Routing, Wavelength Assignment, and Spectrum Allocation Algorithms in Transparent Flexible Optical WDM Networks☆

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Abstract

Current fixed grid wavelength routed networks are limited in terms of spectral efficiency due to the rigid nature of wavelength assignment. We propose the Flexible Optical WDM (FWDM) network architecture for flexible grid optical networks in which the constraint on fixed spectrum allocation to channels is removed and network resources can be dynamically provisioned with an automated control plane. In this paper, we address the routing, wavelength assignment, and spectrum allocation problem (RWSA) in transparent FWDM networks with the objective of maximizing spectral efficiency. We formulate the RWSA problem using an Integer Linear Program (ILP). We also prove the NP-Completeness of the RWSA problem, and propose three efficient polynomial time algorithms; namely the Greedy-Routing, Wavelength Assignment, and Spectrum Allocation algorithm (Greedy-RWSA); the K-Alternate Paths Routing, Wavelength Assignment, and Spectrum Allocation algorithm (KPaths-RWSA); and the Shortest Path Routing, Wavelength Assignment, and Spectrum Allocation algorithm (SP-RWSA). We analyze the

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lower bound on the required spectrum for the given network topology and a set of requests. Simulation results demonstrate that FWDM networks are efficient in terms of spectrum, cost, and energy compared to fixed grid networks. The performance of the proposed algorithms is very close to the lower bound, and approaches to the lower bound as problem size increases.

Keywords: Flexible optical WDM (FWDM) network, Fixed grid network, Routing, Wavelength Assignment, and Spectrum Allocation (RWSA), Lower bound, Flexible grid network

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1. Introduction

Global communication traffic has been growing exponentially and is becoming more dynamic due to emerging applications such as IPTV, online gaming, video on demand, and e-science applications. This growth leads to an increasing demands for better efficiency with respect to resources, scale, energy, and cost. To meet these demands, recent innovations in transmission systems have lead to increasing line rates of WDM channels from 10 Gb/s to 40 Gb/s to 100 Gb/s and beyond with the utilization of advanced modulation formats and multiplexing schemes such as dual polarization multi-level modulation [1] and OFDM [2]. Additionally, advanced modulation schemes, digital signal processing techniques, and transmission impairment mitigation methods have increased the reachability of optical signals through multiple optical spans without signal regeneration. Thus, a transparent transport layer may replace an opaque layer to reduce the requirement of optical-electrical-optical (OEO) convertors and to reduce power consumption. The spacing between adjacent DWDM channels can be reduced from 100 GHz to 50 GHz or narrower, and thus utilization of spectral resources is increased. To increase the utilization of wavelength resources, mixed line rate systems can be deployed to accommodate heterogeneous traffic types at different data rates.

Current DWDM systems with fixed channel grids are not able to take advantages of these advance technologies in an efficient manner. For example, while a 50 GHz channel can accommodate a 100 Gb/s DP-QPSK signal, it is not sufficient to transmit a 400 Gb/s signal within the same spectral spacing. On the other hand, if the channel spacing is changed to 100 GHz to accomodate 400 Gb/s signals, the spectral efficiency is not optimized [3]. In a mixed line rate system, no single fixed channel grid is optimal for all data rates. As a result, there has been growing research interest on WDM systems that are not restricted by the ITU-T channel grid [4], but that offer elastic channel bandwidths to increase spectral efficiency and network flexibility and to reduce capital and operational cost [5] [6]. We refer to such flexible grid systems as the Flexible optical WDM (FWDM) Networks [7]. FWDM networks can allocate variable amount of spectrum to channels in order to support heterogeneous line rates. Additionally, allocated spectral resources to channels can be adapted to support dynamic traffic in FWDM networks.

When implementing the FWDM network, one challenging issue is how to establish lightpaths such that all requests are satisfied with the least amount of spectrum. Since establishing lightpaths consists of subproblems such as routing of lightpaths, assignment of wavelengths, and allocation of spectrum, we refer to the problem as the routing, wavelength assignment, and spectrum allocation problem (RWSA). In [7], we introduced the RWSA problem for the first time and propose an ILP formulations which can solve the problem optimally by addressing all the subproblems at the same time. In [8], authors also propose ILP formulations for the same problem in which the routing, wavelength assignment, and spectrum allocation subproblems are addressed sequentially. The proposed solution in [8] may not result an optimal solution for the RWSA problem. In [9], we also proposed the first algorithm addressing the RWSA problem with the objective of minimizing the connection blocking probability for the dynamic traffic scenario.

In this paper, we study the RWSA problem for the FWDM network architecture in green field optical networks. The RWSA problem is studied with the objective of maximizing the spectrum efficiency in transparent FWDM networks in which all-optical connections are established end-to-end between end users. We also prove the hardness of the RWSA problem, and propose three polynomial time algorithms, the Greedy RWSA algorithm, the K-Alternate Paths RWSA algorithm, and the Shortest Path RWSA algorithm. The proposed algorithms can be used in network planning problems when traffic demands are predetermined. To compare the performance of the proposed algorithms for the larger instance of the problem, we also derive a lower bound for the RWSA problem. To the best of our knowledge this is the first comprehensive study on the RWSA problem.

The paper is organized as follows. In Section 2, we introduce the FWDM architecture. The RWSA problem is described in Section 3. The proposed ILP formulation is described in Section 4, and algorithms are explained in Section 5. Section 6 analyzes the simulation results, and finally the paper is concluded in Section 7.

2. Flexible Optical WDM Network

The flexible optical WDM network architecture consists of three key features which are not concurrently present in existing deployed optical WDM network architectures, such as (1) dynamic provisioning of resources to connections, (2) dynamic provisioning of connections, and (3) automated control plane.

The first key feature is the dynamic provisioning of resources to connections. Unlike fixed spectrum allocation, defined in the fixed grid network by the ITU-T standard [4], spectrum allocation in the FWDM network is flexible to support various line rates. Provisioning connections with the requested data rate in the transparent network avoids inefficient utilization of wavelength resources for the traffic with subwavelength granularity. On the other hand, provisioning connections for the traffic with coarser wavelength granularity gains spectral efficiency due to the nonlinear increment in the required spectral spacing as the line rate increases. Additionally, the flexible channel in FWDM networks may be static or adaptive to dynamic traffic. The line rate and spectral width of the adaptive channel can be changed over time to share the spectral resources in the time domain. These adaptive flexible channels can be realized using variable rate transponders and spectrum variable ROADM nodes. The variable rate transponders can use an OFDM modulation scheme with variable subcarrier assignment or a single carrier modulation scheme with switchable modulation stages and/or variable rate data multiplexer to adjust the line rate according to the traffic demand. The spectrum variable ROADM node uses switching technologies with small pixels, such as liquid crystal on silicon (LCoS) [10] or digital micro electro mechanical systems (MEMS) [11], to obtain finer switching granularity and to perform add/drop/cross-connect with different passband widths [12].

Based on the current transmission technology and products, 10 Gb/s, 40 Gb/s, 100 Gb/s channels require 25 GHz, 50 GHz, and 50 GHz channel spacing respectively. It was recently demonstrated that 75 GHz or less channel spacing is sufficient for 400 Gb/s channel [3]; and it is expected that the required channel spacing for 1 Tb/s channel is likely to be about 150 GHz [5]. Thus, as the capacity of a channel increases the required spectral width per unit line rate decreases. Figure 1 shows a 5-node FWDM network example. Each node is assumed to have a variable rate transponder aggregator and/or spectrum variable ROADM. Requests are routed optically by establishing connections with various line rates using optimum flexible spectral widths

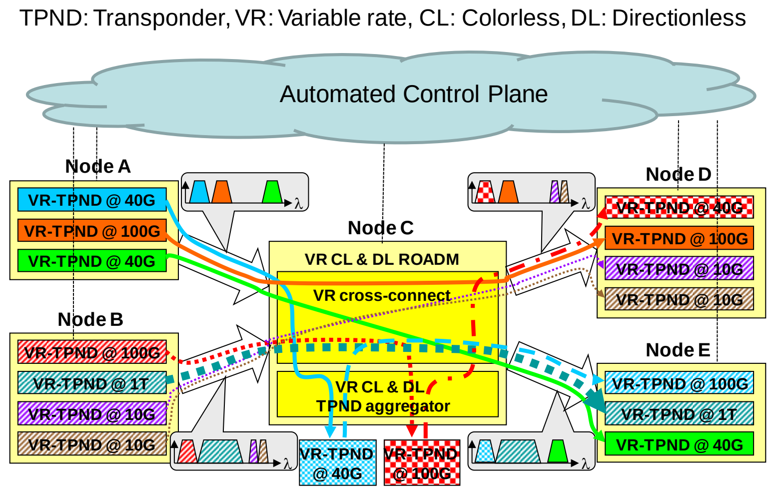

Figure 1: FWDM network architecture.

compared to the fixed grid WDM networks. Instead of provisioning a 40 Gb/s request from A to E using 100 Gb/s connection in 100 Gb/s line rate fixed grid network, a connection with exactly 40 Gbps rate is established using minimum required spectral resources as shown in Fig. 1. This avoids inefficient utilization of wavelength resources. In Fig. 1, instead of provisioning 1 Tb/s demand between node B and E using 10 separate 100 Gb/s connections in the 100 Gb/s line rate fixed grid network, a single connection of 1 Tb/s is established. Thus, instead of allocating 500 GHz of spectrum to connection B-E in the fixed grid networks, the FWDM network requires only 150 GHz spectrum. Thus, FWDM networks are spectral efficient. In case of dynamic traffic patterns, transponders with the maximum transmission capacity need to be deployed at each node; however for static traffic patterns, transponders assignment (selection) can be pre-configured at each node based on scheduled lightpaths.

