Safety Inventories

Chapter 11 of Chopra

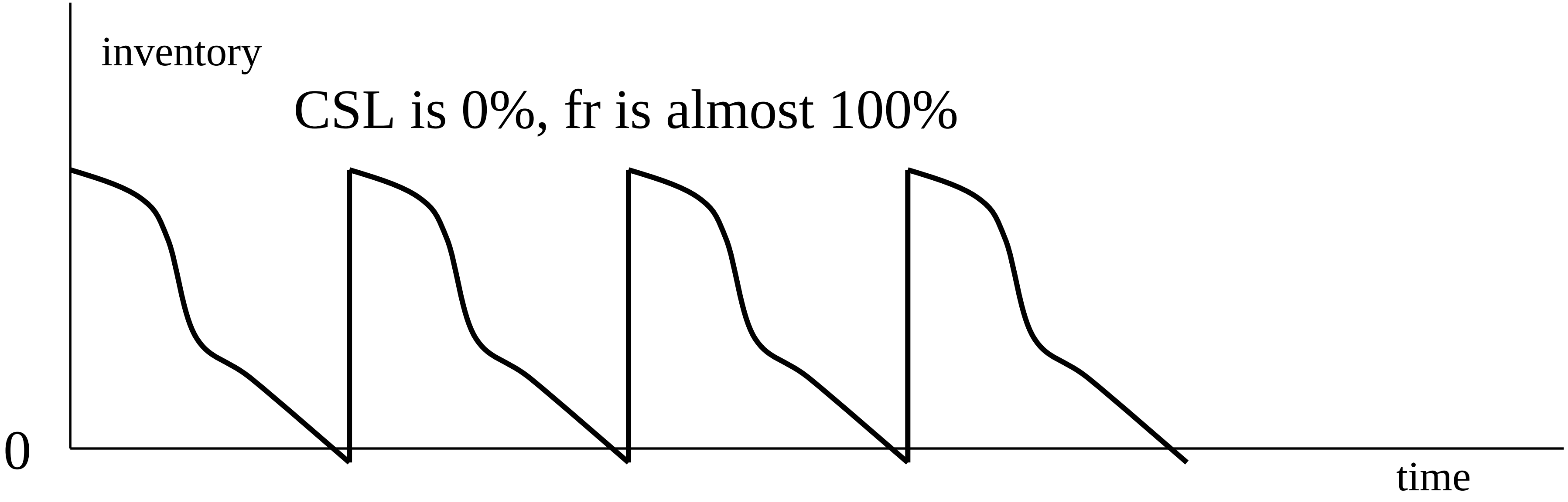
Why to hold Safety Inventory?

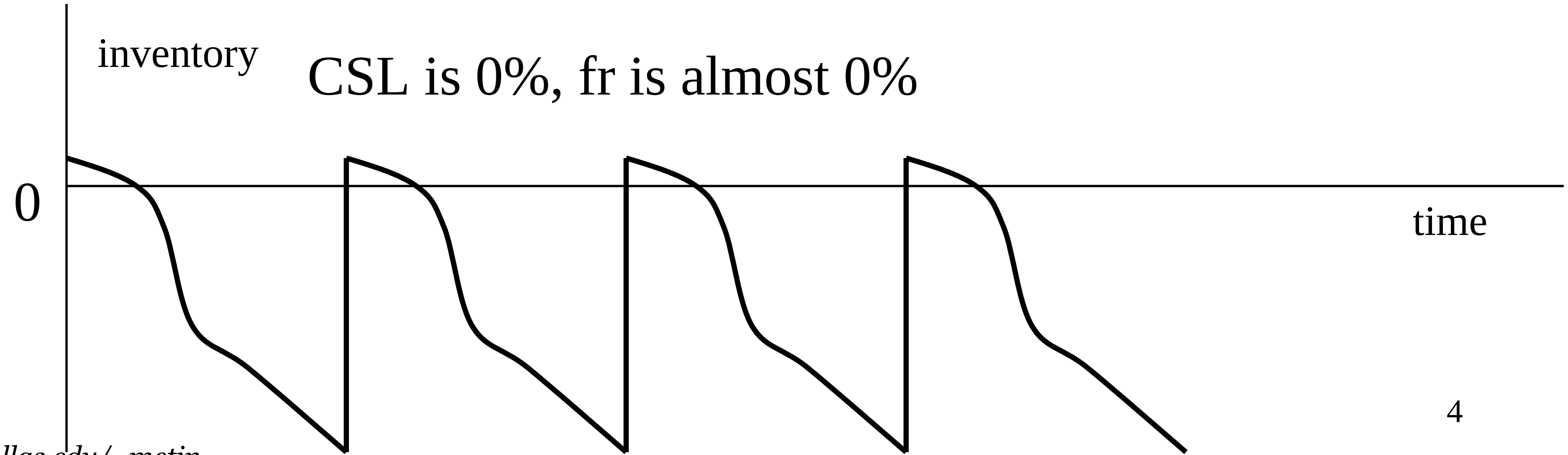
- Desire for product availability
  - Ease of search for another supplier
  - "I want it now" culture
- Demand uncertainty
  - Short product life cycles
- Safety inventory

Measures

- Measures of demand uncertainty
  - Variance of demand
  - Ranges for demand
- Delivery Lead Time, L
- Measures of product availability
  - Stockout?
    - Backorder (patient customer, unique product or big cost advantage) or Lost sales.
  - Cycle service level *(CSL)*, % of cycles
  - Product fill rate (*fr*), % of products
  - Order fill rate, % of orders
    - Equivalent to product fill rate if orders contain one product

Service measures: CSL and fr are different





Replenishment policies

- When to reorder?
- How much to reorder?
  - Most often these decisions are related.

Continuous Review: Order fixed quantity when total inventory drops below Reorder Point *(ROP)*.

ROP meets the demand during the lead time L.

One has to figure out the ROP.

Information technology facilitates continuous review.

