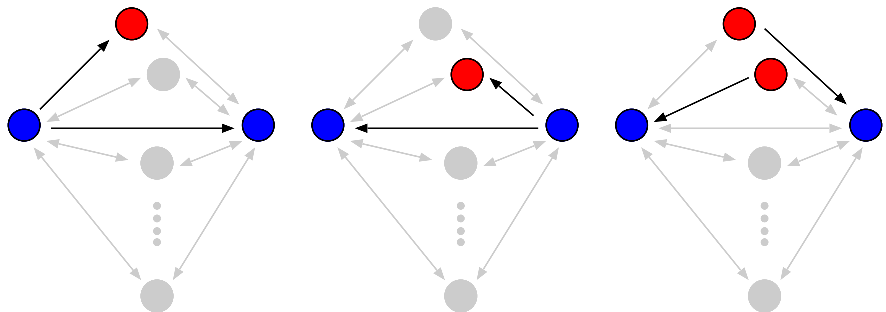

Figure 6.5. An access mode in a 3-phase bi-directional relaying scheme, two relays helping.

6.1.2.2 Two Relays Selection

For the relay-assisted access modes, we release the condition that one relay should help both messages. The sources s_1 and s_2 transmit their messages during Phase 1 and Phase 2. In the Phase 3, a relay is selected from the set of relays that decoded s_1 message to forward s_1 message to s_2 and another relay is selected from the set of relays that decoded s_2 message to forward s_2 message to s_1, Figure 6.5. This can result in two relays active where each is forwarding one of the sources messages. It can happen as well that only one relay is selected to help both sources if that relay decoded both messages and was the optimal relay for both sources.

In Modes i where $i > 1$, a relay or two relays are assumed active helping the communicating nodes. For n relays network, the number of possible access modes are $n(n-1) + n + 1 = n^2 + 1$, where this represents the one non-relay assisted access mode, the n one-relay assisted access modes and $n(n-1)$ possible two-relay assisted access modes.

To calculate the total DMT, the access modes are ordered such that

$$d(r) = d_0(r) + d'_1(r) + \cdots + d'_n(r) + d'_{n+1}(r) + \cdots + d'_{n^2+1} \tag{6.16}$$

where $d_0(r)$ represents the non-relayed direct communication access mode and $d'_i(r)$, $i \in \{1, \ldots, n\}$, is the DMT for the one-relay assisted access modes and the rest of the conditional DMTs are for the two-relay assisted access modes.

The calculation of the conditional DMT for the one-relay assisted access mode, $d'_i(r)$ where $i \in \{0, \ldots, n\}$, is identical to the calculation of the DMTs in the one relay selection problem. For the two-relay assisted access modes, we start by calculating the conditional DMT for mode $n+1$ where this access mode is defined such that Relay 1 decodes the message from s_1 and Relay 2 decodes the message from s_2.

To make the analysis easier we look at the case where $n = 2$, the generalization to $n > 2$ should follow along. The system has a total of 5 access modes in this case. Mode 0 is the direct link, Mode 1 is Relay 1 helping both sources, Mode 2 is Relay 2 helping both sources, Mode 3 is Relay 1 helping s_1 and Relay 2 helping s_2, and Mode 4 is Relay 1 helping s_2 and Relay 2 helping s_1. In access Mode 3, outage is guaranteed to happen if Relay 1 in Mode 1 could not decode the message from Source 1 or if Relay 2 in Mode 2 could not decode the message from Source 2. Note that the conditional outage of Mode 3 assumes that Mode 0 is in outage. As a result, a relay failure to decode in Mode 3 will result in outage since the direct link is already weak. Some other condition can case outage in Mode 1 and 2 and should not affect Mode 3. The conditional outage of Mode 3 is given by

$$\mathbb{P}(\mathcal{O}_3|\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_2) = \sum_{i=1}^{6} \mathbb{P}(\mathcal{O}_3|\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_2, A_i)\mathbb{P}(A_i|\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_2) \tag{6.17}$$

where $\alpha = \frac{\rho^{3r}-1}{\rho}$ and the events A_i where $i \in \{1, \ldots, 6\}$ are defined such that

$$\begin{aligned}
\mathbb{P}(A_1) =& \mathbb{P}(\{|h_{s_1r_1}|^2 < \alpha\}) \\
\mathbb{P}(A_2) =& \mathbb{P}(\{|h_{s_1r_1}|^2 > \alpha\}, \{|h_{r_2s_2}|^2 < \alpha\}) \\
\mathbb{P}(A_3) =& \mathbb{P}(\{|h_{s_1r_1}|^2 > \alpha\}, \{|h_{r_2s_2}|^2 > \alpha\}, \{|h_{r_1s_2}|^2 < \alpha\}, \{|h_{s_2r_2}|^2 < \alpha\}) \\
\mathbb{P}(A_4) =& \mathbb{P}(\{|h_{s_1r_1}|^2 > \alpha\}, \{|h_{r_2s_2}|^2 > \alpha\}, \{|h_{r_1s_2}|^2 < \alpha\}, \{|h_{s_2r_2}|^2 > \alpha\}) \\
\mathbb{P}(A_5) =& \mathbb{P}(\{|h_{s_1r_1}|^2 > \alpha\}, \{|h_{r_2s_2}|^2 > \alpha\}, \{|h_{r_1s_2}|^2 > \alpha\}, \{|h_{s_2r_2}|^2 < \alpha\}) \\
\mathbb{P}(A_6) =& \mathbb{P}(\{|h_{s_1r_1}|^2 > \alpha\}, \{|h_{r_2s_2}|^2 > \alpha\}, \{|h_{r_1s_2}|^2 > \alpha\}, \{|h_{s_2r_2}|^2 > \alpha\})
\end{aligned} \tag{6.18}$$

For the event A_1, one can show that

$$\mathbb{P}(A_1|\mathcal{O}_0,\mathcal{O}_1,\mathcal{O}_2) = \mathbb{P}(A_1|\mathcal{O}_1) = \frac{\mathbb{P}(\mathcal{O}_1|A_1)\mathbb{P}(A_1)}{\mathbb{P}(\mathcal{O}_1)} \doteq 1, \tag{6.19}$$

where it is easy to show that

$$\begin{aligned}
\mathbb{P}(\mathcal{O}_1|A_1) &\doteq \rho^{3r-1} \\
\mathbb{P}(A_1) &\doteq \rho^{3r-1} \\
\mathbb{P}(\mathcal{O}_1) &\doteq \rho^{2(3r-1)}
\end{aligned}$$

The conditional outage $\mathbb{P}(\mathcal{O}_3|\mathcal{O}_0,\mathcal{O}_1,\mathcal{O}_2,A_1)$ is given by

$$\begin{aligned}
\mathbb{P}(\mathcal{O}_3|\mathcal{O}_0,\mathcal{O}_1,\mathcal{O}_2,A_1) &= \mathbb{P}\bigg(\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\mathcal{O}_0,\mathcal{O}_1,\mathcal{O}_2\bigg) \\
&= \mathbb{P}\bigg(\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\mathcal{O}_0\bigg) \\
&\doteq \mathbb{P}\bigg(\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{3r}-1}{\rho}\Big\}\Big|\Big\{|h_{s_1s_2}|^2 < \frac{\rho^{2r}-1}{\rho}\Big\}\bigg) \doteq 1,
\end{aligned} \tag{6.20}$$

where $\frac{\rho^{3r}-1}{\rho} \dot{>} \frac{\rho^{2r}-1}{\rho}$. From (6.17), (6.19) and (6.20), the conditional outage of Mode 3 is given by

$$\mathbb{P}(\mathcal{O}_3|\mathcal{O}_0,\mathcal{O}_1,\mathcal{O}_2) \doteq 1 \tag{6.21}$$

and the conditional DMT $d_3'(r) = 0$. Using the same steps, one can show that access mode 4 would not add any gain to the system, $d_4'(r) = 0$ and the total DMT for two-relay selection is equivalent to the a one-relay selection. The same analysis can be shown to generalize that for the case of n relay.

6.1.2.3 Adaptive Decode and Forward

An Adaptive decode and forward relaying scheme for a half duplex bi-directional relay channel with one relay and direct link between the two sources and various amount of channel state information at the transmitter has been proposed in [55]. In an adaptive decode and forward relaying scheme with no CSI at the transmitter, the first and second phases where s_1 and s_2 transmit to each other and the relay each use a fraction t of the time interval. If the relay fails to decode any of the sources messages in the first two phases, it will transmit nothing in the third phase and the system will be in outage. The third phase where the relay broadcast to both sources uses a fraction $1-2t$ of the transmission interval. Phase one and phase two have the same fractional time allocated to both of them due to the network symmetry, same channel statistics and equal transmit rate requirments for both sources, $r \log \rho$. The allocated The time for each of the phases does not depend on the channel coefficients and depends on the multiplexing gain. For each multiplexing gain, t is kept fixed however. Each of the sources can use a transmit power of $\frac{\rho}{t}$ to not violate the power constraint, but since t is not a function of the transmit power and the fact that the transmit power goes to infinity in the DMT analysis, the DMT is exactly the same when the transmit power is ρ. The same argument applies to the relay transmit power.

Theorem 15 *The DMT for a bi-directional one relay channel scheme with adaptive decode and forward relaying scheme is [55]*

$$d(r) = \begin{cases} 2-5r & r < \frac{1}{5} \\ \frac{2(1-2r)}{1+r} & r \geq \frac{1}{5} \end{cases} \tag{6.22}$$

where the optimal fraction of channel use is

$$t = \begin{cases} \frac{2}{5} & r < \frac{1}{5} \\ \frac{5+2r}{3} & r \geq \frac{1}{5} \end{cases} \tag{6.23}$$

In a network of n relays, we extend the previous scheme to an opportunistic multi-relay bi-directional channel. In the first and second phases, one of the sources tranmists while the other source and the relays listen. The relays try to decode both messages from s_1 and s_2. In the third phase, one of the relays is selected to transmit to both sources. The fractional time of each phase depends on the multiplexing gain and the number of relays. It is however fixed for each multiplexing gain.

Theorem 16 *The DMT for a bi-directional multi-relay channel with opportunistic adaptive decode and forward relay selection is given by*

$$d(r) = \begin{cases} (n+1) - (3n+2)r & r < \frac{n}{3n+2} \\ \frac{(n+1)(1-2r)}{1+nr} & r \geq \frac{n}{3n+2} \end{cases} \tag{6.24}$$

where the optimal fraction time of channel use is

$$t = \begin{cases} \frac{n+1}{3n+2} & r < \frac{n}{3n+2} \\ \frac{1+nr}{n+2} & r \geq \frac{1+nr}{n+2} \end{cases} \tag{6.25}$$

Proof: We define n aceess modes where each access mode represents the use of a different relay through the the previously illustrated three phase communication scheme.

The DMT of a system that switches between n dependent access modes is given by

$$d(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n)}{\log \rho}, \tag{6.26}$$

where the system is in outage if all access modes are in outage. In a manner similar to [28], the probability of outage $\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n)$ can be expressed as follows

$$\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n) \doteq \rho^{-d(r)}. \tag{6.27}$$

Due to Symmetry, we consider the event that the channel does not support the rate for s_1. The probability of outage is given by

$$\mathbb{P}(\mathcal{O}_1, \ldots, \mathcal{O}_n) = \sum_{i=0}^{n} \binom{n}{i} \mathbb{P}(\mathcal{R}_i)\mathbb{P}(\mathcal{O}|\mathcal{R}_i), \tag{6.28}$$

where $\mathbb{P}(\mathcal{R}_i)$ is the probability of i relays decoding the source message and $n-i$ relays not decoding the source message. There are $\binom{n}{i}$ such events. The System outage expresion in Equation (6.28) has n terms each has an exponent d_i where $i \in \{0, \ldots, n\}$. For a given t, the outage exponent from Equations (6.27) and (6.28) can be written as

$$d(r,t) = \min\{d_0, \ldots, d_n\}, \tag{6.29}$$

where d_i represents the exponent of each of the n terms in Equation (6.27).