The second key feature is the dynamic provisioning of connections in the network. The ROADM node in the FWDM network consists of a transponder aggregator to provide a colorless feature where each physical add/drop port is not wavelength specific, and a directionless feature where any transponder is able to be connected to any degree [13]. Both features are achieved without incurring wavelength contention at the node. Thus, dynamic connections can be provisioned by removing the port specific wavelength assignment, port specific routing, and wavelength uniqueness constraints at existing ROADM nodes.

The third key feature is the automated control plane. Network traffic can be provisioned dynamically with the help of an automated control plane which has software controlled optimized routing and resource assignment, dissemination of network state information, provisioning survivability through either protection [14, 15] or restorations, and connection setup and teardown functionalities. Intelligent centralized and distributed multi-constrained path computation algorithms can be incorporated into an existing control plane for various parameter optimization in the FWDM network. Compared to existing fixed grid WDM networks, the FWDM network is required to disseminate additional spectrum availability information of the fiber links which can be realized by updating existing Link Management Protocol (LMP) and Open Shortest Path First-Traffic Engineeting (OSPF-TE) protocols. This information may either be continuous or discrete in frequency domain, and can be maintained either using a centralized approach or a distributed approach. To setup, teardown, and manage the adaptive channels, existing GMPLS control plane can be modified by incorporating channel width and frequency related parameters or establishing lambda paths at subcarrier granularities [6].

3. Routing, Wavelength Assignment, and Spectrum Allocation Problem (RWSA)

For a given FWDM network, and a set of traffic demands, the RWSA problem is to accommodate the given set of traffic demands all optically in the network such that minimum spectrum is required. The RWSA problem considers allocation of an additional parameter, spectrum, compared to the existing routing and wavelength assignment problem (RWA) in existing fixed grid WDM networks. The formal RWSA problem description is as follows.

We are given a physical topology $G(\mathbf{V}, \mathbf{E})$, where $\mathbf{V}$ is a set of ROADM nodes, and $\mathbf{E}$ is a set of fiber edges, and a set of traffic demands $\mathbf{\Lambda}$ in which a request is defined as $R(s, d, \gamma)$, where s is the source, d is the destination, and γ is the requested data rate. The network supports a set of line rates, $\mathbf{L}$. For example, $\mathbf{L} = \{$10 Gb/s, 40 Gb/s, 100 Gb/s, 400 Gb/s, 1 Tb/s$\}$. Each line rate $l_i \in \mathbf{L}$ requires a spectral width of x_{l_i}. For example, a line rate of 100 Gb/s requires 50 GHz of spectrum. Transponders are tunable on a given set of wavelengths, $\mathbf{W}$. The cost and power requirement of a transponder operating at line rate l_i is C_{l_i} and J_{l_i} respectively. The goal of the problem is to find a set of lightpaths to satisfy the traffic demands while minimizing the total spectral width required in the network. For each lightpath, we must find a route over the physical topology, an operating wavelength , and the required spectral width on each link along the route. The spectrum will also determine the line rate of the lightpath. The optical network is assumed to have an ideal transport layer with no wavelength conversion capability at intermediate nodes, and the traffic is not adaptive over time. The network maintains the spectrum availability information centrally in discretized form.

The RWSA problem must follow (1) the wavelength continuity constraint, which is defined as the allocation of the same wavelength on each fiber link along the route of a lightpath, (2) the spectral continuity constraint, which is defined as the allocation of the same continuous spectrum on each fiber link along the route of a lightpath, and (3) the spectral conflict constraint, which is defined as non-overlapping spectrum allocation to two different lightpaths on the same fiber link. In fixed grid networks, each wavelength is allocated the same amount of spectrum. Unlike the RWA problem in fixed grid networks, the RWSA problem in FWDM networks must obey spectral continuity and spectral conflict constrains in addition to wavelength continuity constraint due to the flexible assignment of spectrum.

In order to reduce the complexity of the RWSA problem and to avoid some hardware restrictions such as ultra narrow passband filters, the spectrum is discretized in the frequency domain, and the smallest unit of a spectrum is referred to as a wavelength slot. Thus, the allocated spectrum can also be defined in terms of the number of consecutive wavelength slots. In a fiber section, a wavelength slot can either be in the available state or the occupied state. No more than one lightpath can occupy a wavelength slot (spectrum) at the same time, however a lightpath may occupy more than one consecutive wavelength slots depending on the offered line rate. The state information of wavelength slots on a fiber cable is referred to as the spectrum availability information of a link, and the state information of wavelength slots on a path is referred to as the spectrum availability information of a path. The wavelength of a lightpath is defined as the lowest indexed wavelength slot occupied by the lightpath.

For example, consider a 6-node FWDM network as shown in Fig. 2, which can support various line rates with flexible spectrum assignment. The numbers along edges represent the available wavelength slots, where the spectrum of a wavelength slot is 25 GHz. Consider requests A to F and B to E each requesting 100 Gb/s data rate. The required spectrum for the 100 Gb/s line

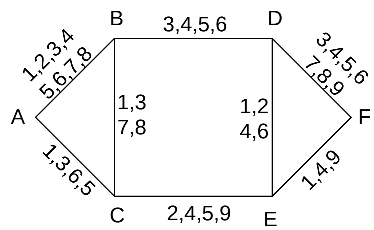

Figure 2: Illustrative example.

rate is 50 GHz. Request A to F can be routed on route A-B-D-F using either wavelength slots (3, 4), (4, 5), or (5, 6). Request B to E cannot be routed on route B-C-E, since no common wavelength slot is available on all links along the path (wavelength continuity constraint), and cannot be routed on routes B-D-E, B-A-C-E, and B-D-F-E, since none of these paths contain two consecutive wavelength slots (spectrum continuity constraint).

The hardness of the RWSA problem can easily be proved by mapping it to the existing RWA problem as follows.

Theorem 1: The RWSA problem is NP-complete.

Proof: For a given identical spectral width for each line rate, the RWSA problem is transformed to the RWA problem in mixed-line rate networks [16], and for a given fixed line rate network, the problem is transformed into the RWA problem in single-line rate WDM networks. With the assumption of having same spectral width and same line rates for all channels in the network, the objective function of minimizing the spectrum in the network is reduced to minimizing the number of wavelengths in the RWA problem. Since the RWA problem is a well known NP-complete problem [17], the RWSA problem is also NP-complete. ■

Thus, finding an optimal solution in a polynomial time for the RWSA problem is as hard as finding an optimal solution in the RWA problem.

4. Integer Linear Program

In this section, we propose an Integer Linear Programming (ILP) formulation for the RWSA problem. The notations used in the formulations are described as follows.