Demand During Lead time

D_i demand in period i. Mostly $D_i \propto Normal(R_i, \sigma_i^2)$

f_i , F_i probabilit y density and cumulative density functions for D_i

$$R_i = E(D_i) = \int D_i f_i(D_i) dD_i \qquad Var(D_i) = \sigma_i^2 = E\{(D_i - R_i)^2\} = \int (D_i - R_i)^2 f_i(D_i) dD_i$$

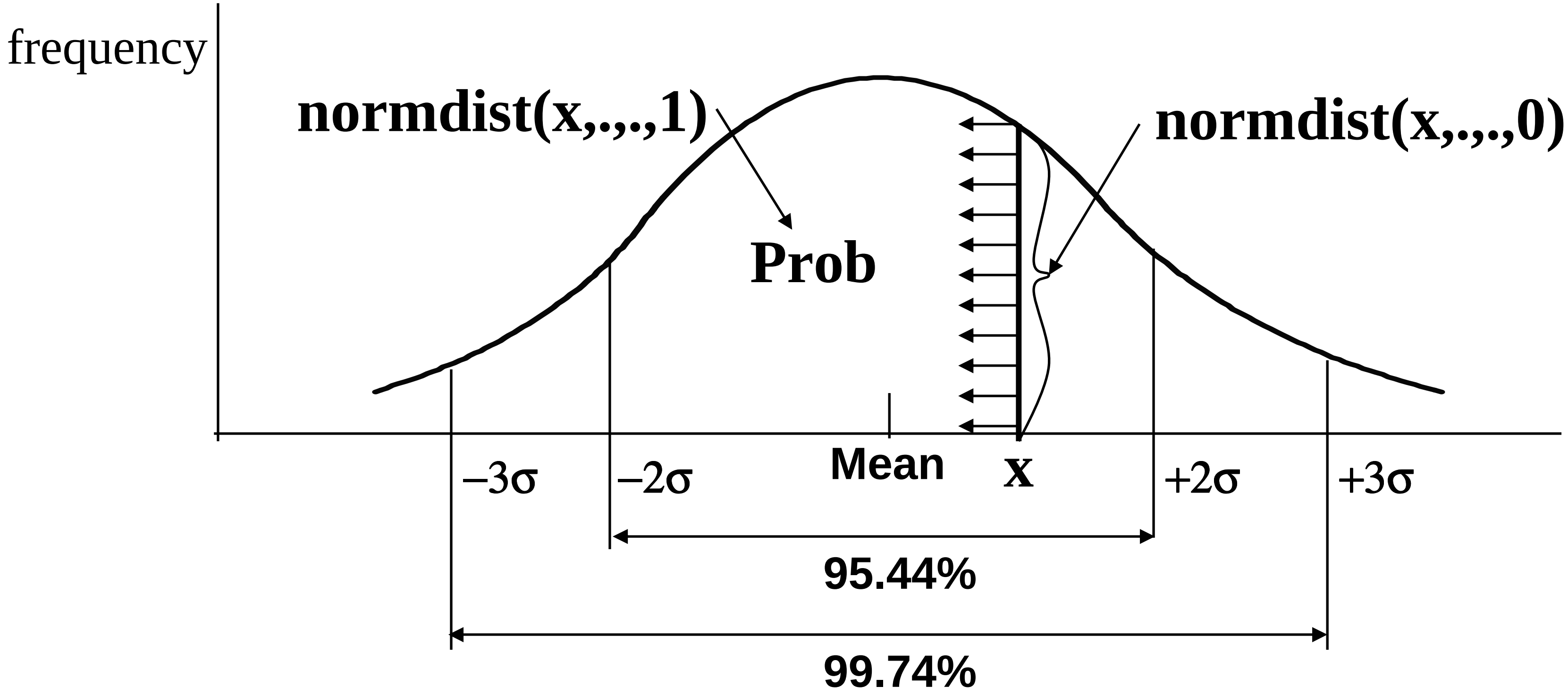
$$\begin{aligned}\operatorname{cov}(D_i, D_j) = \sigma_{i,j}^2 &= E\{(D_i - R_i)(D_j - R_j)\} \\ &= \int\int (D_i - R_i)(D_j - R_j) f_{i,j}(D_i, D_j) dD_i dD_j\end{aligned}$$

$\rho = \sigma_{i,j}^2 / (\sigma_i \sigma_j)$ correlation coefficient

$E(\sum_{i=1}^{L} D_i) = \sum_{i=1}^{L} R_i$ by the linearity of integration

$$Var(\sum_{i=1}^{L} D_i) = \sum_{i=1}^{L} \sum_{j=1}^{L} \operatorname{cov}(D_i, D_j) = \sum_{i=1}^{L} \sigma_i^2 + \sum_{i=1}^{L} \sum_{\substack{j=1 \\ j \neq i}}^{L} \operatorname{cov}(D_i, D_j)$$