The probability of Relay j not decoding the source message is $\mathbb{P}\Big(t\log(1+|h_{s_1r_j}|^2\rho) < r\log\rho\Big) \doteq \rho^{\frac{r}{t}-1}$ and the probability of Relay j decoding is $\mathbb{P}\Big(t\log(1+|h_{s_1r_j}|^2\rho) \geq r\log\rho\Big) \doteq 1$, hence

$$\mathbb{P}(\mathcal{R}_i) \doteq \rho^{(n-i)(\frac{r}{t}-1)}. \tag{6.30}$$

The conditional outage probability $\mathbb{P}(\mathcal{O}|\mathcal{R}_i)$ represents the probability of the whole system being in outage given that there are i relays decoding the source message and $n-i$ relays not decoding the source message. For the first term, non of the relay decoded the signal and the conditional outage is given by

$$\mathbb{P}(\mathcal{O}|\mathcal{R}_0) = \mathbb{P}(t\log(1+|h_{s_1s_2}|^2\rho) < r\log\rho) \doteq \rho^{(\frac{r}{t}-1)}, \tag{6.31}$$

From Equations (6.30) and (6.31), the exponent of the first term in Equation (6.28) is given by

$$d_0 = (n+1)\left(1-\frac{r}{t}\right). \tag{6.32}$$

For the other $n-1$ terms, if any of the relays could not decode the signal from the source, the acces mode relates to this relay would be in outage. The conditional outage for $m > 0$ can be shown to be

$$\begin{aligned}\mathbb{P}(\mathcal{O}|\mathcal{R}_m) = \mathbb{P}\Bigg(&\Big\{f\log(1+|h_{s_1s_2}|^2\rho) + (1-2f)\log(1+|h_{s_1r'_1}|^2\rho) < r\log\rho\Big\}\\ &,\ldots,\Big\{f\log(1+|h_{s_1s_2}|^2\rho) + (1-2f)\log(1+|h_{s_1r'_m}|^2\rho) < r\log\rho\Big\}\Bigg)\end{aligned} \tag{6.33}$$

where the set of relays $\{r'_1,\ldots,r'_m\}$ represents the set of the m relays that decoded the source message. The conditional outage can be expressed as

$$\mathbb{P}(\mathcal{O}|\mathcal{R}_m) \doteq \rho^{-d'_m(r)}, \tag{6.34}$$

$$d'_m = \inf_{(v_1,v_2^{(1)},\ldots,v_2^{(m)})\in O} v_1 + \sum_{i=1}^{m} v_2^{(i)}\ . \tag{6.35}$$

The random variables v_1 and $v_2^{(i)}$ represent the exponential order of $1/|h_{s_1 s_2}|^2,\ 1/|h_{s_1 r'_i}|^2$ and $1/|h_{r'_i s_2}|^2$, respectively. Each of these random variables has a probability density function $p(x)$ that is asymptotically equal to ρ^{-x} for $x \geq 0$ and 0 otherwise [28]. The set O represents the conditional outage event where $O = \mathcal{O}_1^+ \cap \ldots \cap \mathcal{O}_n^+$ and $\mathcal{O}_i^+$ is defined as

$$\mathcal{O}_i^+ = \Big\{ \big(v_1, v_2^{(i)}\big) \in R^{3+} \Big| t(1-v_1)^+ + (1-2t)(1-v_2^{(i)})^+ < r \Big\}, \tag{6.36}$$

where with $r \leq t_i \leq \frac{1}{2}$. The solution to this linear programming problem in (6.35) have the explicit solution

$$d'_m = \begin{cases} (n+1) - \frac{r}{f} & r < t < \frac{1}{3} \\ (n+1) - \frac{nr}{1-2t} & \max\{r, \frac{1}{3}\} \leq t < \frac{1-r}{2} \\ \frac{1-t-r}{t} & t \geq \max\{r, \frac{1}{3}, \frac{1-r}{2}\} \end{cases} \tag{6.37}$$

From Equation (6.30), (6.34), (6.37), the exponent of the i^{th} term in (6.28) for $i > 0$ is given by

$$d_i = \begin{cases} (i+1) - \frac{r}{f} + (n-i)\big(1 - \frac{r}{f}\big) & r < t < \frac{1}{3} \\ (i+1) - \frac{ir}{1-2t} + (n-i)\big(1 - \frac{r}{f}\big) & \max\{r, \frac{1}{3}\} \leq t < \frac{1-r}{2} \\ \frac{1-t-r}{t} + (n-i)\big(1 - \frac{r}{f}\big) & t \geq \max\{r, \frac{1}{3}, \frac{1-r}{2}\} \end{cases} \tag{6.38}$$

Substituting Equations (6.32) and (6.38) in Equation (6.29) and optimizing over $f \in \Big[r, \frac{1}{2}\Big]$, the DMT can be given as

$$d(r) = \sup_{f \in [r, \frac{1}{2}]} \min\{d_0, \ldots, d_n\}. \tag{6.39}$$

Solving the optimaization problem, the DMT of the system can be found. $\square$

Figure 6.6 shows the DMT for an opportunistic adaptive DF bi-directional channel with number of relays varies from 1 to 5. It is shown that as the number of relays increases, the gain achieved at low multiplexing gains is more than that achieved at high multiplexing gains. The adaptive decode and forward does not benifit from increasing the number of relays at high multiplexing gain compared to low multiplexing gains.

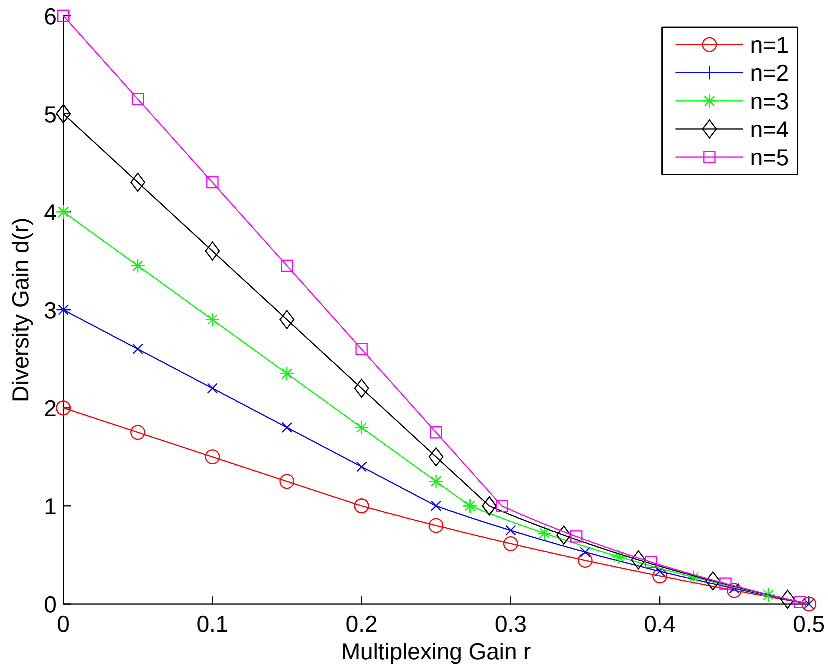

Figure 6.6. DMT for the opportunistic adaptive DF bi-directional multi-relay channel with various number of relays.

6.1.3 Dynamic Decode and Forward Relaying

We introduce the DDF relaying protocol to a bi-directional multi-relay channel with direct link between the sources. We first study a one relay channel and extend the results to the case of n relays. For a one relay bi-directional channel, the communication is done through a three phase communication scheme. In the first phase, s_1 transmits until the relay can decode the message. In the second phase, s_2 transmits until the relay can decode the message. The relay broadcast the superposition of the two decoded message in the third phase. If the relay did not decode any of the sources messages, the receiver of this message should rely only on the information it got from the phase where the source was transmitting.

Theorem 17 *The maximum DMT for a bi-directional relay channel with a viable direct link* *between the two sources and DDF relaying is*

$$d(r) = \begin{cases} 1 & 0 \le r \le \frac{1}{4} \\ 2(1-2r) & \frac{1}{4} < r \le \frac{1}{2} \end{cases} \tag{6.40}$$

Proof: Due to the symmetry of the network and the separation between the signal transmitted from s_1 and s_2, we consider the outage event for one source and it follows that the second source outage is identical. Also because the the similarity between the channels statistics of a message traveling from s_1 to s_2 and a message traveling from s_2 to s_1 and the assumption of equal transmission rates, $r \log \rho$, for both sources, it is valid to assume that both users should equally share the resources. On the other hand, since the fractional time use of the channel for each source is not a function of the channel coefficients and is actually a function of the channel statistics and the transmission rate, $r \log \rho$, Phase 1 and Phase two are assumed to take same fraction of the transmission interval. If t is the relay listening-time ratio in Phase 1, Phase 2 should use the same listening-time ration t and Phase 3 uses $1-2t$ of the whole transmission interval. Neither Phase 1 nor Phase 2 are allowed to take more than half the transmission interval.

In a manner similar to [28], the DMT can be given as follows

$$d(r) = -\lim_{\rho \to \infty} \frac{\log \mathbb{P}(\mathcal{O})}{\log \rho}, \tag{6.41}$$

where the probability of outage $\mathbb{P}(\mathcal{O})$ can be expressed as follows

$$\mathbb{P}(\mathcal{O}) \doteq \rho^{-d(r)}, \tag{6.42}$$

$$d(r) = \inf_{(v_1, v_2, u) \in O} v_1 + v_2 + u \ . \tag{6.43}$$

The random variables v_1, u and v_2 represent the exponential order of $1/|h_{s_1 s_2}|^2$, $1/|h_{s_1 r}|^2$ and $1/|h_{r s_2}|^2$, respectively. Each of these random variables has a probability density function $p(x)$

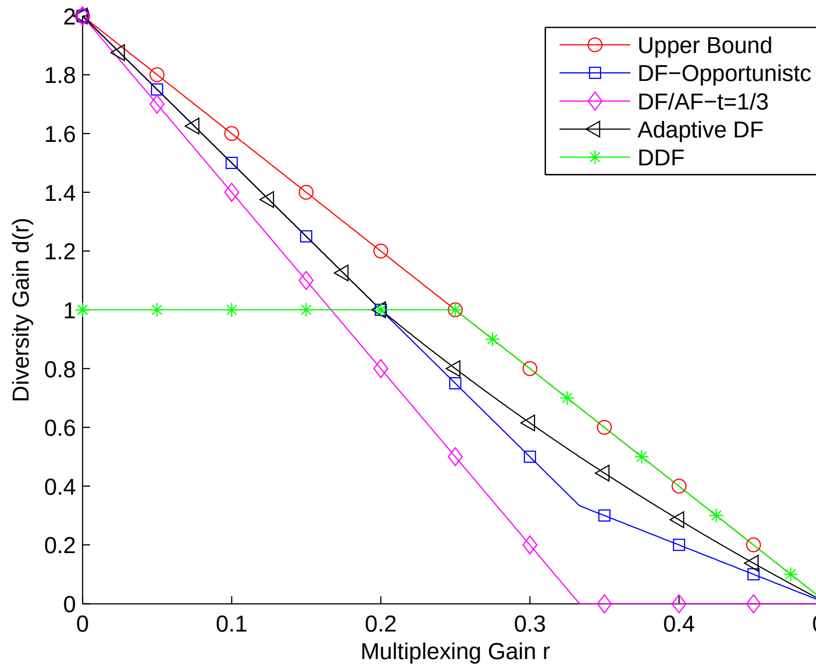

Figure 6.7. DMT for a one relay bi-directional relay channel with various relaying schemes.

that is asymptotically equal to ρ^{-x} for $x \geq 0$ and 0 otherwise. The set O represents the outage event for the network and can be shown to be defined as

$$\mathcal{O}^{+} = \left\{ \left(v_1, v_2, u\right) \in R^{3+} \middle| t(1-v_1)^{+} + (1-2t)(1-v_2)^{+} < r \right\}, \tag{6.44}$$

Solving the optimization problem and taking into account the $u = 1 - \frac{r}{f}$, one can show that Equation (6.40) is the diversity achieved by the DDF. $\square$

Figure 6.7 shows a comparison between some relaying schemes for the bi-directional one-relay channel and the DDF relaying scheme. The DDF scheme is optimal for multiplexing gains higher than .25, however, for low multiplexing gain the diversity is limited to 1. The DMT of a DF or AF protocol with fixed $t = 1/3$ can be easily calculated and gives a DMT $d(r) = 2(1-3r)$. An Opportunistic DF protocol where the relay is allowed to be switched on and off depending on the channel conditions outperforms the regular DF protocol. The DMT of the opportunistic DF protocol can be found by substituting $n = 1$ in Equation (6.5).

An Adaptive DF scheme where t is allowed to be a function of the rate r and optimized to maximize the DMT is proposed in [55]. The adaptive DF outperforms the regular, $t = 1/3$, DF protocol and the DDF for $r < 1/5$ and $r < 1/4$, respectively, while the DDF outperforms the adaptive DF for $r > 1/4$. It is also noted that the opportunistic DF is identical to the adaptive DF for $r < 1/5$ while for $r > 1/5$ the adaptive DF provide more diversity.

The DMT of the DDF is limited to one at low multiplexing gain. The reason for that is that the relay decodes the source message early and the source stops trasnmitting afterwards. Since the source stops transmitting that early and never get the chance to transmits again and the relay does not actually need all the remaining time, the diversity is limited to 1. In the third phase, the relay transmits and the source listen. If we compare this scheme to the one-way DDF relay channel, the source is transmitting througout the transmission interval and never stops transmitting. The half duplex constraint on the source node, where it can not transmit during Phase 2 and Phase 3 (since its listening to s_2 and the relay) is the reason for that loss in performance compared to the one-way DDF performance. On the other hand, at high multiplexing gains, the DDF in the bi-directional channel achieves the upper bound. This does not happen in a one-way DDF scheme. In the bi-directional DDF relaying, sharing the third phase between the two sources is equivalent to giving the channel access to any of the sources for more than half the transmission interval. This gives the relay more time to forward the decoded message to the source.

The adaptive decode and forward outperforms the DDF for $r < .2$ since the optimized time for Phase 1 in the adaptive DF is shown to be fixed and equals to $2/5$. This gives time for the source to transmit to the other source even if the relay already decoded the message compared to the DDF where the time for Phase 1 is the time for the relay to decode.