Notations:

- **V**: A set of ROADM nodes {1, 2, 3, ...,v},

- **W**: A set of wavelength slots,{1, 2, 3, ...w},
- δ: Spectrum of a wavelength slot,
- **L**: A set of line rates, {10 Gb/s, 40 Gb/s, 100 Gb/s, 400 Gb/s, 1 Tb/s},
- x_{l_i}: Spectral width required to support line rate $l_i \in \mathbf{L}$,
- α: Large number, $\alpha > w$,
- $C_{x_{l_i}}^{l_i}$: Cost of a tunable transponder operating at line rate l_i with spectral width x_{l_i},
- $J_{x_{l_i}}^{l_i}$: Power of a tunable transponder operating at line rate l_i with spectral width x_{l_i},
- P_{mn}: Number of fibers from node m to node n,
- P_{mn}^{w}: Number of w wavelengths from nodes m to n,
- $\mathbf{\Lambda}$: Traffic matric,
- Λ^{sd}: Requested data rate from source s to destination d,
- $V_{sd_{x_{l_i}}}^{wl_i}$: 1, if wavelength w is used to establish a lightpath between nodes s and d operating at line rate l_i using spectral width x_{l_i}, 0 otherwise,
- $P_{mn_{x_{l_i}}}^{sd^{wl_i}}$: 1, if lightpath (s,d) is routed through physical link (m,n) using wavelength w, operating at line rate l_i using spectral width x_{l_i}, 0 otherwise,
- $q_{mn_{x_{l_i}}}^{sd^{wl_i}}$: 1, if wavelength w falls within the spectrum x_{l_i} assigned to the lightpath (s,d), operating on line rate l_i, and routed through physical link (m,n), 0 otherwise,
- $U_{mn_{x_{l_i}}}^{sd^{w_1 w_2 l_i}}$: Required spectral spacing (in terms of number of wavelength slots) between lightpaths, which are operating on wavelengths w_1, and w_2, routed through the same fiber (m,n), originating at the same source s and terminated at the same destination d, and operating on the same line rates l_i using the same spectral width x_{l_i}.
- Y: Maximum index of the occupied wavelength slot,

- Z: Minimum index of the occupied wavelength slot,
- C: Cost of a network,
- J: Energy consumption of a network,

$$\sum_{x_{l_i}} \sum_{w} \sum_{l_i \in L} l_i \times V_{sdx_{l_i}}^{wl_i} \geq \Lambda_{sd} \quad \forall s, d \tag{1}$$

$$\sum_{x_{l_i}} \sum_{l_i \in L} V_{sdx_{l_i}}^{wl_i} \leq 1 \quad \forall s, d, w \tag{2}$$

$$\sum_{n} P_{snx_{l_i}}^{sd^{wl_i}} = V_{sd^{x_{l_i}}}^{wl_i} \quad \forall s, d, w, x_{l_i}, l_i \in L \tag{3}$$

$$\sum_{n} P_{ndx_{l_i}}^{sd^{wl_i}} = V_{sd^{x_{l_i}}}^{wl_i} \quad \forall s, d, w, x_{l_i}, l_i \in L \tag{4}$$

$$\sum_{m} P_{mkx_{l_i}}^{sd^{wl_i}} = \sum_{n} P_{knx_{l_i}}^{sd^{wl_i}} \quad \forall s, d, w, x_{l_i}, k, l_i \in L \tag{5}$$

$$\sum_{m} P_{msx_{l_i}}^{sd^{wl_i}} = 0 \quad \forall s, d, w, x_{l_i}, l_i \in L \tag{6}$$

$$\sum_{n} P_{dnx_{l_i}}^{sd^{wl_i}} = 0 \quad \forall s, d, w, x_{l_i}, l_i \in L \tag{7}$$

$$\sum_{x_{l_i}, l_i \in L} \sum_{sd} P_{mnx_{l_i}}^{sd^{wl_i}} \leq P_{mn}^{w} \quad \forall m, n, w \tag{8}$$

$$P_{mn}^{w} = P_{mn} \quad \forall m, n, w \tag{9}$$

$$w_1 \times P_{mnx_{l_i}}^{sd^{w_1 l_i}} - w_2 \times P_{mnx_{l_i}}^{sd^{w_2 l_i}} \geq U_{mnx_{l_i}}^{sd^{w_1 w_2 l_i}} \quad \forall s, d, m, n, w_1, w_2 < w_1, x_{l_i}, l_i \in L \tag{10}$$

$$U_{mnx_{l_i}}^{sd^{w_1 w_2 l_i}} \geq \frac{x_{l_i}}{\delta} \times (P_{mnx_{l_i}}^{sd^{w_1 l_i}} + P_{mnx_{l_i}}^{sd^{w_2 l_i}} - 1) - \alpha \times (2 - P_{mnx_{l_i}}^{sd^{w_1 l_i}} - P_{mnx_{l_i}}^{sd^{w_2 l_i}}) \ \forall s, d, m, n, w_1, w_2, x_{l_i}, l_i \tag{11}$$

$$\sum_{w_1=w}^{w+\frac{x_{l_i}}{\delta}} q_{mnx_{l_i}}^{sd^{w_1 l_i}} \geq \frac{x_{l_i}}{\delta} \times P_{mnx_{l_i}}^{sd^{wl_i}} \quad \forall s, d, m, n, w, x_{l_i}, l_i \tag{12}$$

$$\sum_{x_{l_i}} \sum_{l_i \in L} \sum_{s,d} q_{mnx_{l_i}}^{sd^{wl_i}} \leq 1 \quad \forall m, n, w \tag{13}$$

$$Y \geq q_{mnx_{l_i}}^{sd^{wl_i}} \times w \qquad \forall s, d, w, n, m, x_{l_i}, l_i \in L \tag{14}$$

$$Z \leq q_{mnx_{l_i}}^{sd^{wl_i}} \times w \qquad \forall s, d, w, n, m, x_{l_i}, l_i \in L \tag{15}$$

$$C = \sum_{s,d} \sum_{l_i \in L, x_{l_i}} \sum_{w} C_{x_{l_i}}^{l_i} \times V_{sd^{x_{l_i}}}^{wl_i} \tag{16}$$

$$J = \sum_{s,d} \sum_{l_i \in L, x_{l_i}} \sum_{w} J_{x_{l_i}}^{l_i} \times V_{sd^{x_{l_i}}}^{wl_i} \tag{17}$$

$$S = \min(Y - Z) \tag{18}$$

The given spectrum is slotted in the frequency domain in units of δ GHz. As $\delta \rightarrow 0$, wavelength slots (spectrum) have finer granularities at the cost of higher complexity. Constraint 1 provisions lightpaths with heterogeneous line rates to satisfy the requested data rates between a source and a destination. Assignment of a unique line rate and spectral width to the given lightpath is restricted by Constraint 2. Constraint 2 also specifies that a wavelength cannot be assigned to more than one lightpath between the same source and destination. This limitation of constraint 2 can be addressed by identifying each lightpath uniquely by assigning a unique ID to a lightpath; however this improves the complexity of the problem formulations. Constraints 3 to 7 represent the multi-commodity flow constraints for routing of lightpaths over the physical topology, where wavelength continuity and spectral continuity constraints are represented by Equation 5. Constraints 8 and 9 limit the number of wavelengths used on a fiber link. Constraints 10 and 11 limit the spectral overlapping of lightpaths which coexist on the same physical path with the same originating and terminating nodes, and operate on the same line rate using the same spectral width. Constraint 12 reserves the required spectrum (in terms of number of consecutive wavelength slots) for the a selected lightpath which represents the spectral continuity constraint.

A given wavelength slot, which can be assigned to only a single lightpath, is constrained by Equation 13. Constraint 13 addresses the spectral conflict constraint. Equation 14 finds the maximum assigned wavelength slot in the spectrum, and 15 finds the minimum assigned wavelength slot in the spectrum. The cost and power of the network is defined by Constraint 16 and 17 respectively. The objective function, 18, minimizes the maximum required spectrum for designing a network.

The proposed ILP finds an optimal solution of the RWSA problem using exhaustive search, and helps in analyzing the problem. However the required time to solve the problem increases exponentially as the problem size increases. We have proposed the efficient polynomial-time algorithms in the following section which can approximate the minimum required spectrum in the FWDM networks.

5. RWSA Algorithms

The RWSA problem can be divided into two subproblems. The first subproblem is to find the set of line rates for a request such that total capacity offered by the set of line rates is at least the requested data rate, and the total spectral width required by the set of line rates is minimum. We refer to this problem as the line rate selection problem. We solve the line rate selection problem using dynamic programming [18], and we propose an optimal algorithm named rate selection algorithm (RSA). The second subproblem is how to establish lightpaths at the specific line rates between sources and destinations such that total required spectrum in the network is minimized. We refer to this problem as the routing and channel allocation problem. The second subproblem itself consists of many subproblems, such as how to route the lightpaths over the physical topology, how to assign wavelengths to the lightpaths, and how to assign the spectrum to the lightpaths. We propose three algorithms, the Greedy-RWSA, KPaths-RWSA, and SP-RWSA algorithms, in which the routing and channel selection subproblems are solved together in order to improve the optimization of the objective function. Since the RWSA problem is NP-complete, we also derive a lower bound to analyze the performance of the proposed algorithms for larger instances of the problem.

5.1. Line Rate Selection Problem

The first step to solve the RWSA problem is to select the combination of line rates which requires the minimum amount of spectrum. In this section, we describe the line rate selection problem and propose an optimal solution for the same problem.

For a given set of line rates, $\mathbf{L}$, a required spectral width, x_{l_i} for a line rate $l_i \in \mathbf{L}$, and a requested data rate γ, find a subset of line rates $\mathbf{D_j} \subseteq \mathbf{L}$, and the number of lightpaths, a_{l_i}, operating at such line rate $l_i \in \mathbf{D_j}$ such that,

$$\sum_{l_i \in \mathbf{D_j}} a_{l_i} \times l_i \geq \gamma \tag{19}$$

$$X_\gamma = \min_{\forall j} \sum_{l_i \in \mathbf{D_j}} a_{l_i} \times x_{l_i} \tag{20}$$

Given the assumption that required spectral width monotonically increases with a line rate, the line rate selection problem satisfies optimal sub-structure property in which an optimal solution of a problem can be obtained from an optimal solutions of its sub-problems.