Normal Density Function

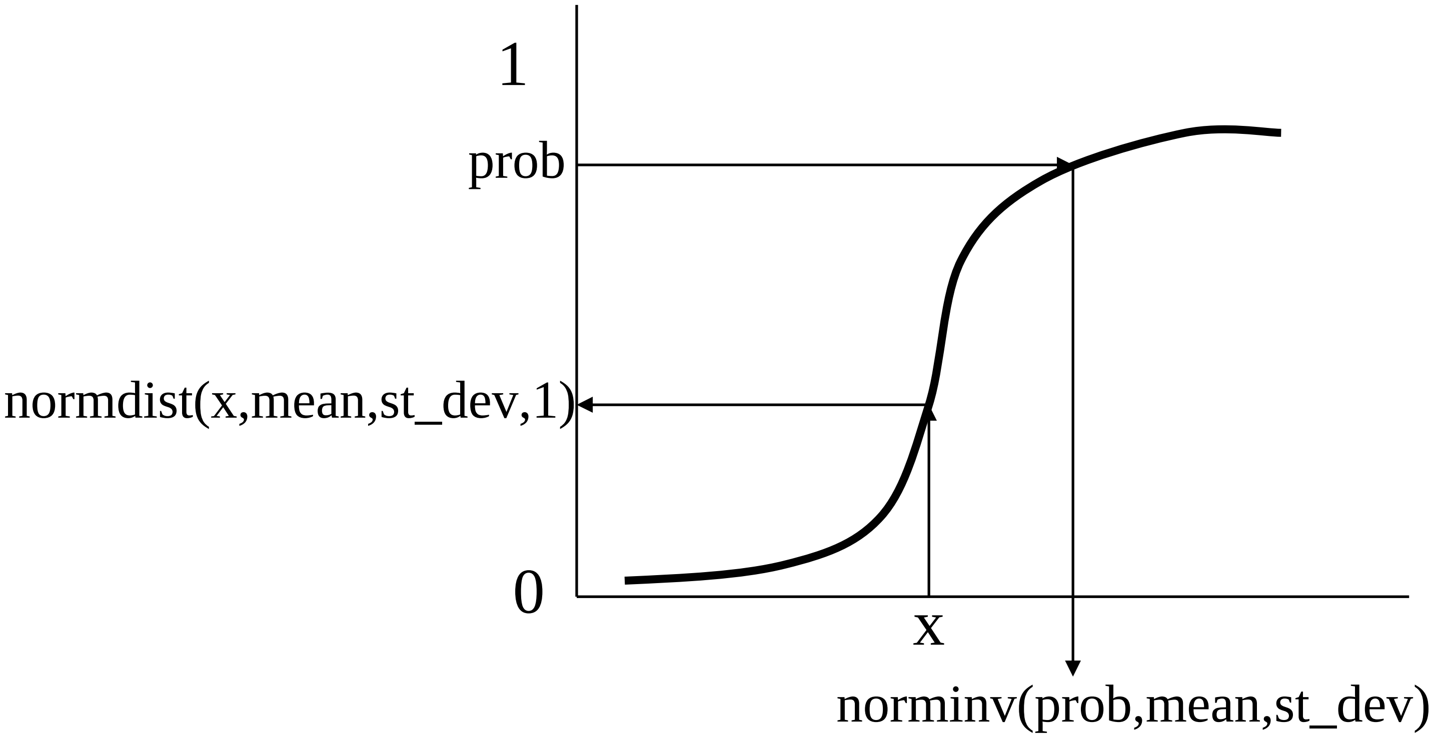


Excel statistical functions :

Density function (pdf) at x : normdist($x, mean, st\_dev$,0)

Cumulative function (cdf) at x : normdist($x, mean, st\_dev$,1)

Cumulative Normal Density



Excel statistical functions :

Cumulative function (cdf) at x : normdist$(x, mean, st_dev, 1)$

Inverse function of cdf at "prob": norminv$(prob, mean, st_dev)$

Demand During Lead Time Determines ROP

Suppose that demands are identically and independently distributed. For identically and independently distributed use iid.

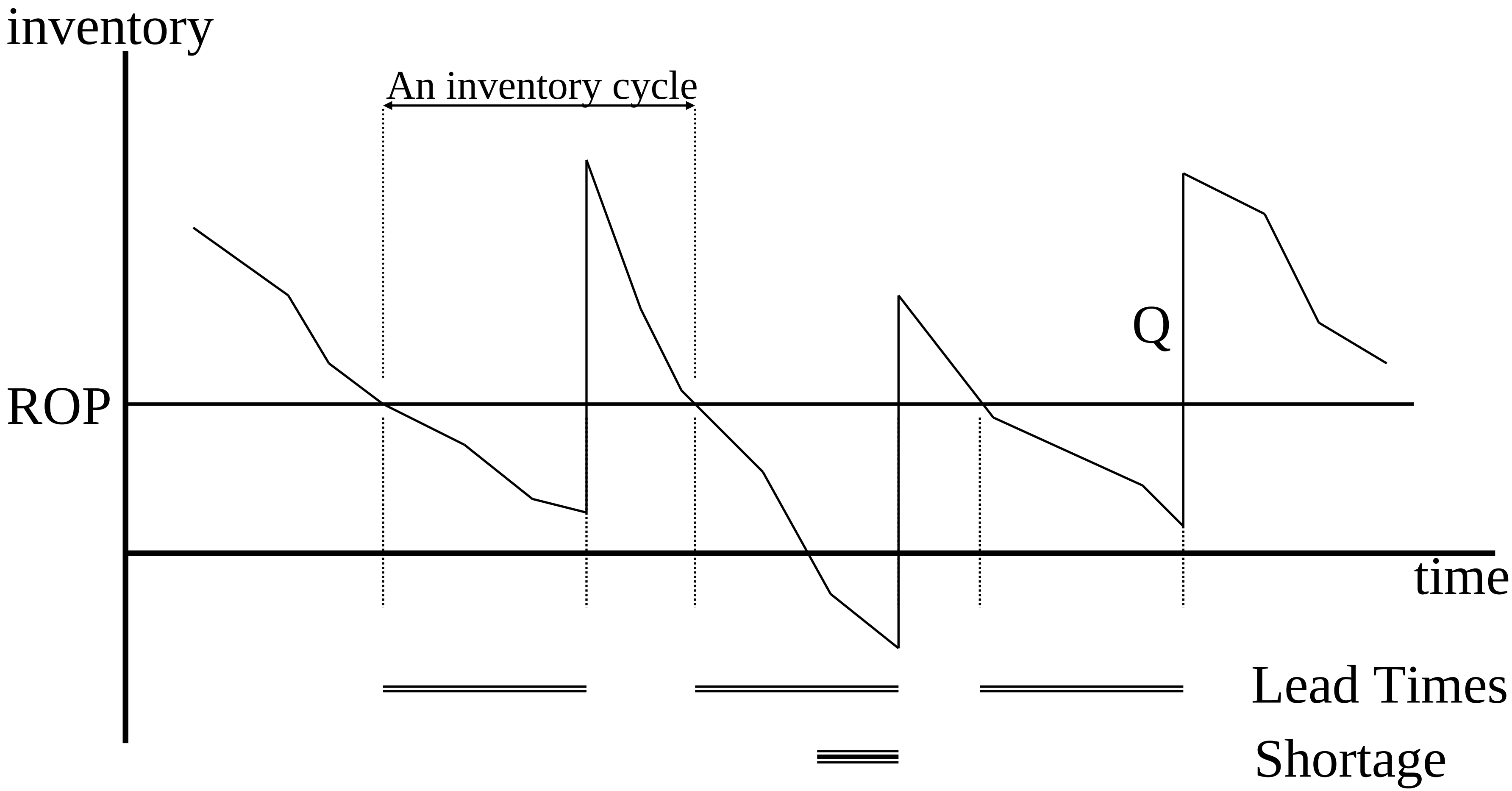
$$E\left(\sum_{i=1}^{L} D_i\right) = LR \quad \text{and} \quad Var\left(\sum_{i=1}^{L} D_i\right) = L\sigma^2$$

$$\text{If } D_i \propto N(R, \sigma^2) \text{ then } \sum_{i=1}^{L} D_i \propto N(LR, L\sigma^2)$$

$$P\left(\sum_{i=1}^{L} D_i \le a\right) = Normdist\left(a, LR, \sqrt{L}\sigma, 1\right) = F\left(a; LR, \sqrt{L}\sigma\right)$$

$$\text{Coefficient of variation of D}: cv = \sqrt{Var(D)} / E(D)$$

Optimal Safety Inventory Levels



Cycle Service Level

Cycle service level: percentage of cycles with stock out

For example consider 10 cycles:

$$CSL = \frac{1+1+0+1+1+1+0+1+0+1}{10} \quad \text{Write 0 if a cycle has stockout, 1 otherwise}$$

$$CSL = 0.7$$

$$CSL = 0.7 = \text{Probability that a single cycle has sufficient inventory}$$

$$[\text{Sufficient inventory}] = [\text{Demand during lead time} \leq \text{ROP}]$$