Theorem 18 *The maximum DMT for a bi-directional multi-relay channel with a viable* *direct link between the two sources and n DDF relays is*

$$d(r) = \begin{cases} n & 0 \leq r \leq \frac{1}{2}\Big(1 - \frac{n}{n+1}\Big) \\ (n+1)(1-2r) & \frac{1}{2}\Big(1 - \frac{n}{n+1}\Big) < r \leq \frac{1}{n+1} \\ \frac{1-2r}{r} & \frac{1}{n+1} < r \leq \frac{1}{2} \end{cases} \tag{6.45}$$

Proof: The DMT of a system that switches between n dependent access modes is given by

$$d(r) = -\lim_{\rho \to \infty} \frac{\log \mathbb{P}(\mathcal{O}_1, \dots, \mathcal{O}_n)}{\log \rho}, \tag{6.46}$$

where the system is in outage if all access modes are in outage. In a manner similar to [28], the probability of outage $\mathbb{P}(\mathcal{O}_1, \dots, \mathcal{O}_n)$ can be expressed as follows

$$\mathbb{P}(\mathcal{O}_1, \dots, \mathcal{O}_n) \doteq \rho^{-d_o(r)}, \tag{6.47}$$

$$d_o(r) = \inf_{(v_1, u^{(1)}, v_2^{(1)}, \dots, u_1^{(n)}, v_2^{(n)}) \in O} v_1 + \sum_{i=1}^{n} \left(u^{(i)} + v_2^{(i)}\right). \tag{6.48}$$

The random variables $v_1, u^{(i)}$ and $v_2^{(i)}$ represent the exponential order of $1/|h_{s_1 s_2}|^2$, $1/|h_{s_1 r_i}|^2$ and $1/|h_{r_i s_2}|^2$, respectively. Each of these random variables has a probability density function $p(x)$ that is asymptotically equal to ρ^{-x} for $x \geq 0$ and 0 otherwise [28]. The set O represents the outage event for the network. The network is considered in outage when no access mode is viable, i.e., $O = \mathcal{O}_1^+ \cap \ldots \cap \mathcal{O}_n^+$. Following simillar steps as [28] and exchanging the variables in the the outage formula, the outage region under DDF for access mode i can be shown to be

$$\mathcal{O}_i^+ = \Big\{ \left(v_1, v_2^{(i)}, u^{(i)}\right) \in R^{3+} \Big| t_i(1 - v_1)^+ + (1 - 2t_i)(1 - v_2^{(i)})^+ < r \Big\}, \tag{6.49}$$

where t_i is the listening-time ratio of the relay i with $r \leq t_i \leq \frac{1}{2}$ and $u = 1 - \frac{r}{f}$. In the following we outline the solution of Equations (6.48) and (6.49) for a two-relay channel. Solving the linear optimization problem, the DMT for DDF is established. □

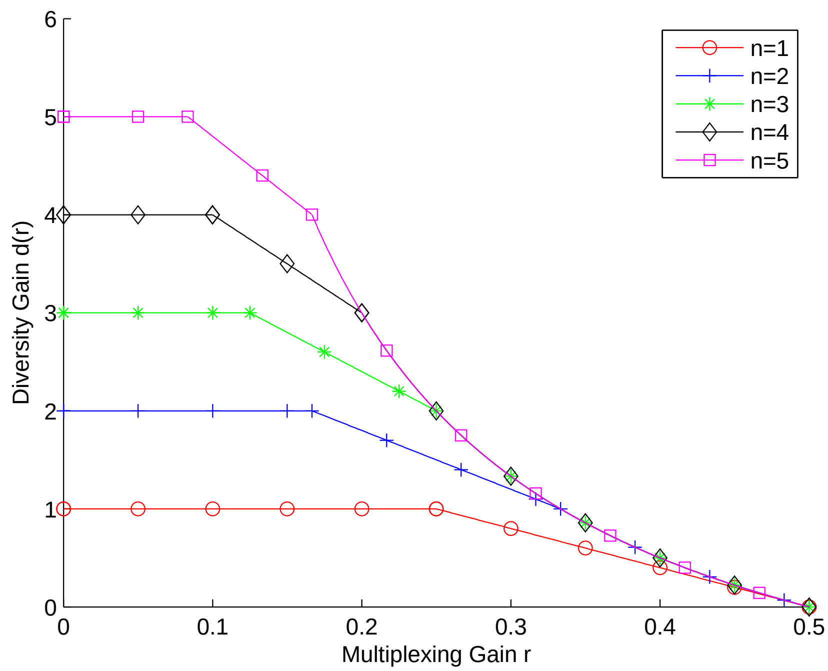

Figure 6.8. DMT for the opportunistic DDF bi-directional multi-relay channel with various number of relays.

Figure 6.8 shows the DMT for an opportunistic DDF bi-directional channel with number of relays from 1 to 5. It is shown that as the number of relays increases in the system, the diversity gain achieved by increasing the number of relays is only seen at low multiplexing gains only. At low multiplexing gain the diversity is limited to the total number of relays in the system. This is as discussed in the one relay bi-directional DDf channel due to the the source stopping trnasmission easrly in the transmission interval. The DMT is always optimal at the mid-multiplexing gains while at high multiplexing gains, the diversity gain is not a function of the number of relays and does not scale with the number of relays in the system.

6.1.4 Amplify and Forward Relaying

The two sources try first to communicate through the direct link using the same procedure discussed before, Mode 0. If Mode 0 is in outage, a three phase communication scheme is used where a relay is helping the two sources. In Phase 1, s_1 transmits and s_2 and the relays listen. In Phase 2, s_2 transmits and s_1 and the relays listen. In Phase 3, the relay adds the two signals received during the first and second phase, amplifies the signal to meet its power constraint and broadcasts it to the two sources. The fractional time of each of these phases is assumed equal, $t = \frac{1}{3}$. Each source can subtract its message from the signal received in the third phase and decode the other source message interference free.

Theorem 19 *The DMT of an opportunistic bi-directional multi-relay channel with a direct link between the two sources and AF relaying is given by*

$$d(r) = (1 - 2r) + N(1 - 3r) \tag{6.50}$$

Proof: We define $N + 1$ access modes where the first access mode is the non-relayed access mode, Mode 0, and the other are the relay-assisted access modes. The DMT of the system is given by equation (6.1). Due to the symmetry of the network , the outage event is calculated for one source only and should be identical for the other source. The DMT of Mode 0 can be shown to be $d_0(r) = 1-2r$. For the relayed access modes, the conditional outage probability can be calculated as follows. For Mode 1, the conditional outage is given by

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = \mathbb{P}(\mathcal{S}_1|\mathcal{O}_0) + \mathbb{P}(\mathcal{S}_2|\mathcal{O}_0) \doteq \mathbb{P}(\mathcal{S}_1|\mathcal{O}_0) \tag{6.51}$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ represent the outage event for s_1 and s_2 in Mode 1. The probability of Mode 1 being in outage given that Mode 0 is in outage is given by

$$\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) \doteq \mathbb{P}\left(\left\{|h_{s_1s_2}|^2 + \frac{|h_{s_1r_1}|^2|h_{r_1s_2}|^2}{|h_{s_1r_1}|^2+|h_{r_1s_2}|^2} < \frac{\rho^{3r}-1}{\rho}\right\}\middle|\left\{|h_{s_1s_2}|^2 < \frac{\rho^{2r}-1}{\rho}\right\}\right) \tag{6.52}$$

In order to calculate the conditional outage probability distribution, we first calculate the conditional density function of $Z = |h_{s_1s_2}|^2 + V$ where $V = \frac{|h_{s_1r_1}|^2|h_{r_1s_2}|^2}{|h_{s_1r_1}|^2+|h_{r_1s_2}|^2}$. The term $\frac{|h_{s_1r_1}|^2|h_{r_1s_2}|^2}{|h_{s_1r_1}|^2+|h_{r_1s_2}|^2}$ represents half the harmonic mean of two independent exponential random variables. Using a result of [54], the harmonic mean of two exponential random variables with parameters λ can be approximated by an exponential random variable with parameter $\lambda + \lambda = 2\lambda$.

Using results from the Appendix I and defining $\alpha = g_1(r,\rho) = \frac{\rho^{2r-1}}{\rho}$ and $g_2(r,\rho) = \frac{\rho^{3r-1}}{\rho}$, the conditional outage probability is

$$\begin{aligned}\mathbb{P}(\mathcal{O}_1|\mathcal{O}_0) = \int_0^{g_2(r,\rho)} f_{Z|B}(z)dz \doteq 2\int_0^{g_1(r,\rho)} \frac{e^{-2z}(e^z-1)}{1-e^{-g_1(r,\rho)}}dz + 2\int_{g_1(r,\rho)}^{g_2(r,\rho)} \frac{e^{-2z}\left(e^{g_1(r,\rho)}-1\right)}{1-e^{-g_1(r,\rho)}}dz \\ = \frac{e^{-2\rho^{3r-1}} - e^{-\rho^{2r-1}} - e^{-2\rho^{3r-1}+\rho^{2r-1}} + 1}{1-e^{-\rho^{2r-1}}} \doteq \rho^{3r-1}.\end{aligned} \tag{6.53}$$

Hence, $d_1'(r) = 1-3r$. To calculate the DMT for the other access modes, we upper bound and lower bound the conditional probability in a matter similar to (6.15). For Mode i where $2 < i \leq n$

$$\begin{aligned}\mathbb{P}\left(\left\{\frac{|h_{s_1r_i}|^2|h_{r_is_2}|^2}{|h_{s_1r_i}|^2+|h_{r_is_2}|^2} < \frac{\rho^{3r}-1}{\rho}\right\}\middle|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}\right) \geq \mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}) \geq \mathbb{P}(\mathcal{O}_i|\mathcal{O}_0) \\ \rho^{3r-1} \geq \mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1}) \geq \rho^{3r-1}\end{aligned} \tag{6.54}$$

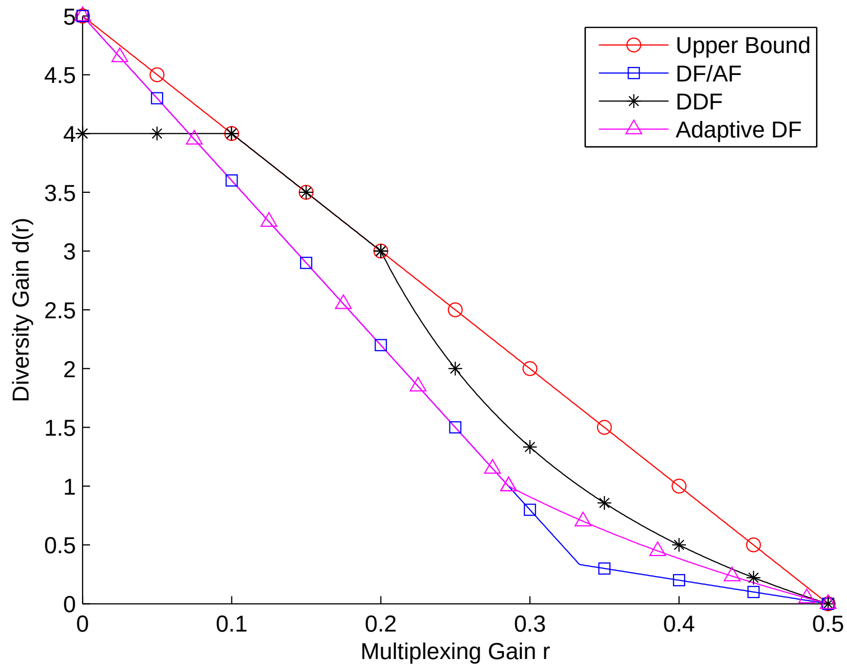

Figure 6.9. DMT for an opportunistic four-relay bi-directional channel with direct link between the sources and various relaying schemes.

Hence, $\mathbb{P}(\mathcal{O}_i|\mathcal{O}_0,\ldots,\mathcal{O}_{i-1})\dot{=}\rho^{3r-1}$ and $d_i'(r) = 1-3r$. The total DMT is the summation of the conditional DMTs and Mode 0 DMT and is given by $d(r) = (1-2r) + M(1-3r)$. $\square$

Figure 6.9 shows a comparison between the DMT of a bi-directional channel with four relays and a direct link between the sources under various relaying protocols. As noted in the one relay DDF channel, the diversity of the DDF is limited by the number of relays, maximum diversity of 4 in the example, at low multiplexing gains. The reason for that is that the source stops transmitting very early and this limits the maximum diversity that can be achieved. However, the DDF is optimal at mid multiplexing gains and outperforms the AF and DF opportunistic relaying as well as the adaptive DF relaying at high multiplexing gains. One can propose a hybrid scheme where at multiplexing gains $r \leq \frac{1}{3n+2}$ the adaptive opportunistic DF, the opportunistic DF or the opportunistic AF relaying scheme is used (all have the same performace) and at multiplexing gains $r > \frac{1}{3n+2}$ the DDF opportunistic scheme is used. This will achieve the maximum diversity gain of all proposed schemes.

6.2 No Direct Link Between the Sources

In case of no viable link between the two sources, the two sources are enforced to communicate through the relays in a 2-phase communicating scheme. In Phase 1, the two sources transmit and the relays listen. In the second phase one of the relays broadcasts and both sources listens, see Figure 6.10. The system will be in outage if one or both sources is not capable of decoding the other source message.