Theorem 2 : The line rate selection problem holds the optimal substructure property.

Proof : Let us assume that for a given data rate, γ, the optimal set of line rates is $\mathbf{D} = \{l_1, l_2, l_3, .., l_n\}$, and the respective number of lightpaths operating at line rates are $a_{l_1}, a_{l_2}, a_{l_3}, .., a_{l_n}$, where $a_{l_i} > 0$, $\forall l_i \in D$. Let $\mathbf{D}^{'} = \{l_1, l_2, l_3, .., l_k\}$ with the respective number of lightpaths $a^{'}_{l_1}, a^{'}_{l_2}, a^{'}_{l_3}, .., a^{'}_{l_k}$ be the optimal solution for the data rate $\gamma^{'} \leq \sum_{l_i \in \mathbf{D}^{'}} a^{'}_{l_i} \times l_i \leq \gamma$ with the spectral width $X^{'}_{\gamma^{'}} = \sum_{l_i \in \mathbf{D}^{'}} a^{'}_{l_i} \times x_{l_i}$, where $\mathbf{D}^{'} \subseteq \mathbf{D}$ and $a^{'}_{l_i} = a_{l_i}$, $\forall l_i \in \mathbf{D}^{'}$.

Suppose there exists either another set of line rates, $\mathbf{D}^{''}$, where $\mathbf{D}^{''} \nsubseteq \mathbf{D}^{'}$, and $\mathbf{D}^{'} \nsubseteq \mathbf{D}^{''}$, or different number of lightpaths $a^{''}_{l_i}$ operating at line rates $l_i \in \mathbf{D}^{''}$, where $a^{''}_{l_i} \neq a^{'}_{l_i}$, or both, such that the required spectral width is $X^{''}_{\gamma^{'}} < X^{'}_{\gamma^{'}}$. Then $\mathbf{D}^{'}$ can be replaced by $\mathbf{D}^{''}$, and $a^{'}_{l_i}$ can be replaced by $a^{''}_{l_i}$, which further improves the spectral width for the required data rate γ. But this violates our assumption that the set $\mathbf{D}$ and respective number lightpaths operating at line rates, a_{l_i}, are the optimal solution for the data rate γ. Thus $\mathbf{D}^{'}$ and the respective number of lightpaths $a^{'}_{l_i}$ operating at line rates $l_i \in \mathbf{D}$ is an optimal solution for the $\gamma^{'}$. ■

5.1.1. Rate Selection Algorithm

Since the line rate selection problem holds the optimal substructure property, we can use the solution of the subproblem to derive the solution of the larger problem. For example, if we know the optimal spectral width, $X_{\gamma-l_i}$, for the data rate $\gamma - l_i$, then we can obtain the optimal spectral width for data rate γ, $X_\gamma = x_{l_i} + X_{\gamma-l_i}$. Initially, we don't know the value of l_i; however it can be easily determined by iterating l_i over each line rate in the given set $\mathbf{L}$. This recursive operation can be represented in terms of mathematical formulation as follows.

$$X_\gamma = 0 \qquad if \quad \gamma \leq 0 \tag{21}$$

$$X_\gamma = \min_{l_i \in \mathbf{L}} [x_{l_i} + X_{\gamma-l_i}] \qquad if \quad \gamma > 0 \tag{22}$$

We use the following notations to describe the pseudocode of the rate selection algorithm. Let us denote $\mathbf{L}^{'}$ as the optimal set of line rates required for the data rate γ, and q_γ as one of the line rates within an optimal set $\mathbf{L}^{'}$ for data rate γ. p_γ denotes the previous data rate from which the data rate γ can be obtained. The optimum spectrum for data rate γ is denoted as X_γ.

```
Algorithm 5.1: RSA(L, γ, x_l)

for n ← −γ to 0
  do {X_n ← 0
for n ← 1 to γ
     ⎧ min ← ∞
     ⎪ for i ← 1 to |L|
     ⎪       ⎧ if x_{l_i} + X_{n−l_i} < min
     ⎪   do  ⎨          ⎧ min ← x_{l_i} + X_{n−l_i}
  do ⎨       ⎪   then   ⎨ rate ← l_i
     ⎪       ⎩          ⎩ prerate ← n − l_i
     ⎪ X_n ← min
     ⎪ q_n ← rate
     ⎩ p_n ← prerate
n ← γ
L' ← φ
while n ≠ 0
  do ⎧ L' ⋃ {q_n}
     ⎩ n ← p_n
return (L', X_γ)
```

Initially, the RSA algorithm initializes the required spectrum, X_n, to zero if the data rate, n, is between $-\gamma$ and 0. After initialization, the RSA algorithm finds the required optimal spectrum and a set of line rates for data rate n iteratively starting from 1 up to γ. For data rate n, the algorithm uses the solution of data rate, $n - l_i$, stored in q_{n-l_i}, p_{n-l_i}, and X_{n-l_i}. In order to find the optimal spectral width for data rate n, the algorithm iterates on all given line rates l_i, and considers the solution which results in minimum spectrum, X_n. The new found line rate l_i is stored in q_n, and the previous data rate, $n - l_i$, is stored into p_n. Finally, after finding X_n for $0 \leq n \leq \gamma$, the final optimal set of line rates $\mathbf{L}'$ is determined by iterating on the stored information, q_n, p_n, and X_n in the backward direction.

Theorem 3: When the RSA algorithm terminates, X_γ contains the optimal spectral width for data rate γ.

Proof: We use induction to prove this theorem. The initialization phase in the algorithm verifies the base case in Equation 21, which is obviously true. Now, $X_1 = \min_{l \in \mathbf{L}}[x_l + X_{1-l}] = \min_{l \in \mathbf{L}}[x_l]$ is the minimum required spectrum to support the unit data rate. Now, if we assume that we have X_i, for $1 \leq i \leq k$, which is the minimum required spectrum for the data rate i, then $X_{k+1} = \min_{l \in \mathbf{L}}[x_l + X_{k+1-l}]$. Since the line rate $l > 0$ is selected in such a way that $[x_l + X_{k+1-l}]$ is minimized, X_{k+1} is the minimum required spectrum for the data rate $k+1$. ■

5.2. Routing and Channel Allocation Algorithms

In this section, we introduce the second subproblem which is referred as the routing and channel allocation problem.

For a given graph, $G(\mathbf{V}, \mathbf{E})$, and a set of requests, λ, in which a request is defined as $R^{'}(s, d, l_i)$, where s is a source node, d is a destination node, and l_i is a line rate of a lightpath, find the routing, wavelength assignment, and spectrum allocation of lightpaths for all requests in the set λ such that the required network spectrum is minimized.

The new set of requests, λ, can be obtained through the rate selection algorithm on a given set of requests, $\mathbf{\Lambda}$. For each request, $R(s, d, \gamma) \in \mathbf{\Lambda}$, a new request, $R^{'}(s, d, l_i) \in \lambda$, is generated for all $l_i \in \mathbf{D}^{\gamma}$, where $\mathbf{D}^{\gamma}$ is the optimal set of line rates obtained through the rate selection algorithm on γ. To address the fairness issues for lightpaths with heterogeneous amounts of spectral requirements, requests in set λ are arranged in descending order of some cost function $U = H^{sd} \times x_{l_i}$, where H^{sd} represents the distance between the source s and destination d. The motivation is that, as the required spectral width and distance between a pair of nodes increases, the probability of finding the wavelength and spectrum continuous route at lower wavelengths decreases. Thus, we want to first schedule those requests that require higher spectral width and longer routes. This ordered set of requests are scheduled in the network one by one using the proposed algorithms. In this initial study, we have neglected the effects of differential delay of bifurcated traffic at destination nodes. The key idea behind the proposed algorithms, Greedy-RWSA, KPaths-RWSA, SP-RWSA, is that lightpaths are scheduled at lower wavelengths as much as possible such that the total occupied spectrum is minimized. The detailed descriptions of the algorithms are as follows.

5.2.1. Greedy Routing, Wavelength Assignment, and Spectrum Allocation Algorithm (Greedy-RWSA)

After finding the new set of requests, λ, and ordering the set of request according to the cost function U, the Greedy-RWSA algorithm solves the routing and channel allocation subproblems at the same time for each request, $R^{'}$, using an auxiliary graph based approach [19]. The total network spectrum is slotted into W wavelength slots, which should be at least $|\lambda| \times \max_i \lceil \frac{x_{l_i}}{\delta} \rceil$ in order to have non blocking system. The discrete spectrum availability information, Z_e^w, is known in advance for each wavelength slot w of a link $e \in E$, which is 1 if the wavelength slot w is available on edge $e \in \mathbf{E}$, and 0 otherwise. The Greedy-RWSA algorithm constructs an auxiliary graph, $G_n(\mathbf{N}, \mathbf{A})$, at each wavelength slot n, where $\mathbf{N}$ represents a set of auxiliary nodes, which is the same as the given set of nodes V, and $\mathbf{A}$ represents a set of auxiliary links. The state of an auxiliary link is denoted as M_e.