ROP: Reorder point

$$\boxed{CSL = \text{Cycle Service Level} = F(ROP; R \cdot L, \sqrt{L}\sigma)}$$

Continuous Review Policy: Safety Inventory and Cycle Service Level

$$CSL = F(ROP, R \cdot L, \sqrt{L}\sigma_R)$$

$$ROP = F^{-1}(CSL, R \cdot L, \sqrt{L}\sigma_R)$$

$$\text{safety stock} = ss := ROP - R \cdot L$$

For normally distributed demand:

$$ss = F^{-1}(CSL, R \cdot L, \sqrt{L}\sigma_R) - R \cdot L$$

$$= F^{-1}(CSL; 0, \sqrt{L}\sigma) = F^{-1}(CSL; 0, 1) \cdot \sqrt{L}\sigma$$

The last two equalities are by properties of the Normal distribution.

Average Inventory = Q/2 + ss

Example Continuous Review Policy

***R* = 2,500 /week; σ= 500**

***L* = 4 weeks; *Q* = 10,000; *ROP* = 16,000**

$$\text{Stdev of demand during lead time} = \sigma\sqrt{L} =$$

$ss = ROP - L\,R =$

Average Inventory =

Average Flow Time =Average inventory/Thruput=

Cycle service level, $F(ROP; L \cdot R, \sqrt{L}\sigma) =$

Finding Safety Inventory for given CSL

***R* = 2,500/week; σ= 500**

L* = 2 weeks; *Q* = 10,000; *CSL = 0.90

$$ss = F^{-1}(CSL;0,1)\sqrt{L}\sigma =$$

$$ROP = L \cdot R + ss =$$

Factors driving safety inventory

- **Replenishment lead time**
- **Demand uncertainty**

Towards the fill rate: Expected shortage per cycle

- **ESC is the expected shortage per cycle**
- **ESC is not a percentage, it is the number of units,** **also see next page**

$$\text{Shortage} = \begin{cases} \text{Demand} - \text{ROP} & \text{if} \quad \text{Demand} \geq \text{ROP} \\ 0 & \text{if} \quad \text{Demand} \leq \text{ROP} \end{cases}$$

$$ESC = E(\max\{\text{Demand during lead time - ROP}, 0\})$$

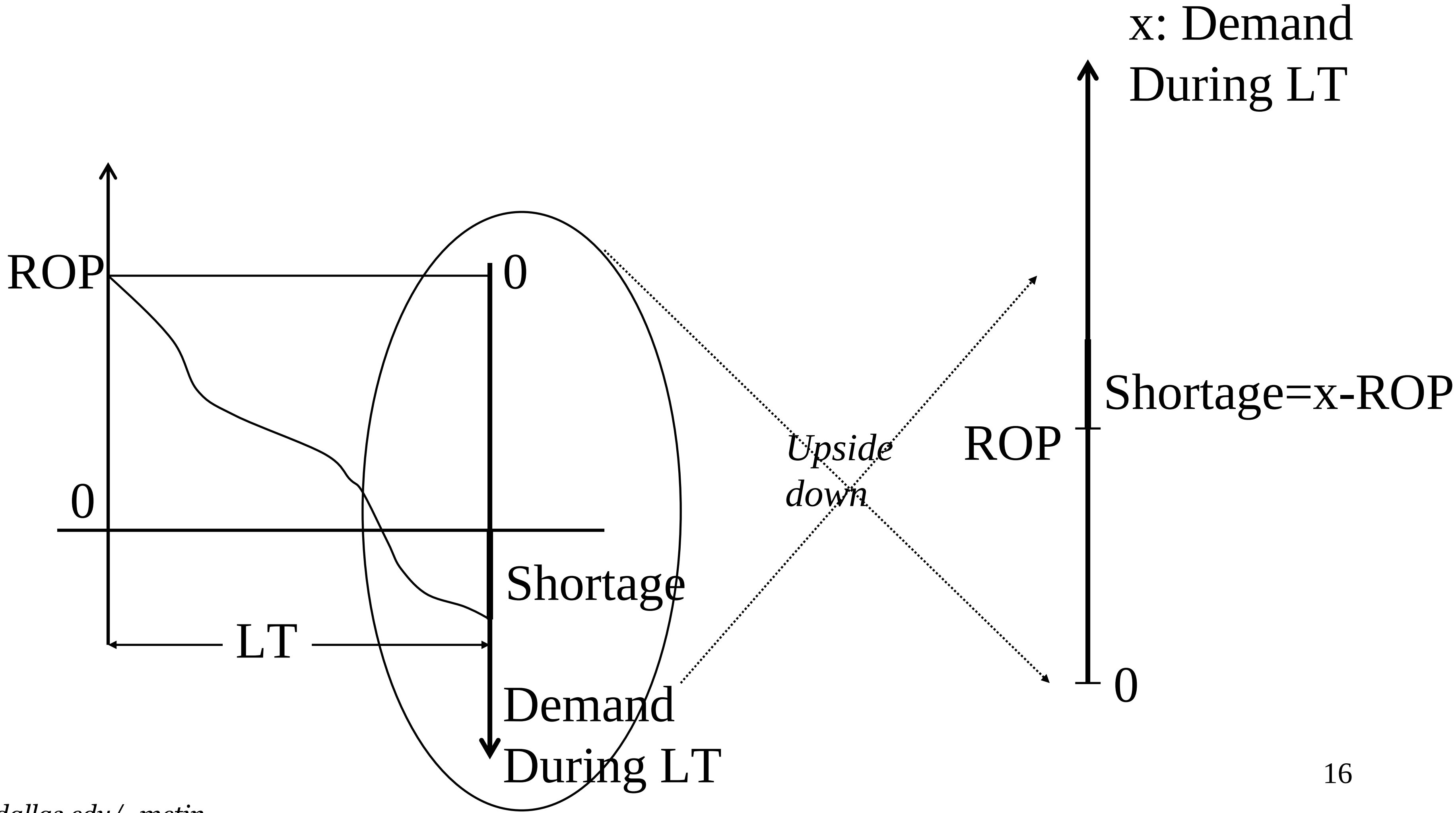
$$ESC = \int_{x=ROP}^{\infty} (x - ROP) f(x) dx$$