For n relays system, one can define n access modes. Mode i represents an access mode where the sources are using the help of Relay i in the two phase scheme described above. The analysis of such case is simple where all the defined access modes are independent (no direct link like to cause dependency between access modes). The DMT of such opportunistic system is given by Equation (6.1) and because of the independency between access modes, the conditional DMTs is replaced by the DMTs of the access modes. Since all access modes are identical, the DMT is given by

$$d(r) = nd_i(r), \tag{6.55}$$

where $d_i(r)$ is the DMT of any Mode $i \in \{1, \ldots, N\}$ since all access modes DMTs are equal in this case.

For one of the access modes, a one relay channel, an easy upper bound can be found by assuming a genie that gives the relay the message of one source. The upper bound for any of the access modes is $1 - 2r$ and by opportunistically switching between n of such access modes, the DMT upper bound of the opportunistic system is $N(1 - 2r)$.

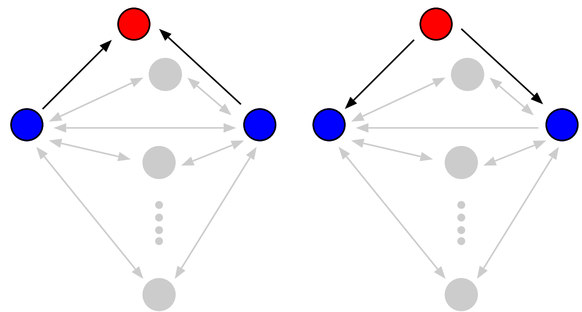

Figure 6.10. 2-phase bi-directional relaying scheme, one relay helping.

6.2.1 Decode and Forward Relaying

The DMT of a single DF bi-directional relay channel with no direct link has been studied in [55]. It was found that the DMT of such system is given by

$$d(r) = \min\left\{1 - \frac{r}{t}, 2 - \frac{4r}{t}, 1 - \frac{r}{1-t}\right\} \tag{6.56}$$

where Phase 1 occupies a fractional of the transmission interval time equals to t and $t \in [\frac{r}{2}, 1-r]$ while Phase 2 occupies a fractional of the transmission interval time equals to $1-t$. If t is assumed to equal to $\frac{1}{2}$, equal time for Phase 1 and Phase 2, the DMT is given by

$$d(r) = \begin{cases} 1 - 2r & 0 \leq r < \frac{1}{6} \\ 2 - 8r & \frac{1}{6} \leq r \leq \frac{1}{3} \end{cases} \tag{6.57}$$

If t is optimized and allowed to depend on the multiplexing gain r, it is shown in [55] that the optimal DMT is given by

$$d(r) = \begin{cases} 1 - 2r & 0 \leq r < \frac{1}{6} \\ 2 - \frac{4r}{t} & \frac{1}{6} \leq r \leq \frac{1}{3} \end{cases} \tag{6.58}$$

and the optimal fractional time t^* is

$$t^*(r) = \begin{cases} \frac{1}{2} & 0 \leq r < \frac{1}{6} \\ \frac{5r+1-\sqrt{(5r+1)^2-16r}}{2} & \frac{1}{6} \leq r \leq \frac{1}{3} \end{cases} \tag{6.59}$$

For the opportunistic system with N relay, if t is fixed to $1/2$ of the transmission interval, the DMT of the opportunity system is

$$d(r) = \begin{cases} N(1 - 2r) & 0 \leq r < \frac{1}{6} \\ N(2 - 8r) & \frac{1}{6} \leq r \leq \frac{1}{3} \end{cases} \tag{6.60}$$

If t is allowed to be a function of the multiplexing gain and optimized to achieve maximum diversity gain, the DMT of the opportunity system is

$$d(r) = \begin{cases} N(1-2r) & 0 \leq r < \frac{1}{6} \\ N(2-\frac{4r}{t*}) & \frac{1}{6} \leq r \leq \frac{1}{3} \end{cases} \tag{6.61}$$

where t^* is defined in (6.59).

6.2.2 Amplify and Forward Relaying

For any of the defined access modes, in the first phase, the two sources transmit simultaneously while the relays listen. In the second phase, the selected relay transmits an amplified version of the received signal at the first phase while the sources listen. Each source can subtract its own message and decode the sent message interference free. It is straight forward to show that for access mode i the DMT of that access mode is given by $d_i(r) = (1-2r)$. The total DMT of the opportunistic system with n AF relays is given by Equation (6.55) and is equal to $d(r) = (n+1)(1-2r)$. This DMT achieves the upper bound and hence selecting more than one relay would give no extra gain.

Figure 6.11 shows the DMT for a 4-relay bi-directional channel with no direct link. The AF outperforms all the DF protocols. DF relaying is constraint by the first phase where the relay decodes both messages transmitted simultaneously by the two sources. In Contrast, AF relaying does not require the relay to decode and the relay forwards all the received signal to the sources without decoding. Each source can easily subtract the interference and decode its message interference free. The adaptive DF as expected outperforms the $t = \frac{1}{2}$ DF protocol.

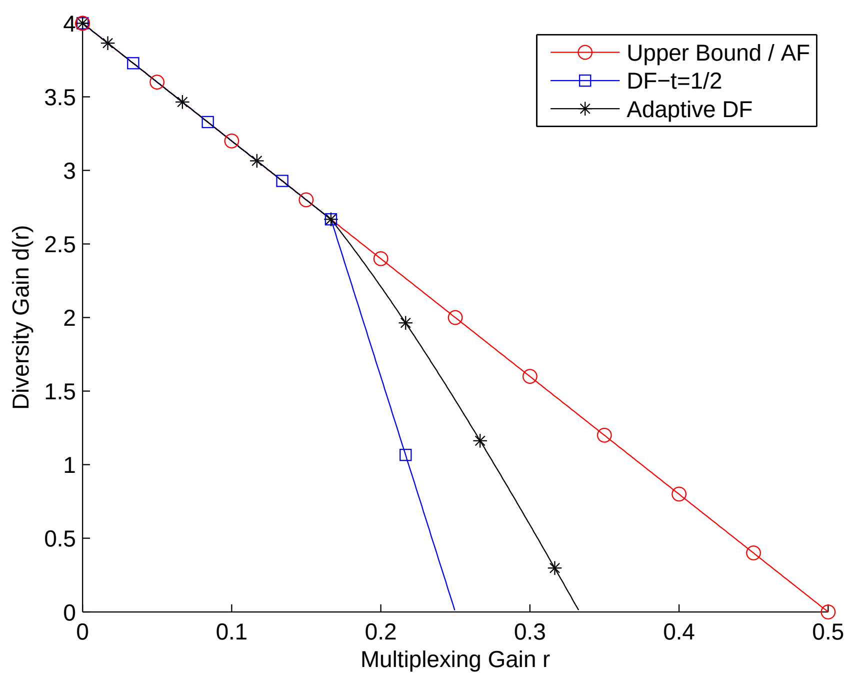

Figure 6.11. DMT for an opportunistic four-relay bi-directional relay channel with no direct link between sources and various relaying schemes.

CHAPTER 7

CONCLUSION

The high-SNR performance of opportunistic relay networks are investigated. Except for a handful of simple relay selection scenarios, there are two main difficulties in the analysis of opportunistic relay networks: (1) the decision variables often depend on more than one link gain, complicating the performance analysis and (2) the opportunistic modes may share links and thus are statistically dependent, which complicates the order statistics that govern the performance of opportunistic systems.

In this work, several relaying geometries are studied and the corresponding DMTs are developed for a number of well-known relaying protocols, including the AF, DF, CF, NAF, and DDF. We studied the simple relay channel, one-source, one-destination and one-relay, in the opportunistic mode. We analyze the network in the opportunistic mode where the relay is allowed to be active or to stay inactive according to the channel conditions. Using opportunistic channel access scheme, we study the interference relay channel, the shared relay channel, the multiple access relay channel, the broadcast relay channel, the X-relay channel, the gateway channel, the parallel relay channel, the heterogeneous relay channel and the bi-directional multi-relay channel. In several instances, selection schemes based on the direct source-destination links are shown to achieve optimal performance, for example the CF multiple access channel. In some network geometries, opportunistic selection using 1-bit feedback is shown to achieve the optimal DMT performance. It is hoped that the approaches developed in this work may be applied towards the analysis of opportunistic communication in a wider class of network geometries.

APPENDIX A

OPPORTUNISTIC DF ORTHOGONAL RELAYING OVER A SIMPLE RELAY CHANNEL

The DMT of the opportunistic orthogonal relaying is given by

$$d(r) = d_1(r) + d_2(r), \tag{A.1}$$

where

$$d_1(r) = \lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_1)}{\log \rho}, \tag{A.2}$$

$$d_2(r) = \lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_2|e_1)}{\log \rho}. \tag{A.3}$$

The events e_1 and e_2 represent the error in the non-relay and the relay-assisted modes, respectively. The non-relay access mode is a simple direct link, whose DMT is $d_1(r) = (1-r)^+$. The DMT of the relay-assisted access mode is known, however, the DMT of the relay channel *conditioned on* the outage event of the direct link requires new calculations.

Recall that the orthogonal DF relaying works as follows: The transmission interval is divided into two halves. In the first half, the source transmits. If the relay cannot decode the source message, it will remain silent and the source will continue to transmit into the second half-interval. If the relay decodes the source message, the relay forwards the decoded message to the destination in the second half of the transmission interval and the source remains silent.

Because of orthogonality and with the use of long codewords, it is trivial to see that error is dominated by outage. The conditional outage probability of the relay-assisted mode is given by

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1) = \mathbb{P}\left(\left\{\frac{1}{2}\log(1+U\rho) < r\log\rho\right\}\middle|\left\{\log(1+|h_{sd}|^2\rho) < r\log\rho\right\}\right) \tag{A.4}$$

$$= \mathbb{P}\left(\left\{U < \frac{\rho^{2r}-1}{\rho}\right\}\middle|\left\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\right\}\right), \tag{A.5}$$

where the random variable U is given by

$$U = \begin{cases} 2|h_{sd}|^2 & |h_{sr}|^2 < \frac{\rho^{2r}-1}{\rho} \\ |h_{sd}|^2 + |h_{rd}|^2 & |h_{sr}|^2 \geq \frac{\rho^{2r}-1}{\rho}. \end{cases} \tag{A.6}$$

The cdf of U is given by

$$F_U(u) = \mathbb{P}\left(|h_{sd}|^2 < \frac{u}{2}\right)\mathbb{P}\left(|h_{sr}|^2 < \frac{\rho^{2r}-1}{\rho}\right) + \mathbb{P}\left(|h_{sd}|^2 + |h_{rd}|^2 < u\right)\mathbb{P}\left(|h_{sr}|^2 \geq \frac{\rho^{2r}-1}{\rho}\right).$$

Hence,

$$\begin{aligned}\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1) =& \mathbb{P}\left(\left\{|h_{sd}|^2 < \frac{1}{2}\frac{\rho^{2r}-1}{\rho}\right\}\middle|\left\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\right\}\right)\mathbb{P}\left(|h_{sr}|^2 < \frac{\rho^{2r}-1}{\rho}\right) \\ &+ \mathbb{P}\left(\left\{|h_{sd}|^2 + |h_{rd}|^2 < \frac{\rho^{2r}-1}{\rho}\right\}\middle|\left\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\right\}\right)\mathbb{P}\left(|h_{sr}|^2 \geq \frac{\rho^{2r}-1}{\rho}\right)\end{aligned} \tag{A.7}$$

One can show that $\frac{1}{2}\frac{\rho^{2r}-1}{\rho} \dot{>} \frac{\rho^r-1}{\rho}$, therefore

$$\mathbb{P}\left(\left\{|h_{sd}|^2 < \frac{1}{2}\frac{\rho^{2r}-1}{\rho}\right\}\middle|\left\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\right\}\right) \doteq 1. \tag{A.8}$$

To analyze the second conditional term in Equation (A.7), we begin with the pdf of $Z = |h_{sd}|^2 + |h_{rd}|^2$ conditioned on the event $B = \left\{|h_{sd}|^2 < \frac{\rho^r-1}{\rho}\right\}$. The channel gain $\gamma \triangleq |h_{sd}|^2$ has the following conditional distribution

$$f_{\gamma|B}(x) = \begin{cases} \frac{e^{-x}}{1-e^{-\frac{\rho^r-1}{\rho}}} & x \leq \frac{\rho^r-1}{\rho}, \\ 0 & x > \frac{\rho^r-1}{\rho}. \end{cases} \tag{A.9}$$

Defining $g_1(r,\rho) \triangleq \frac{\rho^r-1}{\rho}$ and $g_2(r,\rho) \triangleq \frac{\rho^{2r}-1}{\rho}$, the conditional pdf of $Z = |h_{sd}|^2 + |h_{rd}|^2$ is calculated as follows, for $z \leq g_1(r,\rho)$

$$\begin{aligned} f_{Z|B}(z) &= \int_0^z e^{-(z-x)}\frac{e^{-x}}{1-e^{-g_1(r,\rho)}}dx \\ &= \frac{ze^{-z}}{1-e^{-g_1(r,\rho)}}. \end{aligned} \tag{A.10}$$

For $z > g_1(r,\rho)$, the conditional pdf of $Z = |h_{sd}|^2 + |h_{rd}|^2$ is given by

$$\begin{aligned} f_{Z|B}(z) &= \int_0^{g_1(r,\rho)} e^{-(z-x)} \frac{e^{-x}}{1-e^{-g_1(r,\rho)}} dx \\ &= \frac{g_1(r,\rho)e^{-z}}{1-e^{-g_1(r,\rho)}}. \end{aligned} \tag{A.11}$$