An auxiliary link $e \in \mathbf{A}$ exists ($M_e = 1$) if $\lceil \frac{x_{l_i}}{\delta} \rceil$ number of consecutive wavelength slots are available on link e, where $\lceil \frac{x_{l_i}}{\delta} \rceil$ represents the number of consecutive wavelength slots equivalent to the required spectrum for the line rate l_i, otherwise it does not exist ($M_e = 0$). An auxiliary graph is constructed starting from the lowest wavelength slot, $n = 1$. After constructing an auxiliary graph $G(\mathbf{N}, \mathbf{A})$, the connectivity between the source s and destination d is checked using the well-known Breadth First Search algorithm, denoted as $BFS(G(\mathbf{N}, \mathbf{A}), s, d)$. If there exists more than one path, a path is selected with the shortest physical distance. The selected path is considered as one of the potential solutions and stored in P_i, which is referred to as the i^{th} wavelength path, and the wavelength slot, at which the i^{th} wavelength path exists, is stored in q_i. The algorithm increments the wavelength slot, and repeat the same procedure by reconstructing another auxiliary graph at newer wavelength slot. This procedure is terminated when either the algorithm has already found K potential solutions or it reaches at the maximum number of wavelength slots. Finding K potential solutions for each request indicates that the algorithm is finding K routes at K unique wavelengths. Finally, the RWSA algorithm selects a wavelength path whose physical distance is minimum in order to avoid over utilization of spectral resources due to longer path length.

Algorithm 5.2: GREEDY-RWSA(R, Z_e^w, W, K)

```
n ← 1
while j ≤ K  and  n ≤ W
  do { for each e ∈ E
         do { M_e ← 0
       I ← 1
       for each e ∈ E
         do { for j ← n to n + ⌈x_l/δ⌉
                do { if Z_e^j × I = 1
                       then { I ← 1
                       else { I ← 0
              if I = 1
                then { M_e ← 1
       Path ← BFS(G(N, A), s, d)
       if |Path| > 0
         then { P_j ← Path
                q_j ← n
                j ← j + 1
for i ← 1 to K
  do { if |Path| < |P_i|
         then { Path ← P_i
                k ← i
return (Path,  k)
```

K is an input parameter to the Greedy-RWSA algorithm. If $K = 1$, then the algorithm selects a shortest available path on the first available wavelength without taking into account the path length, we refer this routing strategy as the pure adaptive routing. As K increases, among available wavelength paths, the route with shorter physical distance is given higher priority, and thus, the routing becomes a composition of adaptive and shortest path routing. After certain value of K, the physical shortest path always exists among the K wavelength paths, which we refer to as K_{sat}, and the routing is completely converged to pure physical shortest path routing. This analysis indicates that up to K_{sat}, the objective function varies and after K_{sat} the objective function remains constant. Thus, the Greedy-RWSA algorithm is required to execute up to K_{sat}. In this section, The K_{sat} value can be estimated as follows.

Suppose we know the distribution of availability of a wavelength slot i on a link, p_i. We assume that the load is equally distributed in the network, and thus, the distribution of availability of wavelength slots is the same for all links. We denote Q_i as the probability of availability of average consecutive wavelength slots (average requested spectrum) starting from a wavelength slot i, which can be determined as follows.

$$Q_i = \prod_{j=i}^{i+\lceil \frac{x}{\delta} \rceil} p_j, \tag{23}$$

where x is the average spectrum of the requested line rates. If the average physical shortest path distance is h_{avg}, then probability of availability of a shortest path at a wavelength slot i, α_i can be determined as follows.

$$\alpha_i = Q_i^{h_{avg.}} \tag{24}$$

We evaluate α_i for $1 \leq i \leq W$, and finally, we select the value of i with high probability of availability of a physical shortest path, β. The value of i estimates the upper bound on the number of wavelength paths, K_{sat}. Here, we count all wavelength slots up to the i^{th} slot as the number of wavelength paths, but in reality, only those wavelength slots are counted on which there exist at least a path connecting source and destination. Thus our approach for the estimation of K is more conservative, and we consider it as an upper bound.

The estimation of K highly depends on the distribution of the availability of the wavelength slots. Since the Greedy-RWSA algorithm, selects lower wavelengths with high probability, the probability of availability of a wavelength slot increases with the index of a wavelength slot. Thus intuitively, we can conclude that the distribution of availability of a wavelength slot is increasing, but the actual distribution depends on the network topology, the set of traffic demands, the average degree of a node, the variance of the degree of a node, number of paths between a pair of nodes, and the average path length.

Thus, the following formula estimates the worst case required number of wavelength paths.

$$K = \min\left(\{i|\alpha_i \geq \beta\}, |\lambda| \times \max_i \lceil \frac{x_{l_i}}{\delta} \rceil\right) \tag{25}$$

5.2.2. K-Alternate Paths Routing, Wavelength Assignment, and Spectrum Allocation Algorithm (KPaths-RWSA)

In this section, we propose the KPaths-RWSA algorithm. After finding the new set of requests, λ, using the rate selection algorithm, and ordering the set of request according to the cost function U, the KPaths-RWSA algorithm solves the routing and channel allocation subproblems at the same time for each request, $R^{'}$, by confining the routing of a request among the first K shortest paths. In the Greedy-RWSA algorithm, K paths are found on first available K wavelengths, and no wavelength is repeated more than once, while in the KPaths RWSA algorithm, K physical shortest paths are found, and a path is selected which has the lowest available wavelength. Unlike the Greedy-RWSA algorithm, a wavelength may be repeated on more than one path in the KPaths-RWSA algorithm. The KPaths-RWSA first finds K alternate physical shortest paths using a well known K-Shortest Paths algorithm, denoted as $K - ShortestPath(G(\mathbf{V}, \mathbf{E}), s, d, K)$, which takes as an input the given physical topology, $G(\mathbf{V}, \mathbf{E})$, source node s, destination node, d, and number of physical shortest paths, K, and returns K physical shortest paths. The shortest paths are stored in $\mathbf{P}_i$, where $1 \leq i \leq K$. The spectrum availability information for each wavelength slot j on an edge e along the path i is denoted as $Z^j_{e_i}$. In the next step, the algorithm finds the spectrum availability information, Y^j_i, on each wavelength slot j along path i. $Y^j_i = 1$ if the wavelength slot is available on each link e_i of path i, otherwise $Y^j_i = 0$. After finding the spectrum availability information for all K paths, $\lceil \frac{x_{l_i}}{\delta} \rceil$ number of consecutive wavelength slots (equivalent to the required spectrum by the line rate $l_i \in L$) are searched starting from the lowest wavelength slot on the spectrum availability profile, Y^j_i, of each path i. If $\lceil \frac{x_{l_i}}{\delta} \rceil$ number of consecutive wavelength slots are available at some wavelength slot, then the wavelength slot is stored in q_i. Finally a path i is selected on which the available wavelength slot is minimum. In case of a conflict, the shortest path is selected.

Algorithm 5.3: KPATHS-RWSA(R, Z_e^w, W)

```
K − ShortestPath(G(V, E), s, d, K)
I ← 1
for i ← 1 to K
  do {
    q_i ← ∞
    for j ← 0 to W
      do {
        for each e_i ∈ P_i
          do {
            if Z_{e_i}^j × I = 1
              then { I ← 1
              else { I ← 0
          }
        if I = 1
          then { Y_i^j ← 1
          else { Y_i^j ← 0
      }
  }
I ← 1
for i ← 1 to K
  do {
    for j ← 1 to W
      do {
        for m ← j to j + ⌈x_l / δ⌉
          do {
            if Y_i^m × I = 1
              then { I ← 1
              else { I ← 0
          }
        if I = 1
          then { q_i ← j
      }
  }
for i ← 1 to K
  do {
    if min < q_i
      then { min ← q_i
             k ← i
  }
return (P_k,  min)
```

5.2.3. Shortest Path Routing, Wavelength Assignment, and Spectrum Allocation (SP-RWSA)

In this section, we propose the SP-RWSA algorithm. After finding the new set of requests, λ, using the rate selection algorithm, and ordering the set of request according to the cost function U, the SP-RWSA algorithm solves the routing and channel allocation subproblems at the same time for each request, $R^{'}$, by confining the routing of a request on the shortest path. The pseudocode for the SP-RWSA algorithm is the same as the pseudocode of the KPaths-RWSA algorithm when $K = 1$.