- **If demand is normal:**

$$ESC = -ss\left(1 - normdist\left(\frac{ss}{\sqrt{L}\sigma}, 0, 1, 1\right)\right) + \sqrt{L}\sigma \cdot normdist\left(\frac{ss}{\sqrt{L}\sigma}, 0, 1, 0\right)$$

- **Does ESC decrease or increase with ss, L, stdev of Demand?**
- **Does ESC decrease or increase with expected value of demand?**

Shortage and Demand during Lead Time



Fill Rate

- **Fill rate: Proportion of customer demand satisfied from stock**
- ***Q*: Order quantity**

$$\boxed{fr = 1 - \frac{ESC}{Q}}$$

Evaluating Fill Rate

σ= 500; *L* = 2 weeks; ss=1000; *Q* = 10,000;

Fill Rate (*fr*) = ?

$$ESC = -ss\left(1 - normdist\left(\frac{ss}{\sqrt{L}\sigma},0,1,1\right)\right) + \sqrt{L}\sigma \cdot normdist\left(\frac{ss}{\sqrt{L}\sigma},0,1,0\right)$$

$$ESC = -1000(1 - normdist(1000/707,0,1,1)$$

$$+ 707 nomdist(1000/707,0,1,0)$$

$$ESC = 25.13$$

fr = (*Q* - *ESC*)/*Q* = (10,000 - 25.13)/10,000 = 0.9975.

Finding Safety Inventory for a Desired Fill Rate

If desired fill rate is *fr* = 0.975, how much safety inventory should be held?

Clearly ESC = (1 - *fr*)*Q* = 250

Use goal seek of Excel to solve

$$250 = -ss\left[1 - normdist\left(\frac{ss}{707},0,1,1\right)\right] + 707\,normdist\left(\frac{ss}{707},0,1,0\right)$$

Evaluating Safety Inventory For Given Fill Rate

Fill Rate	Safety Inventory
97.5%	67
98.0%	183
98.5%	321
99.0%	499
99.5%	767

Safety inventory is very sensitive to fill rate. Is fr=100% possible?

Factors Affecting Fill Rate

- **Safety inventory: If Safety inventory is up,**
  - **Fill Rate is up**
  - **Cycle Service Level is up.**
- **Lot size: If Lot size is up,**
  - **Cycle Service Level does not change. Reorder point, demand during lead time specify Cycle Service Level.**
  - **Expected shortage per cycle does not change. Safety stock and the variability of the demand during the lead time specify the Expected Shortage per Cycle. Fill rate is up.**

To Cut Down the Safety Inventory

- Reduce the Supplier Lead Time
  - Faster transportation
    - » Air shipped semiconductors from Taiwan
  - Better coordination, information exchange, advance retailer demand information to prepare the supplier
    - » Textiles; Obermeyer case
  - Space out orders equally as much as possible
- Reduce uncertainty of the demand
  - Contracts
  - Better forecasting to reduce demand variability

Lead Time Variability

Supplier's lead time may be uncertain:

$L =$ Average lead time. $\quad s^2 =$ Variance of lead time

$$E(\sum_{i=1}^{L} D_i) = LR \qquad Var(\sum_{i=1}^{L} D_i) = L \cdot \sigma^2 + R^2 \cdot s^2 =: \sigma_L^2$$

The formulae do not change:

$$ss = F^{-1}(CSL;0,1) \cdot \sigma_L = F^{-1}(CSL;0,1) \cdot \sqrt{L\sigma^2 + R^2 s^2}$$

$$ESC = -ss\left[1 - normdist\left(\frac{ss}{\sigma_L};0,1,1\right)\right] + \sigma_L nomdist\left(\frac{ss}{\sigma_L};0,1,0\right)$$

Impact of Lead Time Variability, s

R = 2,500/day; σ= 500

L = 7 days; Q = 10,000; CSL = 0.90

s	ss	Jump in ss
0	1695	-
1	3625	1930
2	6628	3003
3	9760	3132
4	12927	3167
5	16109	3182
6	19298	3189

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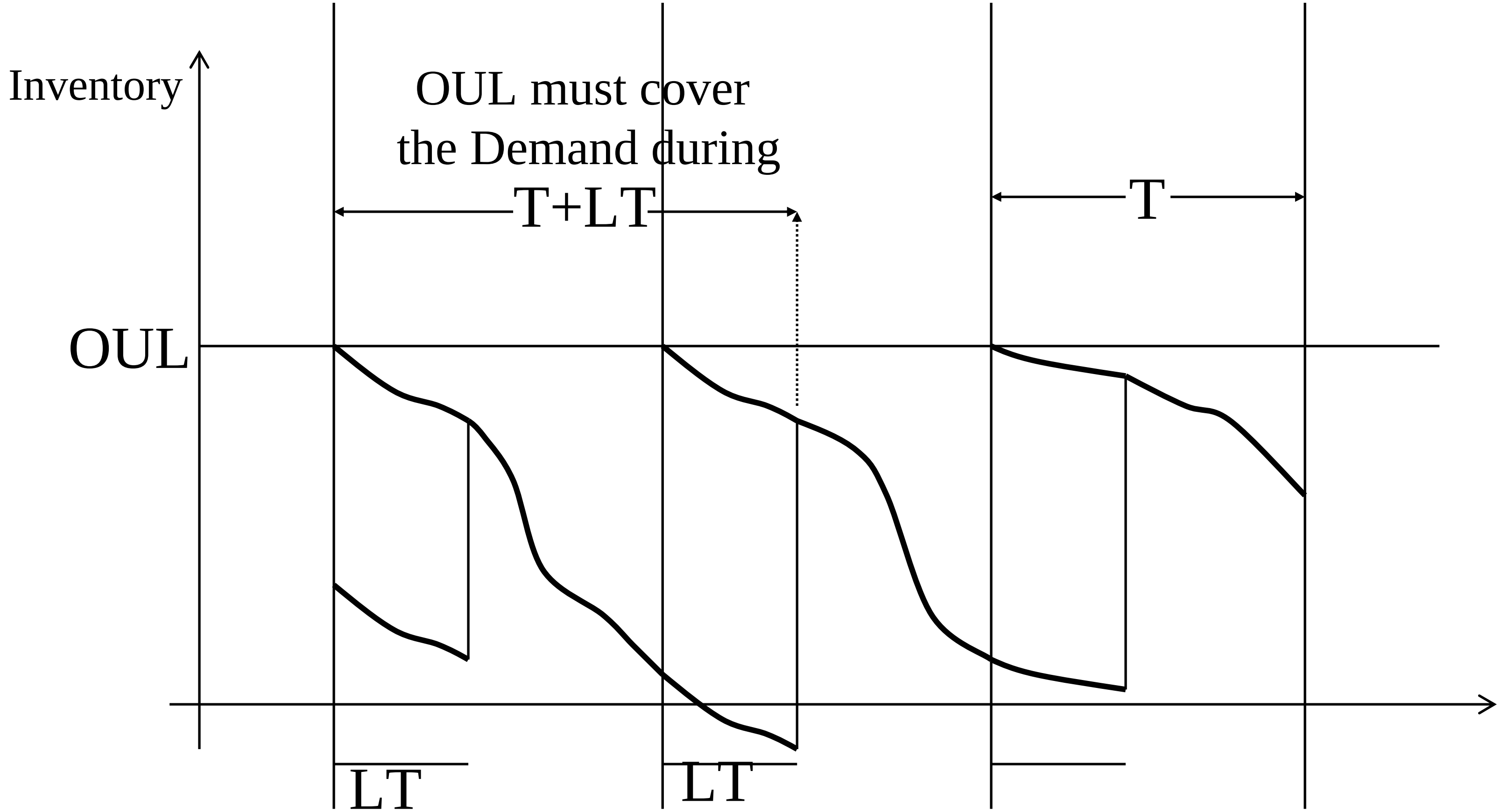
Periodic Review