The conditional probability of outage is calculated as follows

$$\begin{aligned} &\mathbb{P}\Big(\Big\{|h_{sd}|^2 + |h_{rd}|^2 < g_2(r,\rho)\Big\}\Big|\Big\{|h_{sd}|^2 < g_1(r,\rho)\Big\}\Big) \\ &\qquad = \int_0^{g_1(r,\rho)} \frac{ze^{-z}}{1-e^{-g_1(r,\rho)}} dz + \int_{g_1(r,\rho)}^{g_2(r,\rho)} \frac{g_1(r,\rho)e^{-z}}{1-e^{-g_1(r,\rho)}} dz \\ &\qquad = \frac{1-e^{-g_1(r,\rho)} - g_1(r,\rho)e^{-g_2(r,\rho)}}{1-e^{-g_1(r,\rho)}} \\ &\qquad \doteq 1 - \frac{\rho^{r-1}e^{-\rho^{2r-1}}}{1-e^{-\rho^{r-1}}} \doteq \rho^{2r-1}. \end{aligned} \tag{A.12}$$

Substituting (A.8) and (A.12) into (A.5), the conditional probability of outage is given by

$$\begin{aligned} P(\mathcal{O}_2|\mathcal{O}_1) &\doteq \rho^{(2r-1)} + \rho^{(2r-1)}(1-\rho^{(2r-1)}) \\ &\doteq \rho^{(2r-1)}. \end{aligned} \tag{A.13}$$

Using Equations (A.1), (A.2), (A.3) and (A.13), the DMT of the orthogonal opportunistic DF relaying is given by

$$d(r) = (1-r)^+ + (1-2r)^+. \tag{A.14}$$

APPENDIX B

OPPORTUNISTIC AF ORTHOGONAL RELAYING OVER A SIMPLE RELAY CHANNEL

The outage probability of the relay-assisted mode, given that the non-relay mode is in outage is given by

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1) = \mathbb{P}\Bigg(\Big\{\frac{1}{2}\log\Big(1+|h_{sd}|^2\rho + f\big(|h_{sr}|^2\rho, |h_{rd}|^2\rho\big)\Big) < r\log\rho\Big\}\Big|\Big\{\log(1+|h_{sd}|^2\rho) < r\log\rho\Big\}\Bigg) \tag{B.1}$$

$$= \mathbb{P}\Bigg(\Big\{|h_{sd}|^2 + \frac{1}{\rho} f\big(|h_{sr}|^2\rho, |h_{rd}|^2\rho\big) < \frac{\rho^{2r}-1}{\rho}\Big\}\Big|\Big\{|h_{sd}|^2 < \frac{\rho^r - 1}{\rho}\Big\}\Bigg), \tag{B.2}$$

At high SNR, Equation (B.2) can be approximated by

$$\mathbb{P}(\mathcal{O}_2|\mathcal{O}_1) = \mathbb{P}\Bigg(\Big\{|h_{sd}|^2 + \frac{|h_{sr}|^2|h_{rd}|^2}{|h_{sr}|^2+|h_{rd}|^2} < \frac{\rho^{2r}-1}{\rho}\Big\}\Big|\Big\{|h_{sd}|^2 < \frac{\rho^r - 1}{\rho}\Big\}\Bigg) \tag{B.3}$$

where $\frac{|h_{sr}|^2|h_{rd}|^2}{|h_{sr}|^2+|h_{rs}|^2}$ represents the harmonic mean of two independent exponential random variables. Using the result of [54], the harmonic mean of two exponential random variables with exponential parameters λ can be approximated by an exponential random variable with exponential parameter 2λ.

In order to calculate the conditional outage probability distribution, we first calculate the conditional density function of $Z = |h_{sd}|^2 + V$ where $V = \frac{|h_{sr}|^2|h_{rd}|^2}{|h_{sr}|^2+|h_{rs}|^2}$. Again, we are assuming $g_1(r,\rho) = \frac{\rho^r-1}{\rho}$, $g_2(r,\rho) = \frac{\rho^{2r}-1}{\rho}$, and conditioning is over the event $B = \Big\{h_{sd}| < \frac{\rho^r-1}{\rho}\Big\}$. The conditional probability density function of $Z = |h_{sd}|^2 + V$ is given by

$$f_{Z|B}(z) = \begin{cases} \frac{2e^{-2z}(e^z-1)}{1-e^{-g_1(r,\rho)}} & z \le g_1(r,\rho) \\ \frac{2e^{-2z}\left(e^{g_1(r,\rho)}-1\right)}{1-e^{-g_1(r,\rho)}} & z > g_1(r,\rho). \end{cases} \tag{B.4}$$

The conditional probability of outage is calculated as follows

$$
\begin{aligned}
&\mathbb{P}\Big(|h_{sd}|^2+|h_{rd}|^2<g_2(r,\rho)\Big||h_{sd}|^2<g_1(r,\rho)\Big)\\
&\qquad = 2\int_0^{g_1(r,\rho)}\frac{e^{-2z}(e^z-1)}{1-e^{-g_1(r,\rho)}}dz+2\int_{g_1(r,\rho)}^{g_2(r,\rho)}\frac{e^{-2z}\big(e^{g_1(r,\rho)}-1\big)}{1-e^{-g_1(r,\rho)}}dz\\
&\qquad = \frac{e^{-2g_2(r,\rho)}-e^{-g_1(r,\rho)}-e^{-2g_2(r,\rho)+g_1(r,\rho)}+1}{1-e^{-g_1(r,\rho)}}\\
&\qquad \doteq 1+e^{-2\rho^{2r-1}}\frac{1-e^{\rho^{r-1}}}{1-e^{-\rho^{r-1}}}\doteq\rho^{2r-1}.
\end{aligned}
\tag{B.5}
$$

Using Equations (A.1), (A.2), (A.3), and (B.5), the DMT of the orthogonal opportunistic AF relaying is given by

$$
d(r)=(1-r)^{+}+(1-2r)^{+}. \tag{B.6}
$$

APPENDIX C

DMT UPPER BOUND FOR THE SHARED RELAY CHANNEL

In this appendix, we prove Theorem 2 for $n = 2$; the generalization for $n > 2$ follows along the same lines. Using Lemma 1, we can upper bound the DMT of the opportunistic shared relay network as follows

$$d(r) \leq d_1(r) + d_2(r) + d_3(r), \tag{C.1}$$

where

$$d_1(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_1)}{\log \rho}, \tag{C.2}$$

$$d_2(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_2|e_1)}{\log \rho}, \tag{C.3}$$

$$d_3(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_3|e_1, e_2)}{\log \rho}. \tag{C.4}$$

The events e_1, e_2 and e_3 are the error events in the three access modes (a), (b) and (c) shown in Figure 3.5. The error events e_2 and e_1 are independent, and the error events in the network is dominated by the outage events, therefore we can calculate $d_1(r), d_2(r)$ and $d_3(r)$ as follows

$$d_1(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(\mathcal{O}_1)}{\log \rho}, \tag{C.5}$$

$$d_2(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(\mathcal{O}_2)}{\log \rho}, \tag{C.6}$$

$$d_3(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(\mathcal{O}_3|\mathcal{O}_1, \mathcal{O}_2)}{\log \rho}, \tag{C.7}$$

where $\mathcal{O}_1, \mathcal{O}_2$ and $\mathcal{O}_3$ are the outage events for the three access modes (a), (b) and (c). Access mode (c) is a parallel Rayleigh channel. The outage of a parallel Rayleigh channel,

$\mathbb{P}(\mathcal{O}_3)$, is given by

$$\mathbb{P}(\mathcal{O}_3) = \mathbb{P}(\mathcal{O}_{31}) + \mathbb{P}(\mathcal{O}_{32}) + \mathbb{P}(\mathcal{O}_{33}), \tag{C.8}$$

where $\mathcal{O}_{31}, \mathcal{O}_{32}$ and $\mathcal{O}_{33}$ partition the outage event $\mathcal{O}_3$ according to whether the first, the second, or both direct links are in outage.

$$\mathbb{P}(\mathcal{O}_{31}) = \mathbb{P}\Big(\log(1+|h_{11}|^2\rho) < \frac{r}{2}\log\rho\Big)\mathbb{P}\Big(\log(1+|h_{22}|^2)\rho \geq \frac{r}{2}\log\rho\Big) \tag{C.9}$$

$$\mathbb{P}(\mathcal{O}_{32}) = \mathbb{P}\Big(\log(1+|h_{11}|^2\rho) \geq \frac{r}{2}\log\rho\Big)\mathbb{P}\Big(\log(1+|h_{22}|^2)\rho < \frac{r}{2}\log\rho\Big) \tag{C.10}$$

$$\mathbb{P}(\mathcal{O}_{33}) = \mathbb{P}\Big(\log(1+|h_{11}|^2\rho) < \frac{r}{2}\log\rho\Big)\mathbb{P}\Big(\log(1+|h_{22}|^2)\rho < \frac{r}{2}\log\rho\Big). \tag{C.11}$$

The outage event $\mathcal{O}_3$ is independent of the channel gains $|h_{ir}|^2$ and $|h_{ri}|^2$ where $i \in \{1, 2\}$, hence

$$\mathbb{P}(\mathcal{O}_3 \mid \mathcal{O}_1, \mathcal{O}_2) \geq \mathbb{P}(\mathcal{O}_{31} \mid \mathcal{O}_1, \mathcal{O}_2) = \mathbb{P}(\mathcal{O}_{31} \mid \mathcal{O}_1) \geq \mathbb{P}(\mathcal{O}_{31}) \doteq \rho^{r/2-1}, \tag{C.12}$$

We are now ready to assemble the components of inequality (C.1) to get the final result. The inequality (C.12) shows that the conditional outage of mode (c) is dominated by its unconditional outage, therefore $d_3(r) \leq (1-\frac{r}{2})^+$. The remaining two modes are independent of each other and each can be upper bounded using the MISO upper bound of $2(1-r)^+$. Together, we have

$$d(r) \leq 2(1-r)^+ + 2(1-r)^+ + (1-r/2)^+, \tag{C.13}$$

Since the total number of paths between the inputs and the outputs are 4, then the diversity order cannot be greater than 4. This constraint together with the above inequality completes the proof.

APPENDIX D

GENIE-AIDED DMT UPPER BOUND FOR THE SHARED RELAY CHANNEL

The indexing of the access modes does not affect the problem, therefore we can order the conditional events in Lemma 1, 2 arbitrarily. In the following, we index the outage events according to the order of selection that is described below, which is designed to sort out the dependencies in a way to make computations tractable.

The selection algorithm is as follows: If the non-relayed configuration (shown in Figure 3.5 part (c)) can support the required rate $R = r \log \rho$, it is selected. We shall call this *Mode 1* in the remainder of appendices. If *Mode 1* is in outage (an event denoted by $\mathcal{U}_1$) we will check to see if either of the two direct links can individually support half the rate, i.e., $R = \frac{r}{2} \log \rho$. If one of the direct links can support this reduced rate, we consider the relayed mode sharing that direct link. (If none of the direct links can even support half the rate, we can consider either one at random.) This relayed mode shall be called *Mode 2*. If *Mode 2* can support the full required rate, it is selected. The outage of *Mode 2* is denoted $\mathcal{U}_2$. If both *Modes 1, 2* are in outage, the remaining relayed mode, which will be denoted *Mode 3*, is selected. The outage of *Mode 3* is denoted $\mathcal{U}_3$ in this and the following appendices. The error events corresponding to the three modes are denoted e'_1, e'_2, e'_3 in this and subsequent appendices.

The total DMT of the genie-aided system is

$$d(r) = d'_1(r) + d'_2(r) + d'_3(r), \tag{D.1}$$

where

$$d_1'(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_1')}{\log \rho}, \tag{D.2}$$

$$d_2'(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_2'|e_1')}{\log \rho}, \tag{D.3}$$

$$d_3'(r) = -\lim_{\rho\to\infty} \frac{\log \mathbb{P}(e_3'|e_2', e_1')}{\log \rho}. \tag{D.4}$$

Although the expressions above are in terms of error events, in the remainder of this appendix the diversities are expressed in terms of outage events instead of error events due to the fact that the genie-aided modes are equivalent to MISO channels and the codewords are assumed to be long enough.