5.3. Complexity Analysis of the RWSA Algorithms

In this section, we analyze the time complexity of the Greedy-RWSA, KPaths-RWSA, and SP-RWSA algorithms. For a given graph, $G(\mathbf{V},\mathbf{E})$, where $\mathbf{V}$ is a set of node and $\mathbf{E}$ is a set of edges, a given set of requests, $\mathbf{\Lambda}$, and a given set of line rates, $\mathbf{L}$, supported by the FWDM network, the asymptotic time complexity can be determined as follow.

If the maximum data rate supported by the FWDM network is γ, the total time required for the rate selection algorithm is $\gamma|\mathbf{L}|$. Complexity of sorting the new set of requests λ, obtained through rate selection algorithm, is $|\lambda| log|\lambda|$. The worst case number of wavelength slots required to ensure a non-blocking system is $W = \max_{l_i \in L}\lceil\frac{x_{l_i}}{\delta}\rceil|\lambda|$. Since the Greedy-RWSA algorithm searches for $\rho = \lceil\frac{x_{l_i}}{\delta}\rceil$ consecutive wavelength slots starting from each wavelength slot for all links in the graph, the complexity of constructing an auxiliary graph is $O(\rho|\mathbf{E}|)$. Once the auxiliary graph is constructed, the connectivity between the source and destination is checked using the BFS algorithm. The complexity of the BFS algorithm at each wavelength slot is $O(|\mathbf{V}| + |\mathbf{E}|)$. This operation is performed until K wavelength paths are found. Thus the total complexity of the RWSA algorithm is $O(K[(\rho+1)|\mathbf{E}| + |\mathbf{V}|] + \gamma|\mathbf{L}| + |\lambda| log|\lambda|)$. In the worst case, the number of wavelength paths K is $W - \rho$.

In the KPaths-RWSA algorithm, the complexity of finding the shortest K paths is $O(|\mathbf{E}| + |\mathbf{V}| log|\mathbf{V}| + K)$ [20]. After finding the K-shortest paths, the complexity of generating the spectrum availability profile for each path is $O(|\mathbf{E}||\mathbf{W}|)$. The complexity of finding the requested wavelength slots is $O((W - \rho)\rho)$. Thus the total complexity is $O(|\mathbf{E}| + |\mathbf{V}| log|\mathbf{V}| + KW|\mathbf{E}| + K(W - \rho)\rho + \gamma|\mathbf{L}| + |\lambda| log|\lambda|)$.

The complexity of the SP-RWSA algorithm is the same as the KPaths RWSA algorithm when $K = 1$, which is $O(|\mathbf{V}| + |\mathbf{E}| + W|\mathbf{E}| + (W - \rho)\rho + \gamma|\mathbf{L}| + |\lambda| log|\lambda|)$. Among the proposed algorithms, the complexity of the SP-RWSA algorithm is the lowest, and the complexity of the KPaths-RWSA algorithm is the highest.

5.4. Lower Bound

To analyze the performance of the proposed algorithms, we analyze the lower bound on the required spectrum for a given physical topology and a set of requests using the concept of a cut theory [21]. Let us consider a given network topology, $G(\mathbf{V},\mathbf{E})$, and a set of requests, $\mathbf{\Lambda}$. The rate selection algorithm finds a new set of requests, λ, in which a request is defined as $R^{'}(s,d,l_i)$, where $l_i \in \mathbf{D}^{\gamma}$=RSA ($\gamma$), from a given request $R(s,d,\gamma) \in \mathbf{\Lambda}$. We already prove that $\mathbf{D}^{\gamma}$ is the optimum set of line rates for which the required spectrum to support the data rate γ is minimum. Let us denote $\lambda^{\theta} \subseteq \lambda$ as the set of requests crossing the cut θ. N_{θ} is the number of edges crossing the θ cut. The minimum required spectrum, S can be given by the following formula.

$$S = \max_{\forall\theta} \lceil \frac{\sum_{R'(s,d,l_i)\in\lambda^{\theta}} x_{l_i}}{N_{\theta}} \rceil, \tag{26}$$

where the numerator is the total spectrum required by all the requests crossing the cut θ.

Theorem 4: The amount of spectrum, required by the set of requests $\mathbf{\Lambda}$ on a given topology, $G(\mathbf{V},\mathbf{E})$ is at least S.

Lemma 1: The amount of spectrum required by any set of line rates for a given request $R(s,d,\gamma)$ is at least the the amount of spectrum required by a set of line rates, $\mathbf{D}^{\gamma}$, obtained through rate selection algorithm RSA.

Proof: Since the RSA algorithm finds the optimal set of line rates for which the required spectrum is minimum for the given data rates, the Lemma 1 is obviously true. ■

Lemma 2: The maximum spectrum per link is minimized if the given spectrum is distributed equally among all links across the cut.

Proof: If, T is the total spectrum we want to distribute among N links, then the amount of spectrum allocated per link, w_i, through equal distribution is $\frac{T}{N}$, such that $\sum_{i=1}^{N} w_i = T$. Let's consider some distribution which requires the maximum spectrum per link, $w_i^{'}$ less than $\frac{T}{N}$. But $\sum_{i=1}^{N} w_i^{'} < T$, which violates the given condition. Thus the maximum spectrum per link is minimized with equal distribution of spectrum among links. ■

Lemma 3: The amount of spectrum required in the network with wavelength and spectral continuity constraint is at least the amount of spectrum required with relaxed wavelength and spectral continuity constraint.

Proof: The problem with the relaxed wavelength and spectral continuity constraints is more generalized than the problem with wavelength and spectral continuity constraints. Since any solution in the network with wavelength and spectral continuity constraints is also the solution of the network with relaxed wavelength and spectral continuity constraints, the Lemma 3 is proved. ■

Thus, we first use the RSA algorithm to find the optimum required spectrum for the given requested data rates, and then this spectrum is equally distributed among the links across the cut with the relaxation of the wavelength and spectral continuity constraints. Since the procedure follows the Lemma 1, 2, and 3, the required spectrum across the cut is minimum. Finally we find the maximum spectrum among all possible cuts, which is the lower limit on the spectrum required by the given instance of the problem. ■

The proposed solutions are applicable to FWDM networks irrespective of the granularity at which the spectrum is slotted. Furthermore, the algorithms can also be applied to the special cases such as RWA problem with mixed and single line rate systems in fixed grid networks.

6. Numerical Results

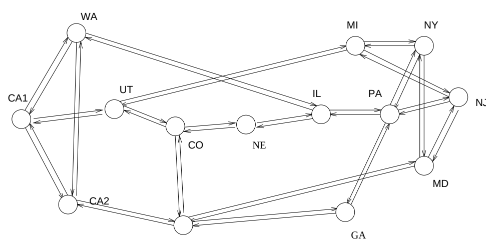

Figure 3: 14-node NSFNET topology

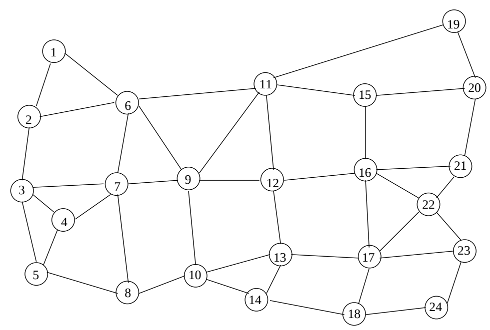

Figure 4: 24-node NSFNET topology

We simulate the proposed algorithms on a 6-node network (as shown in Fig. 2), the 14-node NSF network, and the 24-node NSF network topologies. Requests are uniformly distributed over the network, and requested data rates are uniformly distributed between 1 Gb/s to 1 Tb/s with any granularity. The network can support 10 Gb/s, 40 Gb/s, 100 Gb/s, 400 Gb/s, and 1 Tb/s line rates with required spectral width of 25 GHz, 50 GHz, 50 GHz, 75 GHz [3], 150 GHz [5] respectively, and the cost of transponders operating at respective line rates are assumed to be 1x, 2.5x, 3.75x [16], 5.5x, and 6.75x respectively. The estimated power consumption of the 10 Gb/s, 40Gb/s, 100 Gb/s, 400 Gb/s, and 1Tb/s transponders are assumed to be 47 W [22],167 W [23], 300 W, 450 W, and 550 W respectively. The cost, and energy parameters for the 400 Gb/s and 1 Tb/s are obtained by extending the current trend of cost and energy of the standardized line rates using a logarithmic function. We assume that sufficient spectrum is available to support the network traffic, traffic between the same source-destination pairs is aggregated (groomed) at source nodes, and transponders are tunable to any wavelength. For comparison purposes, we use the proposed algorithms for a fixed grid network in which the line rates of lightpaths are identical.