Order at fixed time intervals (T apart) to raise total inventory (on hand + on order) to Order up to Level *(OUL)*



Periodic Review Policy: Safety Inventory

T: Reorder interval

σ_R: Standard deviation of demand per unit time

σ_{L+T}: Standard deviation of demand during L+T periods

***OUL*: Order up to level**

$$\boxed{\begin{aligned} R_{T+L} &= (T+L)R \\ \sigma_{T+L} &= \sigma\sqrt{L+T} \\ ss &= F^{-1}(CSL;0,1)\cdot\sigma_{T+L} \\ OUL &= R_{T+L} + ss \end{aligned}}$$

Example: Periodic Review Policy

***R* = 2,500/week; σ_R = 500**

***L* = 2 weeks; *T* = 4 weeks; *CSL* = 0.90**

What is the required safety inventory?

$$\boxed{ss = F^{-1}(CSL;0,1) \cdot \sigma_{T+L} = 1570}$$

Factors driving safety inventory

- **Demand uncertainty**
- **Replenishment lead time**
- **Reorder interval**

Periodic vs Continuous Review

- Periodic review ss covers the uncertainty over [0,T+L], T periods more than ss in continuous case.
- Periodic review ss is larger.
- Continuous review is harder to implement, use it for high-sales-value per time products

Methods of Accurate Response to Variability

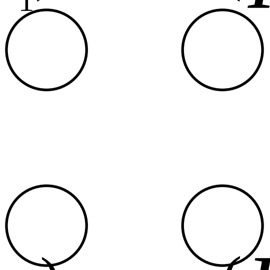
- Physical Centralization
- Information centralization
  - Virtual aggregation, Barnes&Nobles stores
- Specialization, what to aggregate
- Product substitution
- Raw material commonality - postponement

Inventory Pooling

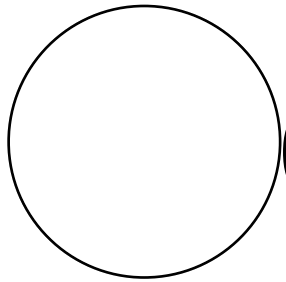
Which of two systems provides a higher level of service for a given safety stock?

Consider demands:

(R_1, σ_1) (R_2, σ_2)



(R_3, σ_3) (R_4, σ_4)



(R^C, σ_R^C)

$$R^C = \sum_{i=1}^{k} R_i; \quad (\sigma_R^C)^2 = \sum_{i=1}^{k} \sigma_i^2 + 2\sum_{i \neq j} \operatorname{cov}(i, j)$$

Sum of Identical Random Variables

$$\sigma_R^C \leq \sum_{i=1}^{k} \sigma_i$$ When they are independent

$$\sigma_R^C = \sum_{i=1}^{k} \sigma_i$$ When they are perfectly positively correlated

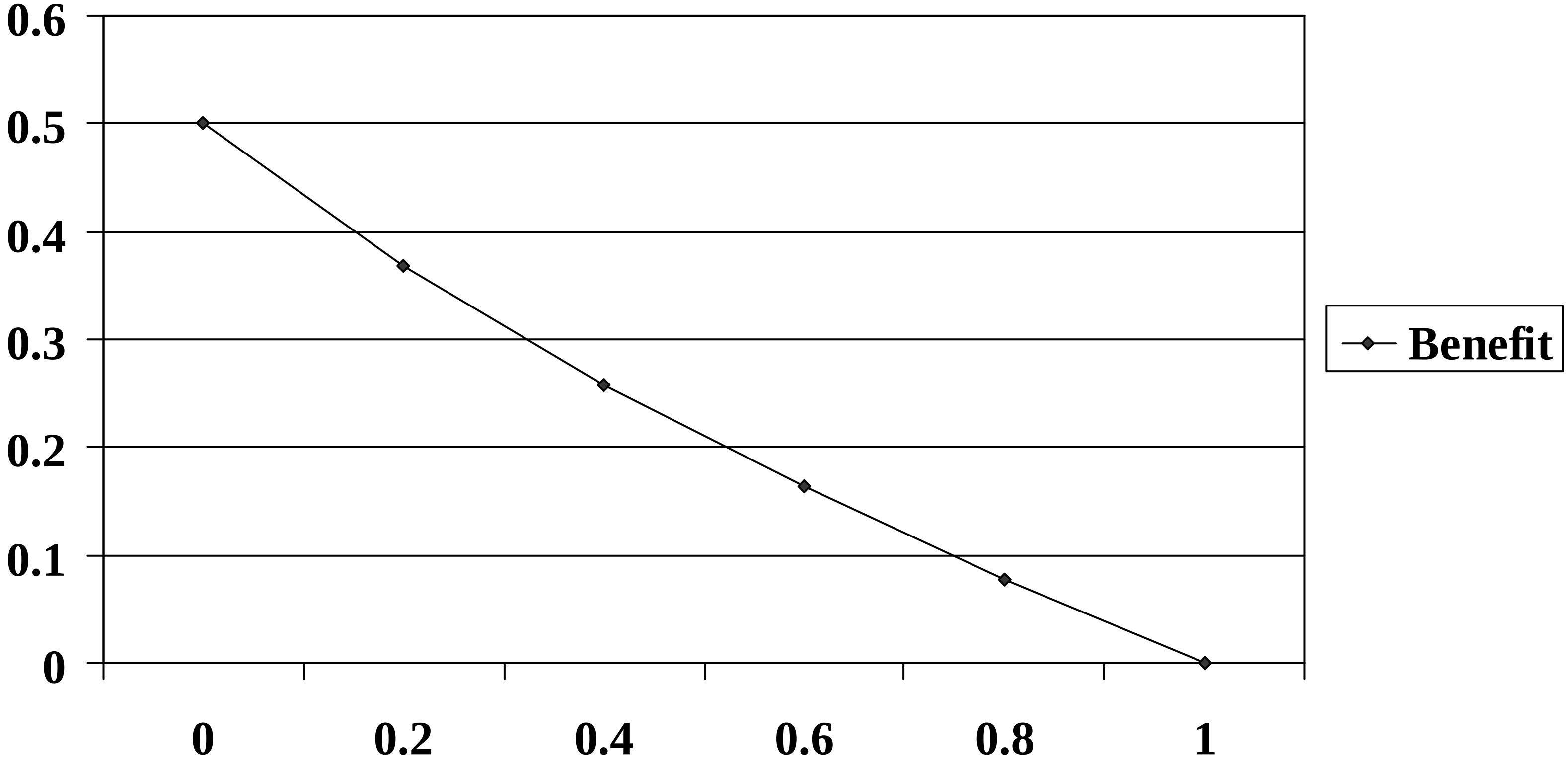
$$\sigma_R^C = 0 \leq \sum_{i=1}^{k} \sigma_i$$ When they are perfectly negatively correlated