Mode 1, access mode (c), represents a parallel Rayleigh channel. The outage of a parallel Rayleigh channel, $\mathbb{P}(\mathcal{O}_3)$, is given by Equation (C.8). Therefore, in the asymptote of high SNR:

$$P(\mathcal{O}_3) \doteq \rho^{r/2-1} + \rho^{r/2-1} + \rho^{r-2} \doteq \rho^{r/2-1} \tag{D.5}$$

The unconditional DMT of the non-relayed mode

$$d_1'(r) = (1 - \frac{r}{2})^+. \tag{D.6}$$

To calculate $d_2'(r)$ and $d_3'(r)$, we study the outage of the respective access modes. We start by calculating the conditional outage of Mode 3 and use the result to calculate the conditional outage for Mode 2.

$$\begin{aligned}
\mathbb{P}(\mathcal{U}_3|\mathcal{U}_2, \mathcal{U}_1) &= \mathbb{P}\Bigg(\Big\{ \log\big(1 + (|h_{ii}|^2 + |h_{ri}|^2)\rho\big) < r\log\rho \Big\} \;\Big|\; \\
&\qquad \Big\{|h_{jj}|^2 < f_2^{-1}(R, |h_{r,d_j}|^2)\Big\}, \; \Big\{|h_{11}|^2 < \frac{\rho^{r/2}-1}{\rho}\Big\}, \Big\{|h_{22}|^2 < \frac{\rho^{r/2}-1}{\rho}\Big\}\Bigg) \\
&\doteq \mathbb{P}\Bigg(\Big\{|h_{ii}|^2 + |h_{ri}|^2 < \frac{\rho^r - 1}{\rho}\Big\} \;\Big|\; \Big\{|h_{ii}|^2 < \frac{\rho^{r/2}-1}{\rho}\Big\}\Bigg),
\end{aligned} \tag{D.7}$$

where i is the index of the source selected in Mode 3 and j is the index of the source selected in Mode 2. The channel gain $\gamma_{ii} \triangleq |h_{ii}|^2$, conditioned on the event $B = \left\{|h_{ii}| < \frac{\rho^{r/2}-1}{\rho}\right\}$ has the following conditional distribution

$$f_{\gamma_{ii}|B}(x) = \begin{cases} \frac{e^{-x}}{1-e^{-\frac{\rho^{r/2}-1}{\rho}}} & x \leq \frac{\rho^{r/2}-1}{\rho} \\ 0 & x > \frac{\rho^{r/2}-1}{\rho} \end{cases} \tag{D.8}$$

Defining $g_1(r,\rho) \triangleq \frac{\rho^{r/2}-1}{\rho}$ and $g_2(r,\rho) \triangleq \frac{\rho^{r}-1}{\rho}$, the conditional probability density function of $Z = |h_{ii}|^2 + |h_{ri}|^2$ is

$$f_{Z|B}(z) = \begin{cases} \frac{ze^{-\lambda z}}{1-e^{-g_1(r,\rho)}} & z \leq g_1(r,\rho) \\ \frac{g_1(r,\rho)e^{-z}}{1-e^{-g_1(r,\rho)}} & z > g_1(r,\rho). \end{cases} \tag{D.9}$$

The probability of outage $\mathbb{P}(\mathcal{U}_3|\mathcal{U}_2,\mathcal{U}_1)$ can be calculated as follows

$$\begin{aligned} \mathbb{P}(\mathcal{U}_3|\mathcal{U}_2,\mathcal{U}_1) &= \int_0^{g_1(r,\rho)} \frac{ze^{-z}}{1-e^{-g_1(r,\rho)}}dz + \int_{g_1(r,\rho)}^{g_2(r,\rho)} \frac{g_1(r,\rho)e^{-z}}{1-e^{-g_1(r,\rho)}}dz \\ &= \frac{1-e^{-g_1(r,\rho)} - g_1(r,\rho)e^{-g_2(r,\rho)}}{1-e^{-g_1(r,\rho)}} \\ &\doteq 1 - \frac{\rho^{r/2-1}e^{-\rho^{r-1}}}{1-e^{-\rho^{r/2-1}}} \doteq \rho^{r-1}. \end{aligned} \tag{D.10}$$

To facilitate the analysis of the conditional outage of Mode 2, we introduce a partition of $\mathcal{U}_1$. Define $\mathcal{V}$ as the event that *at least* one of the direct links can support half the desired rate, i.e. $\frac{r}{2}\log\rho$, and introduce:

$$\mathcal{V}_1 = \mathcal{V} \cap \mathcal{U}_1 \qquad\qquad \mathcal{V}_2 = \bar{\mathcal{V}} \cap \mathcal{U}_1 \tag{D.11}$$

Thus, $\mathcal{V}_1$ is the event that the non-relayed *Mode 1* is in outage, and yet at least one of the two direct links can support at least half the desired rate, i.e., $\frac{r}{2}\log\rho$.

$$\begin{aligned} \mathbb{P}(\mathcal{U}_2|\mathcal{U}_1) &= \frac{\mathbb{P}(\mathcal{U}_2,\mathcal{U}_1)}{\mathbb{P}(\mathcal{U}_1)} \\ &= \frac{\mathbb{P}(\mathcal{U}_2|\mathcal{V}_1)\mathbb{P}(\mathcal{V}_1) + \mathbb{P}(\mathcal{U}_2|\mathcal{V}_2)\mathbb{P}(\mathcal{V}_2)}{\mathbb{P}(\mathcal{V}_1)+\mathbb{P}(\mathcal{V}_2)} \\ &\doteq \frac{\rho^{2(r-1)}2\rho^{(\frac{r}{2}-1)} + \rho^{(r-1)}\rho^{2(\frac{r}{2}-1)}}{2\rho^{(\frac{r}{2}-1)} + \rho^{2(\frac{r}{2}-1)}} \\ &\doteq \rho^{2(r-1)}, \end{aligned} \tag{D.12}$$

where $\mathbb{P}(\mathcal{V}_1) = \mathbb{P}(\mathcal{O}_{31}) + \mathbb{P}(\mathcal{O}_{32}) \doteq 2\rho^{(r/2-1)}$ from Equation (C.9) and (C.10), $\mathbb{P}(\mathcal{V}_2) = \mathbb{P}(\mathcal{O}_{33}) \doteq \rho^{2(r/2-1)}$ from Equation (C.11). The probability of U_2 conditioned on $\mathcal{V}_2$ is equivalent to Equation (D.7) and hence $\mathbb{P}(\mathcal{U}_2|\mathcal{V}_2) \doteq \rho^{(r-1)}$. The conditional probability $\mathbb{P}(\mathcal{U}_2|\mathcal{V}_1)$ is given by

$$\mathbb{P}(\mathcal{U}_2|\mathcal{V}_1) = \mathbb{P}\Bigg(\Big\{|h_{ii}|^2 + |h_{ri}|^2 < \frac{\rho^r - 1}{\rho}\Big\} \Bigm| \Big\{|h_{ii}|^2 > \frac{\rho^{r/2} - 1}{\rho}\Big\}\Bigg). \tag{D.13}$$

We notice that

$$\begin{aligned}\mathbb{P}\Bigg(\Big\{|h_{ii}|^2+|h_{ri}|^2 < \frac{\rho^r - 1}{\rho}\Big\}\Bigg) &\\ = \mathbb{P}\Bigg(\Big\{|h_{ii}|^2 + |h_{ri}|^2 < \frac{\rho^r - 1}{\rho}\Big\} \Bigm| \Big\{|h_{ii}|^2 < \frac{\rho^{r/2} - 1}{\rho}\Big\}\Bigg)&\mathbb{P}\Bigg(\Big\{|h_{ii}|^2 < \frac{\rho^{r/2} - 1}{\rho}\Big\}\Bigg)\\ + \mathbb{P}\Bigg(\Big\{|h_{ii}|^2 + |h_{ri}|^2 < \frac{\rho^r - 1}{\rho}\Big\} \Bigm| \Big\{|h_{ii}|^2 > \frac{\rho^{r/2} - 1}{\rho}\Big\}\Bigg)&\mathbb{P}\Bigg(\Big\{|h_{ii}|^2 > \frac{\rho^{r/2} - 1}{\rho}\Big\}\Bigg).\end{aligned} \tag{D.14}$$

At high SNR, using result from Equation (D.10), Equation (D.14) leads to

$$\rho^{2(r-1)} \doteq \rho^{(r-1)}\rho^{(r/2-1)} + \mathbb{P}\Bigg(\Big\{|h_{ii}|^2 + |h_{ri}|^2 < \frac{\rho^r - 1}{\rho}\Big\} \Bigm| \Big\{|h_{ii}|^2 > \frac{\rho^{r/2} - 1}{\rho}\Big\}\Bigg), \tag{D.15}$$

where the random variable $|h_{ii}|^2 + |h_{ri}|^2$ has Gamma distribution. Using Equation (D.13) and (D.15), one can see that $\mathbb{P}(\mathcal{U}_2|\mathcal{V}_1) = \rho^{2(r-1)}$.

Equations (D.12) and (D.10) indicate that

$$d_2'(r) = 2(1 - r)^+, \tag{D.16}$$

$$d_3'(r) = (1 - r)^+. \tag{D.17}$$

The DMT of the genie aided system is given by

$$d(r) = (1 - \frac{r}{2})^+ + 2(1 - r)^+ + (1 - r)^+ \tag{D.18}$$

$$= \begin{cases} 4 - \frac{7}{2}r, & 0 \le r \le 1 \\ (1 - \frac{r}{2}), & 1 < r \le 2. \end{cases} \tag{D.19}$$

APPENDIX E

NAF ACHIEVABLE DMT FOR THE SHARED RELAY CHANNEL

The DMT of the NAF protocol for the shared relay channel will be calculated according to the selection algorithm developed in Appendix D, which we invite the reader to review before continuing with the present appendix.

The overall diversity is governed by Equation (D.1), and we need to calculate $d'_1(r), d'_2(r), d'_3(r)$.

To begin with, the DMT of the non-relayed mode does not depend on the relaying protocol, so there is no need to calculate it again: it is $d'_1(r) = (1 - \frac{r}{2})^+$ as calculated in expression (D.6).

For calculating $d'_3(r)$, the equivalence of error and outage analysis is nontrivial and will be relegated to Appendix F. In this appendix we analyze the conditional outage of *Mode 3*:

$$\begin{aligned}\mathbb{P}(\mathcal{U}_3|\mathcal{U}_2,\mathcal{U}_1) &\doteq \frac{1}{2}\mathbb{P}\Big(I_1 < R \mid \bar{\mathcal{V}}, I_2 < R\Big) + \frac{1}{2}\mathbb{P}\Big(I_2 < R \mid \bar{\mathcal{V}}, I_1 < R\Big) \\ &\doteq \mathbb{P}\Big(I_1 < R \Big| \Big\{|h_{11}|^2 < \frac{\rho^{r/2} - 1}{\rho}\Big\}\Big),\end{aligned} \tag{E.1}$$

where I_1 and I_2 are the instantaneous mutual information of the simple relay channel for User 1 and User 2, respectively. The symmetry arguments has been used to simplify the expression. We will use the exponential order of channel gains, defined thus

$$v = -\lim_{\rho\to\infty} \frac{\log |h|^2}{\log \rho}, \tag{E.2}$$

where v itself is a random variable. Recall that the conditional pdf of the source-destination channel gain $|h_{11}|^2$, subject to h_{11} not supporting rate $\frac{r}{2}\log\rho$, is given by Equation (D.8).

The exponential order of this *conditional* random variable is denoted v_1 whose pdf can be calculated as follows

$$f(v_1) = \begin{cases} \ln\rho\, \rho^{-v_1} \frac{e^{-\rho^{-v_1}}}{1-e^{-\frac{\rho^{r/2-1}}{\rho}}} & v_1 \geq 1-\frac{r}{2}, \\ 0 & v_1 < 1-\frac{r}{2}. \end{cases} \tag{E.3}$$

As $\rho \to \infty$ we can show that

$$f(v_1) \doteq \begin{cases} \rho^{-v_1-(r/2-1)} & v_1 \geq 1-\frac{r}{2}, \\ 0 & v_1 < 1-\frac{r}{2}. \end{cases} \tag{E.4}$$

Also, the channel gains $|h_{r1}|^2$ and $|h_{1r}|^2$ (exponentially distributed, unconditioned) have exponential orders that are denoted v_2 and v_3, respectively. Furthermore, the pdf of v_1, v_2, v_3 are in turn characterized by their asymptotic exponential orders $f(v_i) \doteq \rho^{-u_i}$, over their respective regions of support.

In a manner similar to [28], the outage region is more conveniently addressed in the space of the exponential orders, i.e.

$$O = \{(v_1, v_2, v_3)\ :\ I < r\log\rho\}, \tag{E.5}$$

We can now calculate:

$$\begin{aligned} \mathbb{P}(I < r\log\rho) &= \iiint_O f(v_1, v_2, v_3)\, dv_1\, dv_2\, dv_3 \\ &= \iiint_{O'} \log\rho\, \rho^{-v_1} \frac{e^{-\rho^{-v_1}}}{1-e^{-\frac{\rho^{r/2-1}}{\rho}}} \log\rho\, \rho^{-v_2} e^{-\rho^{-v_2}} \\ &\quad \times \log\rho\, \rho^{-v_3} e^{-\rho^{-v_3}} dv_1\, dv_2\, dv_3 \\ &\doteq \iiint_{O'} \rho^{-\sum u_i} dv_1\, dv_2\, dv_3 \\ &\doteq \rho^{-d_o}, \end{aligned} \tag{E.6}$$

where O' is the intersection of O and the support of $f(v_1, v_2, v_3)$, and

$$\begin{aligned} d_o &= \inf_{(v_1,v_2,v_3)\in O'} \quad \sum_{j=1}^{n} u_i, \\ &= \inf_{(v_1,v_2,v_3)\in O'} v_1 + (r/2-1) + v_2 + v_3 \end{aligned} \tag{E.7}$$

Following the same steps as those used in the proof of [28, Theorem 2],

$$O' = \left\{(v_1, v_2, v_3) \in R^{3+}, v_1 \geq \left(1 - \frac{r}{2}\right) \ , \ \left[\max\left((1 - v_1), \frac{1}{2}(1 - (v_2 + v_3)\right)\right]^+ < r\right\} \quad \text{(E.8)}$$

Solving (E.7), we can show that

$$d_0 = (1 - 2r)^+ \quad \text{(E.9)}$$

It remains to show that $d_3'(r) = d_0$, which will be done in Appendix F.