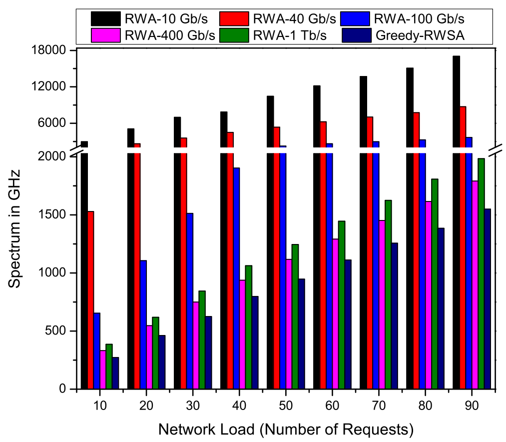

Figure 5: Spectral width vs. Load.

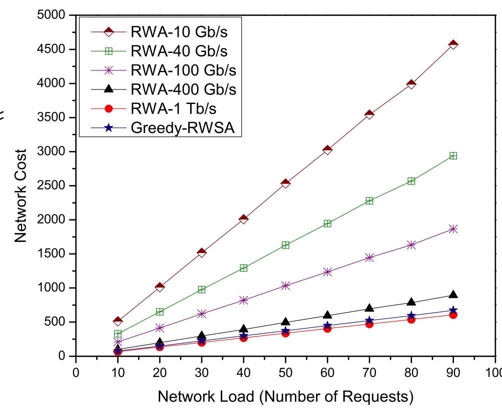

Figure 6: Cost vs. Load.

The required spectrum by the FWDM network, obtained using the Greedy-RWSA algorithm, and the required spectrum for fixed grid networks with various line rates are compared in Fig. 5 for the 14-node NSF network (shown in Fig. 3). Spectrum utilization in the FWDM network is improved by 91%, 82%, 58%, 17%, and 29% in the worst case compared to 10 Gb/s, 40 Gb/s, 100 Gb/s, 400 Gb/s, and 1 Tb/s fixed grid networks respectively. The reason is that as the requested data rate increases, the selection of higher line rate achieves spectral gain compared to selecting multiple lower line rates. Similarly for requests with lower data rate, selection of a lower line rate increases the spectral utilization. Since the FWDM network establishes lightpaths at any line rate from the given set, the spectral efficiency is improved significantly. Since 400 Gb/s is the closest line rate to the average requested data rate (500 Gb/s), the required spectrum in the 400 Gb/s-fixed grid network is the closest to the required spectrum in the FWDM network.

Additionally, we observe a significant cost improvement in the FWDM network compared to fixed grid networks for the 14-node network topology as shown in Fig. 6. The cost in the FWDM network is improved by 85%, 77%, 64%, and 26% in the worst case compared to 10 Gb/s, 40 Gb/s, 100 Gb/s, and 400 Gb/s fixed grid networks respectively. However, the network cost in the FWDM network is increased by only 10% compared to the 1 Tb/s fixed grid network. The reason is that the 1 Tb/s fixed grid network requires the fewest transponders, and the cost per unit capacity of the 1 Tb/s transponder is less than that of any other line rates. In this experiment, we only consider the cost of the transponder, since the cost of the transponder is sensitive to the line rate. Figure 7 compares the energy consumption of the FWDM network with that of fixed grid networks for the 14-node network topology. The energy consumption in the 10 Gb/s, 40 Gb/s, 100 Gb/s, and 400 Gb/s is higher than the energy consumption of the FWDM network by 75%, 72%, 63%, and 27% in the worst case. The FWDM Network consumes 9% more power than the 1 Tb/s fixed grid network. The reason is that the power consumption per unit capacity of the 1 Tb/s transponder is less than the 10 Gb/s, 40 Gb/s, 100 Gb/s, and 400 Gb/s transponders, and the 1 Tb/s fixed grid network requires the fewest transponders compared to the all other networks. The cost and power of the optical switches and amplifiers is fixed and is less sensitive to the selection of line rate. Thus, we omit the cost and power of intermediate switches and amplifiers.

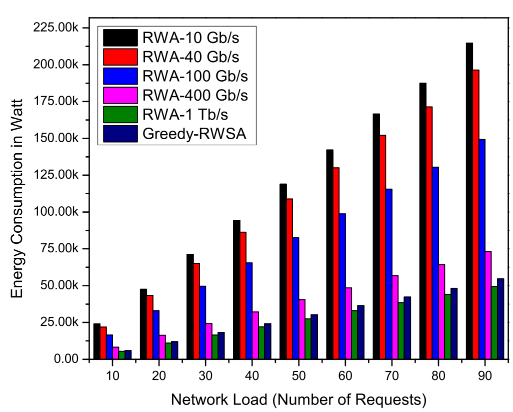

Figure 7: Power consumption vs. Load.

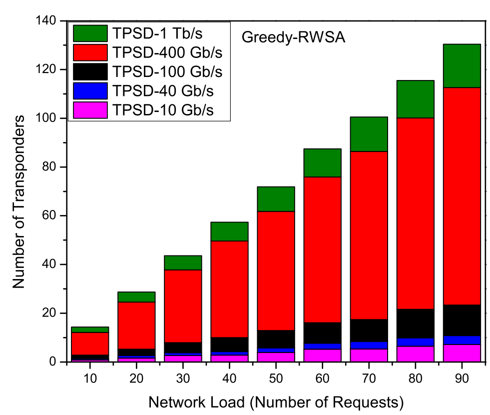

Figure 8: Number of transponders vs. Load.

Figure 8 demonstrates the average number of transponders of type 10 Gb/s, 40Gb/s, 100 Gb/s, 400 Gb/s, and 1 Tb/s used in the Greedy-RWSA algorithm. More than 65% of the transponders are of type 400 Gb/s because the average requested data rate is 500 Gb/s. An average of 15% of transponders are of type 1 Tb/s, and an average of 20% of the transponders are of types 10 Gb/s, 40 Gb/s, and 100 Gb/s.

To evaluate the performance of the proposed algorithms, we compare the required spectral width in the Greedy-RWSA approach, KPaths-RWSA approach, and SP-RWSA approach with the ILP and lower bound in Fig. 9 for 6-node network topology (shown in Fig. 2). At low load, the spectral width difference of the lower bound and the ILP is up to 15%, and as load increases, this difference decreases. At high load, the spectral width difference is reduced up to 2%, which indicates that the proposed lower bound is tight enough to evaluate the performance of the algorithms for the larger instance of the problem. The spectral width of the Greedy-RWSA, and KPaths-RWSA algorithms are within 9% of the ILP results, and, as the load increases, this difference is also decreased to 3%. The performance of the Greedy-RWSA and KPaths-RWSA algorithms are very similar. The SP-RWSA algorithm is within 30% of the ILP results. The results indicate that the performance of the proposed algorithms is very close to the optimal results.

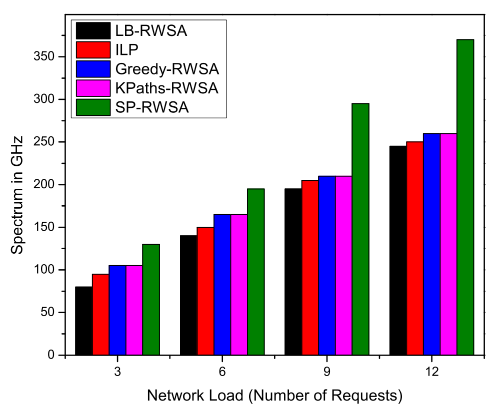

Figure 9: Spectrum (6-node net.)

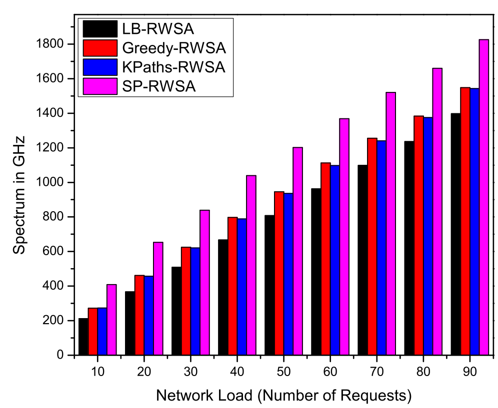

Figure 10: Spectrum (14-node net.)