Factors Affecting Value of Aggregation

- Demand Correlation, aggregation is helpful almost always except when products are positively correlated
- Coefficient of Variation of demand: Do not bother to aggregate if the variance is relatively small to begin with.

Impact of Correlation on Aggregated Safety Inventory (4 outlets)

Benefit=SS after aggregation / SS before aggregation



EX 11.8: W.W. Grainger a supplier of Maintenance and Repair products

- About 1600 stores in the US
- Produces large electric motors and industrial cleaners
- Each motor costs \$500; Demand is Normal(20,40x40)
- Each cleaning can costs \$30; Demand is Normal(1000,100x100)
- Which demand has a larger coefficient of variation?
- How much savings if motors inventoried centrally? How much savings if cleaners inventoried centrally?

EX. 11.8: Specialization: Impact of *cv* on Benefit From 1600-Store Aggregation , h=0.25

	Motors	***Cleaner***
Mean demand/wk	20	1,000
SD of demand	40	100
Disaggregate cv	2	0.1
Value/Unit	$500	$30
Disaggregate ss value	$105,600,000	$15,792,000
Aggregate cv	0.05	0.0025
Aggregate ss value	$2,632,000	$394,770
Inventory cost savings	102,968,000	15,397,230
Holding Cost Saving	$25,742,000	$3,849,308
Saving / Unit	$7.74	$0.046

Slow vs Fast Moving Items

- Low demand = Slow moving items, vice versa.
- Slow moving items have high coefficient of variation, vice versa.
- Stock slow moving items at a central store

Buying a best seller at Amazon.com vs. a Supply Chain book vs. a Banach spaces book, which has a shorter delivery time?

- "Case Interview" books are not in our sku list. You must check with our central stores.

  - Store keeper at Barnes and Nobles at Collin Creek, March 2002.

Product Substitution

- Manufacturer driven
- Customer driven

Consider: The price of the products substituted for each other and the demand correlations

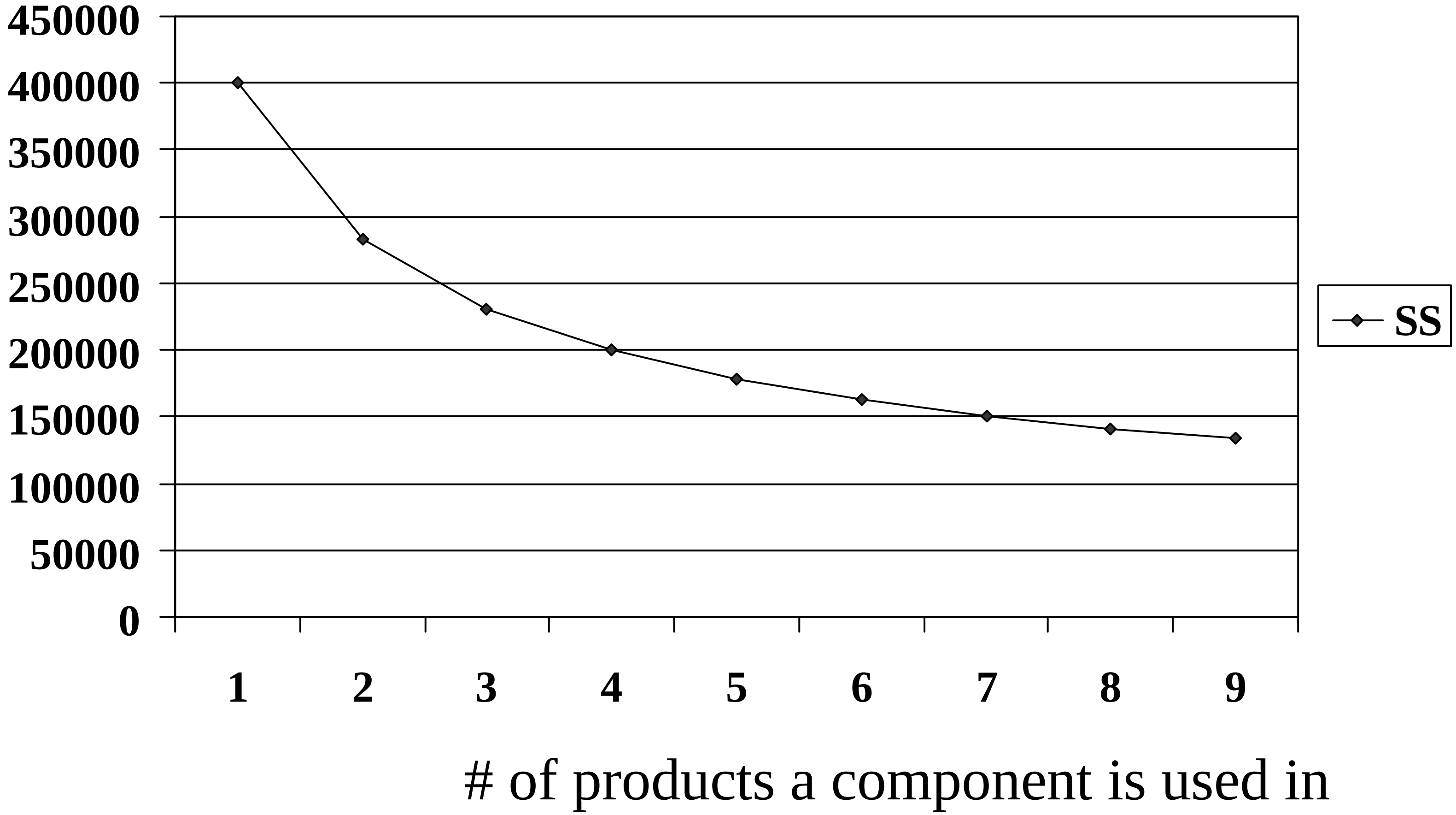
- One-way substitution
  - Army boots. Aggregate?
- Two-way substitution:
  - Grainger motors; water pumps model DN vs IT.
  - Similar products, can customer detect specifications.