For calculating $d_2'(r)$, we follow steps essentially similar to those leading to Equation (D.12), except this time we need to make explicit the relationship between outage and error events.

$$\begin{aligned}
\mathbb{P}(e_2'|e_1') &\doteq \mathbb{P}(e_2'|\mathcal{U}_1) && \text{(E.10)}\\
&= \frac{\mathbb{P}(e_2', \mathcal{U}_1)}{\mathbb{P}(\mathcal{U}_1)}\\
&= \frac{\mathbb{P}(e_2'|\mathcal{V}_1)\mathbb{P}(\mathcal{V}_1) + \mathbb{P}(e_2'|\mathcal{V}_2)\mathbb{P}(\mathcal{V}_2)}{\mathbb{P}(\mathcal{V}_1) + \mathbb{P}(\mathcal{V}_2)}\\
&\doteq \frac{\rho^{(r-1)}\rho^{(2r-1)}2\rho^{(\frac{r}{2}-1)} + \rho^{(2r-1)}\rho^{2(\frac{r}{2}-1)}}{2\rho^{(\frac{r}{2}-1)} + \rho^{2(\frac{r}{2}-1)}} && \text{(E.11)}\\
&\doteq \rho^{-(1-r)^+ - (1-2r)^+}. && \text{(E.12)}
\end{aligned}$$

where (E.10) is true because e_1' is the error of a non-relayed link therefore, with long codewords, it is exponentially equivalent to the outage event $\mathcal{U}_1$. Equation (E.11) is derived by substituting the known error exponents and noting that the third term is dominated by the first two in both the numerator and denominator. Overall, $d_2'(r) = (1 - r)^+ + (1 - 2r)^+$ can be obtained.

To summarize, we have calculated $d_1'(r), d_2'(r)$ and $d_3(r)$.

APPENDIX F

RELATION OF OUTAGE AND ERROR EVENTS FOR THE SHARED RELAY CHANNEL

In this appendix, we show that the outage and error events have the same exponential order. The approach follows [28, Theorem 3] and is adapted to the specific case at hand. We need to show $\mathbb{P}(e)\dot{\leq}\mathbb{P}(\mathcal{O})$ and $\mathbb{P}(e)\dot{\geq}\mathbb{P}(\mathcal{O})$. The former is a straightforward application of [18, Lemma 5]. For showing the latter inequality, note that

$$\begin{aligned}\mathbb{P}(e) &= \mathbb{P}(\mathcal{O})\mathbb{P}(e|\mathcal{O}) + \mathbb{P}(e,\bar{\mathcal{O}}) \\ &\leq \mathbb{P}(\mathcal{O}) + \mathbb{P}(e,\bar{\mathcal{O}}) \\ &\doteq \mathbb{P}(\mathcal{O}) \end{aligned} \tag{F.1}$$

where the last equation is valid whenever $\mathbb{P}(e,\bar{\mathcal{O}})\dot{\leq}\mathbb{P}(\mathcal{O})$, whose verification is the subject of the remainder of this appendix. The pairwise error probability conditioned on the channel coefficients is given by

$$P_{\mathbf{c}\to\mathbf{e}|h_{sd},h_{sr},h_{rd}} \leq \det\left(\mathbf{I}_2 + \frac{1}{2}\mathbf{\Sigma}_s\mathbf{\Sigma}_n^{-1}\right)^{-\ell/2} \tag{F.2}$$

where ℓ is the codebook codeword length and $\mathbf{\Sigma}_s$ and $\mathbf{\Sigma}_n$ are the covariance matrices of the received signal and the noise, respectively. The pair wise error probability is given by

$$P_{\mathbf{c}\to\mathbf{e}|v_1,v_2,v_3} \dot{\leq} \rho^{-\frac{l}{2}\max(2(1-v_1),1-(v_2+v_3))^+}, \tag{F.3}$$

where

$$(v_1,v_2,v_3) \in \ R^{3+} \cap \left\{v_1 \geq \left(1-\frac{r}{2}\right)\right\}. \tag{F.4}$$

The total probability of error is

$$P_{e|v_1,v_2,v_3} \dot{\leq} \rho^{-\frac{l}{2}([\max(2(1-v_1),1-(v_2+v_3))]^+-2r)}, \tag{F.5}$$

The probability of error while no outage $\mathbb{P}(e,\bar{\mathcal{O}})$ satisfies

$$\begin{aligned}\mathbb{P}(e,\bar{\mathcal{O}}) &\dot{\leq} \iiint_{O''} P_{e|v_1,v_2,v_3}\mathbb{P}((v_1,v_2,v_3)\in\bar{\mathcal{O}})\, dv_1\, dv_2\, dv_3 \\ &= \iiint_{O''} \rho^{-\frac{l}{2}([\max(2(1-v_1),1-(v_2+v_3))]^+-2r)+v_1+(\frac{r}{2}-1)+v_2+v_3}\, dv_1\, dv_2\, dv_3.\end{aligned} \tag{F.6}$$

where $O'' = \{(v_1,v_2,v_3) \in R^+ : (v_1,v_2,v_3) \notin O'\}$, the area in the positive quadrant that is the complement of O'. Recall that O' is the outage region in the space of exponents, as defined in (E.8). The integral is dominated by the minimum value of the SNR exponent over $\bar{\mathcal{O}}$, i.e,

$$\mathbb{P}(e,\bar{\mathcal{O}}) \dot{\leq} \rho^{-d_1(r)}, \tag{F.7}$$

where

$$d_1(r) = \inf_{v_1,v_2,v_3\in O''} \frac{\ell}{2}\Big(\big[\max(2(1-v_1),1-(v_2+v_3))\big]^+ - 2r\Big) + v_1 + (r/2-1) + v_2 + v_3. \tag{F.8}$$

Note that the multiplier of ℓ is positive throughout the region O''. Now recall from the previous appendix that the outage probability is:

$$\mathbb{P}(\mathcal{O}) \doteq \rho^{-d_0(r)}, \tag{F.9}$$

where

$$d_0(r) = \inf_{(v_1,v_2,v_3)\in O'} v_1 + (r/2-1) + v_2 + v_3 \tag{F.10}$$

The expression for $d_1(r)$ has one extra term compared with $d_0(r)$ which, as mentioned above, is positive and can be made as large as desired by choosing ℓ to be large enough. Therefore the condition $\mathbb{P}(e,\bar{\mathcal{O}}) \dot{\leq} \mathbb{P}(\mathcal{O})$ is established, therefore we have $\mathbb{P}(e) \dot{\leq} \mathbb{P}(\mathcal{O})$, which completes the proof that the probability of error and outage events are exponentially equivalent.

APPENDIX G

DMT FOR DDF OPPORTUNISTIC SHARED RELAY CHANNEL

We derive an achievable DMT for the DDF opportunistic shared relay channel, employing the mode selection rule defined in Appendix D. The DMT is given by Equations (D.1), (D.2), (D.3) and (D.4). The reader is referred to Appendix D for the definition of the access modes as well as the selection rule.

The DMT for *Mode 1* is not affected by the relay and is given by $d_1'(r) = (1 - r/2)^+$, as seen in previous appendices. For *Mode 2* one can employ the techniques of Appendix D to show that outage is dominated by the event of one link being in outage, hence using results from [28], one can prove that

$$d_2'(r) = \begin{cases} 2(1-r) & 0 \leq r \leq \frac{1}{2} \\ \frac{1-r}{r} & \frac{1}{2} \leq 1 \end{cases}$$

To calculate $d_3'(r)$, we consider the conditional outage of *Mode 3*; the equivalence of error and outage analysis can be shown in a manner similar to Appendix F and [28]. In the following we directly derive diversity from the outage events. The conditional outage of *Mode 3* was calculated in Equation (E.1):

$$\mathbb{P}(\mathcal{U}_3|\mathcal{U}_2,\mathcal{U}_1) \doteq \mathbb{P}\Big(I_1 < R \;\Big|\; \Big\{|h_{11}|^2 < \frac{\rho^{r/2}-1}{\rho}\Big\}\Big)$$

Given that $|h_{11}|^2 < \frac{\rho^{r/2-1}}{\rho}$, the exponential order of $|h_{11}|^2$ is proved in (E.4) to have the following distribution at high SNR

$$f(v_i) \doteq \begin{cases} \rho^{-v_i-(r/2-1)} & v_i \geq 1 - \frac{r}{2}, \\ 0 & v_i < 1 - \frac{r}{2}. \end{cases} \tag{G.1}$$

The outage as shown in Equation (E.6) is given by

$$\mathbb{P}(\mathcal{U}_3|\mathcal{U}_2,\mathcal{U}_1) \doteq \rho^{-d_3'(r)}, \tag{G.2}$$

where

$$d_3'(r) = \inf_{(v_1,v_2,v_3)\in O'} v_1 + (r/2-1) + v_2 + v_3. \tag{G.3}$$

Following the same steps of the proof of [28, Theorem 5], the outage event O' is defined as

$$O' = \Big\{(v_1,v_2,v_3) \in R^{3+}\ ,\ v_1 \geq (1-r/2)\ ,\ t(1-v_1)^+ + (1-t)\big(1-\min(v_1,v_2)\big)^+ \leq r\Big\}, \tag{G.4}$$

where t is the listening-time ratio of the half-duplex relay, with $r \leq t \leq 1$.

To get the DMT, we need to solve the optimization problem of (G.2), (G.4). Solving the above optimizations and combining the results, the DMT is given by

$$d_3'(r) = \begin{cases} 1 - \frac{r}{1-r}\big(1-\frac{r}{2}\big), & 0 \leq r \leq 0.5 \\ \frac{(1-r)}{r} - \big(1-\frac{r}{2}\big), & 0.5 < r \leq 2-\sqrt{2} \\ 0, & 2-\sqrt{2} < r \leq 1. \end{cases} \tag{G.5}$$

Adding $d_1'(r)$, $d_2'(r)$ and $d_3'(r)$ completes the proof.

APPENDIX H

DMT FOR CF OPPORTUNISTIC SHARED RELAY CHANNEL

The methods of this appendix closely follow [30], with the notable exception of implementing the effects of our selection algorithm and the dependence between the nodes.

We use the selection criterion defined in Appendix D, and the DMT is given by Equations (D.1), (D.2), (D.3) and (D.4). The DMT of non-relayed *Mode 3* is given by $d'_1(r) = (1 - r/2)^+$, as seen several times already, since it is not contingent on the relay protocol.

To calculate $d'_2(r)$ and $d'_3(r)$, we borrow the following result from [30]. For the random half-duplex single-antenna relay channel, the dynamic-state CF protocol is DMT optimal and by random here we mean that the random binary state of the relay (listen/transmit) is used as a channel input and used in designing codebooks to convey information through the state of the relay.

For *Mode 2*, one can employ the techniques of Appendix D to show that outage is dominated by the event of one link being in outage, hence using results from [30], one can prove that

$$d'_2(r) = 2(1 - r)^+$$

For *Mode 3*, the DMT is given by

$$d'_3(r) = \max_t \min(d_{MAC}(r, t), d_{BC}(r, t)), \tag{H.1}$$

where

$$d_{BC} = -\lim_{\rho\to\infty} \frac{\log \min_{p(x_s,x_r|q)} \mathbb{P}\big(I_{BC} < r\log\rho|\mathcal{U}_2,\mathcal{U}_1\big)}{\log\rho},$$
$$d_{MAC} = -\lim_{\rho\to\infty} \frac{\log \min_{p(x_s,x_r|q)} \mathbb{P}\big(I_{MAC} < r\log\rho|\mathcal{U}_2,\mathcal{U}_1\big)}{\log\rho},$$

where q represents the state of the relay (listening vs. transmitting), $p(x_s, x_r|q)$ is the probability density of the codebooks generated for the source and the relay, and I_{BC} and I_{MAC} represent the total mutual information across the source cutset and the destination cutset, respectively. It can be shown [30] that

$$I_{BC} \leq (1-t)\log(1 + (|h_{s^*d^*}|^2 + |h_{s^*r}|^2)\rho) + t\log(1 + |h_{s^*d^*}|^2\rho)$$
$$I_{MAC} \leq (1-t)\log(1 + |h_{s^*d^*}|^2\rho) + t\log(1 + (|h_{s^*d^*}|^2 + |h_{rd^*}|^2)\rho)$$

where s^* and d^* are the selected source and destination for Mode 3. Using the same technique as in Appendix G, we have

$$\mathbb{P}\big(I_{BC} < r\log\rho|\mathcal{U}_2,\mathcal{U}_1\big) \doteq \rho^{-d_{BC}(r)}, \tag{H.2}$$

where

$$d_{BC}(r) = \inf_{(v_1,v_3)\in O'} v_1 + (r/2-1) + v_3, \tag{H.3}$$

and the outage event O' is defined as

$$O' = \Big\{(v_1,v_3) \in R^{2+}, v_1 \geq (1-r/2)\,,\, (1-t)(1-v_1)^+ + t\big(1-\min(v_1,v_3)\big)^+ \leq r\Big\}. \tag{H.4}$$

Solving the optimization problem, the DMT for the source cutset is given by

$$d_{BC} = \begin{cases} 1 - \frac{r}{t}\Big(1 - \frac{1-t}{2}\Big) & t > \frac{1}{2},\ r \leq \frac{1-(1-t)}{1-(1-t)/2} \\ 0 & t > \frac{1}{2},\ r > \frac{1-(1-t)}{1-(1-t)/2} \\ 1 - r\Big(\frac{1}{t} - \frac{1}{2}\Big) & t \leq \frac{1}{2},\ r \leq t \\ \frac{1-r}{1-t} + \frac{r}{2} - 1 & t \leq \frac{1}{2},\ \frac{1-(1-t)}{1-(1-t)/2} \geq r > t \\ 0 & t \leq \frac{1}{2},\ r > \frac{1-(1-t)}{1-(1-t)/2}. \end{cases} \tag{H.5}$$

Similarly, The DMT for the destination cutset is given by

$$d_{MAC} = \begin{cases} 1 - \frac{r}{1-t}\left(1 - \frac{t}{2}\right) & t < \frac{1}{2},\ r \leq \frac{1-t}{1-t/2} \\ 0 & t < \frac{1}{2},\ r > \frac{1-t}{1-t/2} \\ 1 - r\left(\frac{1}{1-t} - \frac{1}{2}\right) & t \geq \frac{1}{2},\ r \leq (1-t) \\ \frac{1-r}{t} + \frac{r}{2} - 1 & t \geq \frac{1}{2},\ \frac{1-t}{1-t/2} \geq r > (1-t) \\ 0 & t \geq \frac{1}{2},\ r > \frac{1-t}{1-t/2}. \end{cases} \tag{H.6}$$

The two functions are equal at $t = \frac{1}{2}$ and that gives the maximum DMT. The DMT is given by

$$d_3'(r) = \left(1 - \frac{3}{2}r\right)^+. \tag{H.7}$$

Adding the DMT of the three modes completes the proof.