Figure 10 compares the required spectral width in the proposed algorithms as the load increases for the 14-node NSFNET. The required spectrum in the SP-RWSA algorithm is higher than the required spectrum in other approaches. The reason is that in the SP-RWSA algorithm, each request is routed on the fixed shortest path. Thus, even though there exist other paths on which some of the spectrum along the shortest path can be reutilized, due to fixed routing scheme, the SP-RWSA algorithm cannot utilize the existing spectrum efficiently. Additionally, due to higher spectral utilization of the bottleneck links, the spectral continuity constraint, and spectral conflict constraint, probability of finding requested spectrum on the shortest paths is low at the lower wavelengths in the network. Thus, as load increases, the new request is most likely scheduled on the new unutilized spectrum in the network. On the other hand, in the Greedy-RWSA, a combination of adaptive routing and shortest path routing is used. The adaptive routing does not prefer a path that requires to reserve new spectrum if there exist some available paths on which existing spectrum can be utilized. Thus the objective of the adaptive routing is to maximize the utilization of the spectrum. Additionally, to minimize the over-utilization of a spectrum by longer paths, the Greedy-RWSA algorithm selects a path with shorter physical distance. Similarly, in the KPaths-RWSA algorithm, a path is selected out of K shortest paths on which the requested spectrum is available at lower wavelengths. The utilization of spectrum in the KPaths-RWSA algorithm is also high. Thus the required spectrum in the Greedy-RWSA and KPaths-RWSA algorithm is higher than the SP-RWSA algorithm. The Greedy-RWSA and KPaths-RWSA algorithms are within 22% of the lower bound at low load and as the load increases this difference decreases to 9%.

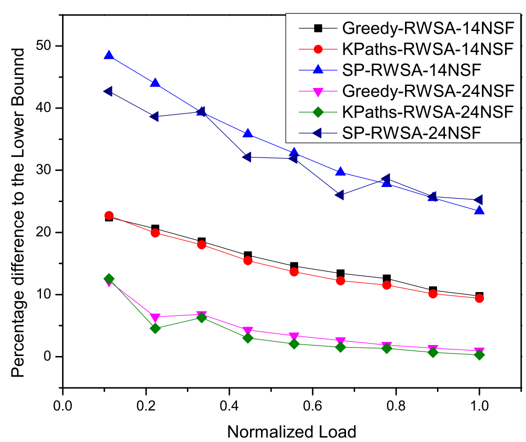

Figure 11: % difference to the lower bound.

The percentage difference between the proposed algorithms and the lower bound is shown in the Fig. 11 for the 14-node NSF and 24-node NSF networks (shown in Fig. 4)as load increases. As the load increases, the percentage difference between the proposed algorithm and the lower bound decreases for both network topologies, which indicates that the proposed algorithms are approach the lower bound as load increases. Additionally, as the network size increases from 14 nodes to 24 nodes, the percentage difference between the proposed algorithm and the lower bound further decreases significantly.

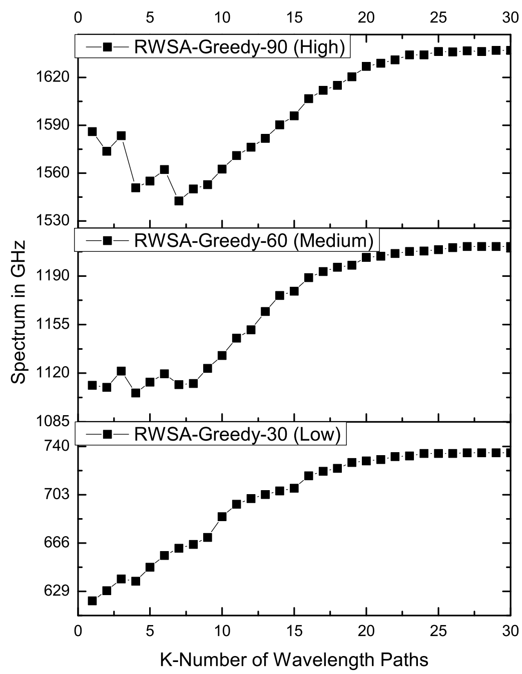

Figure 12: Wavelength path sensitivity.

Figures 12 shows the spectral width in the Greedy-RWSA algorithm as the number of wavelength paths increases for the 14-node NSF network at the low, moderate, and high loads respectively. When K is equal to 1, then the routing is pure adaptive routing, and as K increases, the routing is tending towards the shortest path routing. When the spectrum reaches at saturation point then the routing becomes pure shortest path routing. At low load, the minima occurs at very low value of K, which indicates that at low load pure adaptive routing can minimize the required spectrum in the network by routing and allocating spectral resources evenly over all links in the network.

As K increases, the probability of routing a connection over a shortest path increases, which leads to an uneven distribution of the occupied spectral resources at a given traffic load in the network. Thus, essentially the required spectrum in the network increases with K. On the other hand, at moderate and high load, we observe the non-monotonic nature of the required spectrum as K increases. The reason is that at low value of K, the algorithm selects a route among the available routes on the lowest K wavelengths. This route may occupy a large amount of spectrum if the length of a route in terms of number of hops is high. As K increases, the likelihood of the algorithm selecting a shortest route increases, which essentially reduces the amount of spectrum occupied by the connections, and thus, we observe non-monotonic nature of the spectrum at lower value of K. On the other hand, after certain value of K, we observe monotonically increasing nature of the required spectrum as K increases, since the distribution of the occupied spectrum becomes uneven thereafter. Thus, at moderate and high load, the minima occurs between $K = 1$ and $K = K_{sat}$, which indicates that at moderate and high loads, some composition of adaptive and shortest path routing can minimize the required spectrum. Hence, the optimum value of K balances the spectral resource consumption and distribution of occupied spectrum by the given set of traffic demands in the network. We also observe that as the load increases, the average distance in terms of hop count decreases in the Greedy-RWSA approach, which indicated that the pure adaptive routing is converging towards the shortest path routing.

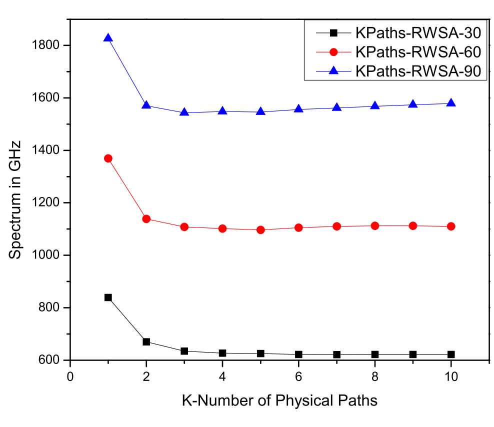

Figure 13: Number of physical paths

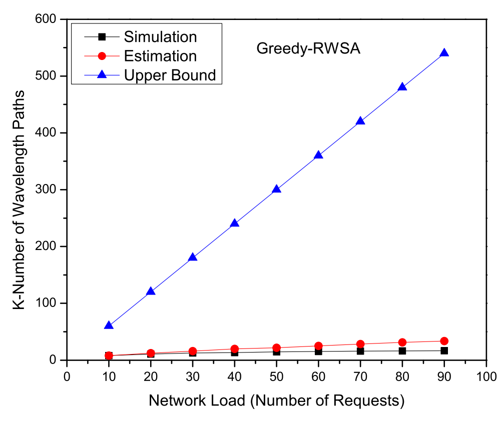

Figure 14: Number of wavelength paths.

Figures 13 shows the spectral width in the KPaths-RWSA algorithm as the number of physical paths increases for the 14-node NSF network at low, moderate, and high loads. As K increases, the spectrum decreases, and after certain value of K, the spectrum reach at saturation point. The results indicates that the $K = 5$ are sufficient to obtain the minimum spectrum in the KPaths-RWSA algorithm.

The theoretical estimation of the number of wavelength paths in the Greedy-RWSA algorithm is compared with the experimental value and the upper bound in Fig. 14 for 14-node NSFNET topology. We observe that the distribution of the spectrum availability is increasing approximately linearly as the index of the wavelength slot increases for the 6-node network, 14-node NSF network, and 24-node NSF network. We denote the distribution of spectrum availability as $Y = aX + b$, where X represents an index of the wavelength slot, and Y represents the probability of availability of the wavelength slot. The coefficients a and b are changing with the load. The results shows that the theoretical estimation of number of wavelength paths improves the estimation by 86% from the worst case upper bound.

7. Conclusion

We introduce the flexible optical WDM network architecture for the next generation network, and address the routing, wavelength assignment, and spectrum allocation problem in the transparent FWDM network. We prove the NP-completeness of the RWSA problem, and We propose three algorithms, namely Greedy-RWSA algorithm, KPaths-RWSA algorithm, and SP-RWSA algorithm. We also analyze the lower bound for the same problem. The simulation results demonstrate that the FWDM network is efficient in terms of spectrum, cost, and energy compared to fixed grid networks. Among the proposed algorithms, the KPaths-RWSA algorithm provides a more optimal solution at the cost of higher time complexity. On the other hand, the time complexity of the Greedy-RWSA algorithm is lower than that of the KPaths-RWSA algorithm, but the spectral optimization is marginally sacrificed. The performance of the proposed algorithms is close to the lower bound, and approaches to the lower bound as the network size and traffic load increases which verifies the efficiency of the algorithms. The proposed solutions efficiently utilize the the available spectrum, and thus enhances the traffic load carrying capacity of the flexible optical WDM networks when applied into the control plane of the flexible optical WDM networks.

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