APPENDIX I

SUM OF AN EXPONENTIAL AND A CLIPPED EXPONENTIAL RANDOM VARIABLES

Define $Z = X + Y$, where X and Y are exponential random variables with exponential parameters λ_1 and λ_2, respectively. Conditioned on the event $B = \{Y < \alpha\}$, the conditional pdf of Y is given by

$$f_{Y|B}(y) = \begin{cases} \lambda_2 \frac{e^{-\lambda_2 y}}{1-e^{-\lambda_2 \alpha}} & y \leq \alpha \\ 0 & y > \alpha. \end{cases} \tag{I.1}$$

The pdf of $Z = X + Y$ conditioned on the event B is as follows.

$$f_{Z|B}(z) = \begin{cases} \int_0^z \lambda_1 e^{-\lambda_1(z-x)} \frac{\lambda_2 e^{-\lambda_2 x}}{1-e^{-\lambda_2 \alpha}} dx & z \leq \alpha \\ \int_0^\alpha \lambda_1 e^{-\lambda_1(z-x)} \frac{\lambda_2 e^{-\lambda_2 x}}{1-e^{-\lambda_2 \alpha}} dx & z > \alpha. \end{cases} \tag{I.2}$$

Hence, if $\lambda_1 \neq \lambda_2$, the conditional pdf of $Z = X + Y$ is

$$f_{Z|B}(z) = \begin{cases} \lambda_1 \lambda_2 \frac{e^{-\lambda_1 z}(e^{(\lambda_1-\lambda_2)z}-1)}{(\lambda_1-\lambda_2)(1-e^{-\lambda_2 \alpha})} & z \leq \alpha \\ \lambda_1 \lambda_2 \frac{e^{-\lambda_1 z}(e^{(\lambda_1-\lambda_2)\alpha}-1)}{(\lambda_1-\lambda_2)(1-e^{-\lambda_2 \alpha})} & z > \alpha. \end{cases} \tag{I.3}$$

If $\lambda_1 = \lambda_2 = \lambda$, the conditional pdf of Z is

$$f_{Z|B}(z) = \begin{cases} \lambda^2 \frac{z e^{-\lambda z}}{(1-e^{-\lambda \alpha})} & z \leq \alpha \\ \lambda^2 \frac{\alpha e^{-\lambda z}}{(1-e^{-\lambda \alpha})} & z > \alpha. \end{cases} \tag{I.4}$$

REFERENCES

[1] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *International conference on communications, ICC*, Seattle, WA, Jun. 1995, pp. 331–335.

[2] T. E. Hunter and A. Nosratinia, "Distributed protocols for user cooperation in multi-user wireless networks," in *IEEE Global Telecommunications Conference, GLOBECOM*, vol. 6, Nov./Dec. 2004, pp. 3788–3792.

[3] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 659–672, Mar. 2006.

[4] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450–3460, Sep. 2007.

[5] ——, "Outage optimality of opportunistic amplify-and-forward relaying," *IEEE Commun. Lett.*, vol. 11, no. 3, pp. 261–263, Mar. 2007.

[6] Y. Jing and H. Jafarkhani, "Single and multiple relay selection schemes and their achievable diversity orders," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1414–1423, Mar. 2009.

[7] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 235–237, Apr. 2008.

[8] I. Krikidis, J. Thompson, S. Mclaughlin, and N. Goertz, "Max-min relay selection for legacy amplify-and-forward systems with interference," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3016–3027, Jun. 2009.

[9] R. Vaze and R. W. Heath, "To code in space and time or not in multihop relay channels," *IEEE Trans. Signal Processing*, vol. 57, no. 7, pp. 2736–2747, Jul. 2009.

[10] A. Nosratinia and T. E. Hunter, "Grouping and partner selection in cooperative wireless networks," *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 369–378, Feb. 2007.

[11] M. M. Fareed and M. Uysal, "On relay selection for decode-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3341–3346, Jul. 2009.

[12] E. Beres and R. Adve, "Selection cooperation in multi-source cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 118–127, Jan. 2008.

[13] S. Lee, M. Han, and D. Hong, "Average SNR and ergodic capacity analysis for opportunistic DF relaying with outage over Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2807–2812, Jun. 2009.

[14] T. J. Oechtering and H. Boche, "Bidirectional regenerative half-duplex relaying using relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1879–1888, May 2008.

[15] Z. Yi and I.-M. Kim, "Diversity order analysis of the decode-and-forward cooperative networks with relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1792–1799, May 2008.

[16] K. S. Hwang, Y. C. Ko, and M. S. Alouini, "Outage probability of cooperative diversity systems with opportunistic relaying based on decode-and-forward," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5100–5107, Dec. 2008.

[17] R. Tannious and A. Nosratinia, "The interference channel with MIMO relay: Degrees of freedom," in *IEEE International Symposium on Information Theory, ISIT*, 2008, pp. 1908–1912.

[18] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[19] D. N. C. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1859–1874, Sep. 2004.

[20] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[21] A. Carleial, "Interference channels," *IEEE Trans. Inform. Theory*, vol. 24, no. 1, pp. 60–70, Jan. 1978.

[22] H. Sato, "The capacity of the Gaussian interference channel under strong interference (corresp.)," *IEEE Trans. Inform. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.

[23] O. Sahin and E. Erkip, "Achievable rates for the Gaussian interference relay channel," in *IEEE Global Telecommunications Conference, GLOBECOM*, 2007, pp. 1627–1631.

[24] S. Sridharan, S. Vishwanath, S. Jafar, and S. Shamai, "On the capacity of cognitive relay assisted Gaussian interference channel," in *IEEE International Symposium on Information Theory, ISIT*, 2008, pp. 549–553.

[25] I. Maric, R. Dabora, and A. Goldsmith, "On the capacity of the interference channel with a relay," in *IEEE International Symposium on Information Theory, ISIT*, 2008, pp. 554–558.

[26] M. O. Hasna and M. S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-Fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[27] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Select. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug. 2004.

[28] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec. 2005.

[29] M. Abouelseoud and A. Nosratinia, "Diversity multiplexing trade-off for opportunistic multiple access relay channel," in *Proc. Asilomar Conference on Signals, Systems and Computers*, 2008.

[30] M. Yuksel and E. Erkip, "Multiple-antenna cooperative wireless systems: A diversity multiplexing tradeoff perspective," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3371–3393, Oct. 2007.

[31] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[32] S. Pawar, A. S., and T. D., "Diversity multiplexing tradeoff of the half-duplex relay channel," in *Proc. Allerton Conference on Communications, Control, and Computing*, Monticello, IL, Sep. 2008.

[33] D. Gunduz, A. Glodsmith, and V. Poor, "MIMO two-way relay channel: Diversity-multiplexing tradeoff analysis," in *Proc. Asilomar Conference on Signals, Systems and Computers*, Pasific Grove, CA., 2008.

[34] A. Tajer and A. Nosratinia, “A broadcasting relay for orthogonal multiuser channels,” in *IEEE Global Telecommunications Conference, GLOBECOM*, San Francisco, CA, Nov. 2006, pp. 1–5.

[35] M. Khojastepour and X. Wang, “Capacity bounds for MIMO shared relay channel with half-duplex constraint,” in *IEEE International Symposium on Information Theory, ISIT*, Jul. 2008, pp. 1918–1922.

[36] S. A. Jafar and S. Shamai, “Degrees of freedom region of the MIMO X channel.” *IEEE Trans. Inform. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.

[37] M. Abouelseoud and A. Nosratina, “The gateway channel: Outage analysis,” in *IEEE Global Telecommunications Conference, GLOBECOM*, New Orleans, LA, Dec. 2008, pp. 1–5.

[38] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication.* Cambridge University Press, 2005.

[39] G. Kramer and A. Van Wijngaarden, “On the white Gaussian multiple-access relay channel,” in *IEEE International Symposium on Information Theory, ISIT*, Jun. 2000, p. 40.

[40] K. Azarian, H. El Gamal, and P. Schniter, “On the optimality of the ARQ-DDF protocol,” *IEEE Trans. Inform. Theory*, vol. 54, no. 4, pp. 1718–1724, Apr. 2008.

[41] D. Chen and J. Laneman, “The diversity-multiplexing tradeoff for the multiaccess relay channel,” in *40th Annual Conference on Information Sciences and Systems*, Mar. 2006, pp. 1324–1328.

[42] D. Chen, K. Azarian, and J. N. Laneman, “A case for amplify-forward relaying in the block-fading multi-access channel,” *IEEE Trans. Inform. Theory*, Submitted Dec. 2007, accepted for publication.

[43] G. Kramer, P. Gupta, and M. Gastpar, “Information-theoretic multihopping for relay networks,” in *International Zurich Seminar on Communications,*, 2004, pp. 192–195.

[44] Y. Liang and V. V. Veeravalli, “The impact of relaying on the capacity of broadcast channels,” in *IEEE International Symposium on Information Theory, ISIT*, Jun./Jul. 2004.

[45] Y. Jiang and M. Varanasi, “Diversity-multiplexing tradeoff of MIMO system with antenna selection,” in *IEEE International Symposium on Information Theory, ISIT*, Nice, France, Jun. 2007, pp. 2836–2840.

[46] R. Tannious and A. Nosratinia, "Spectrally-efficient relay selection with limited feedback," *IEEE J. Select. Areas Commun.*, vol. 26, no. 8, pp. 1419–1428, Aug. 2008.

[47] Y. Li, B. Vucetic, Z. Chen, and J. Yuan, "An improved relay selection scheme with hybrid relaying protocols," in *IEEE Global Telecommunications Conference (GLOBECOM)*, Nov. 2007, pp. 3704 –3708.

[48] J. Fricke, M. Butt, and P. Hoeher, "Quality-oriented adaptive forwarding for wireless relaying," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 200 –202, Mar. 2008.

[49] T. Liu, L. Song, Y. Li, Q. Huo, and B. Jiao, "Performance analysis of hybrid relay selection in cooperative wireless systems," *IEEE Trans. Commun.*, vol. 60, no. 3, pp. 779 –788, Mar. 2012.

[50] X. Bao and J. Li, "Efficient message relaying for wireless user cooperation : Decode-amplify-forward (DAF) and hybrid DAF and coded-cooperation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 11, pp. 3975–3984, Nov. 2007.

[51] M. Jeong, S. Hwang, D. Kim, and J. Yang, "Performance analysis of cooperative communication with heterogeneous relays," in *IEEE Personal Indoor and Mobile Radio Communications (PIMRC)*, Sep. 2011, pp. 1104 –1108.

[52] P. Lusina, R. Schober, and L. Lampe, "Diversity-multiplexing trade-off of the hybrid non-orthogonal amplify-decode and forward protocol," in *IEEE International Symposium on Information Theory*, Jul. 2008, pp. 2375 –2379.

[53] S. Yang and J.-C. Belfiore, "Towards the optimal amplify-and-forward cooperative diversity scheme," *IEEE Trans. Inform. Theory*, vol. 53, no. 9, pp. 3114 –3126, Sep. 2007.

[54] K. Seddik, A. Sadek, W. Su, and K. Liu, "Outage analysis of multi-node amplify-and-forward relay networks," in *Wireless Communications and Networking Conference, WCNC*, Oct. 2006, pp. 1184 – 1188.

[55] T. Kim and H. Poor, "Diversity-multiplexing trade-off in adaptive two-way relaying," *Information Theory, IEEE Transactions on*, vol. 57, no. 7, pp. 4235 –4254, july 2011.

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