If products are very similar, why not to eliminate?

Component commonality. Ex. 11.9

- Dell producing 27 products with 3 components (processor, memory, hard drive)
- No product commonality: A component is used in **only** 1 product. 27 component versions are required for each component. A total of 3*27 = 81 distinct components is required.
- Max component commonality. Only three distinct versions for each component. Each combination of components is a distinct product. A component is used in **9** products.
- Component commonality provides inventory aggregation.

Example 11.9: Value of Component Commonality in Safety Inventory Reduction



Advantages of Standardization

- Fewer parts to deal with in inventory & manufacturing
  - Less costly to fill orders from inventory
- Reduced training costs and time
- More routine purchasing, handling, and inspection procedures
- Opportunities for long production runs, automation
- Need for fewer parts justifies increased expenditures on perfecting designs and improving quality control procedures.

Disadvantages of Standardization

- Decreased variety results in less consumer appeal.
- Designs may be frozen with too many imperfections remaining.
  - US wireless communication standards vs Europe's
- High cost of design changes increases resistance to improvements
  - Keyboards: We are using a suboptimal qwerty keyboard. Even more interestingly countries using languages other than English also mostly use qwerty keyboard.

Mass Customization

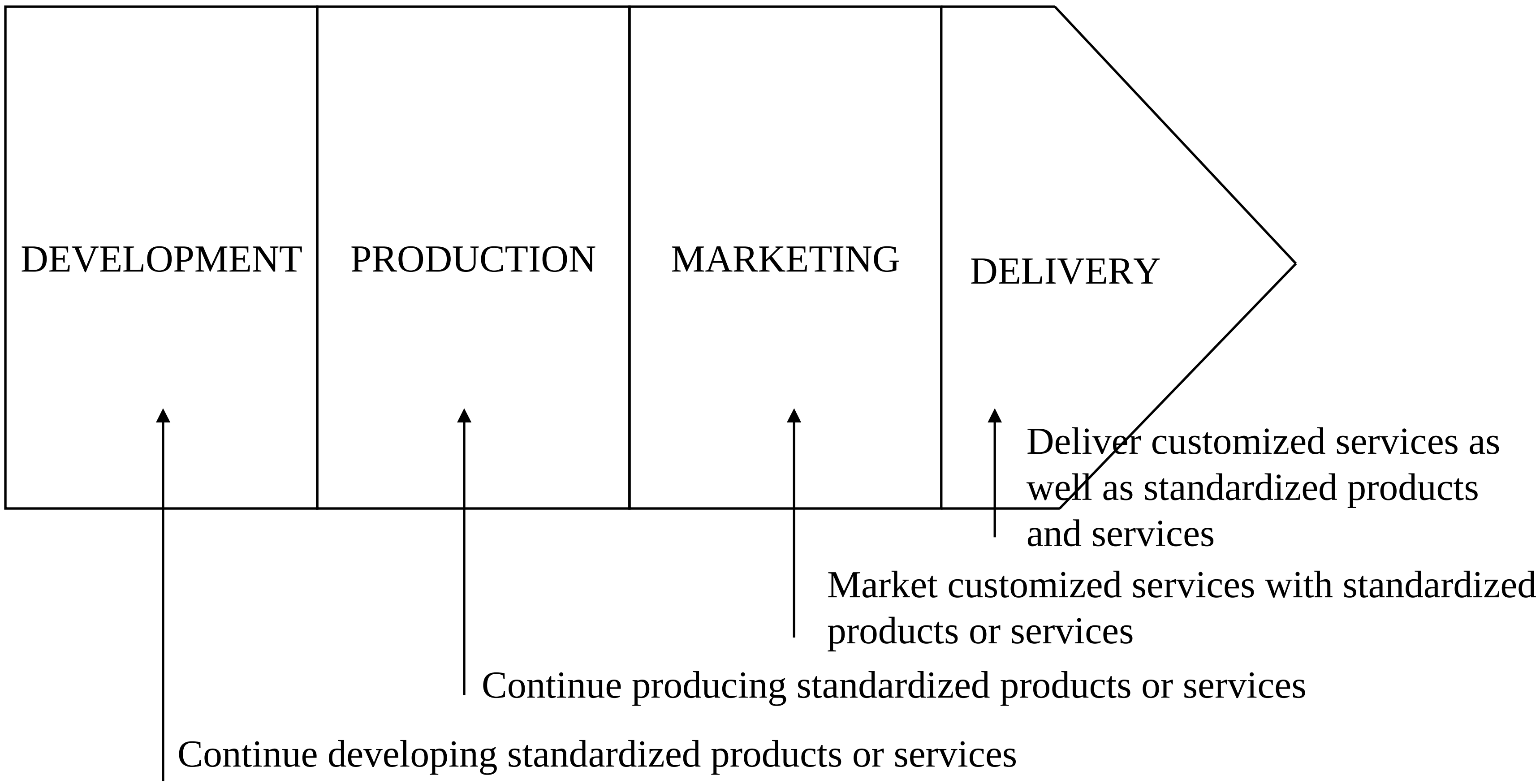
Mass customization:

- A strategy of producing standardized goods or services, but incorporating some degree of customization
- Modular design
- Delayed differentiation

Mass Customization I: Customize Services Around Standardized Products

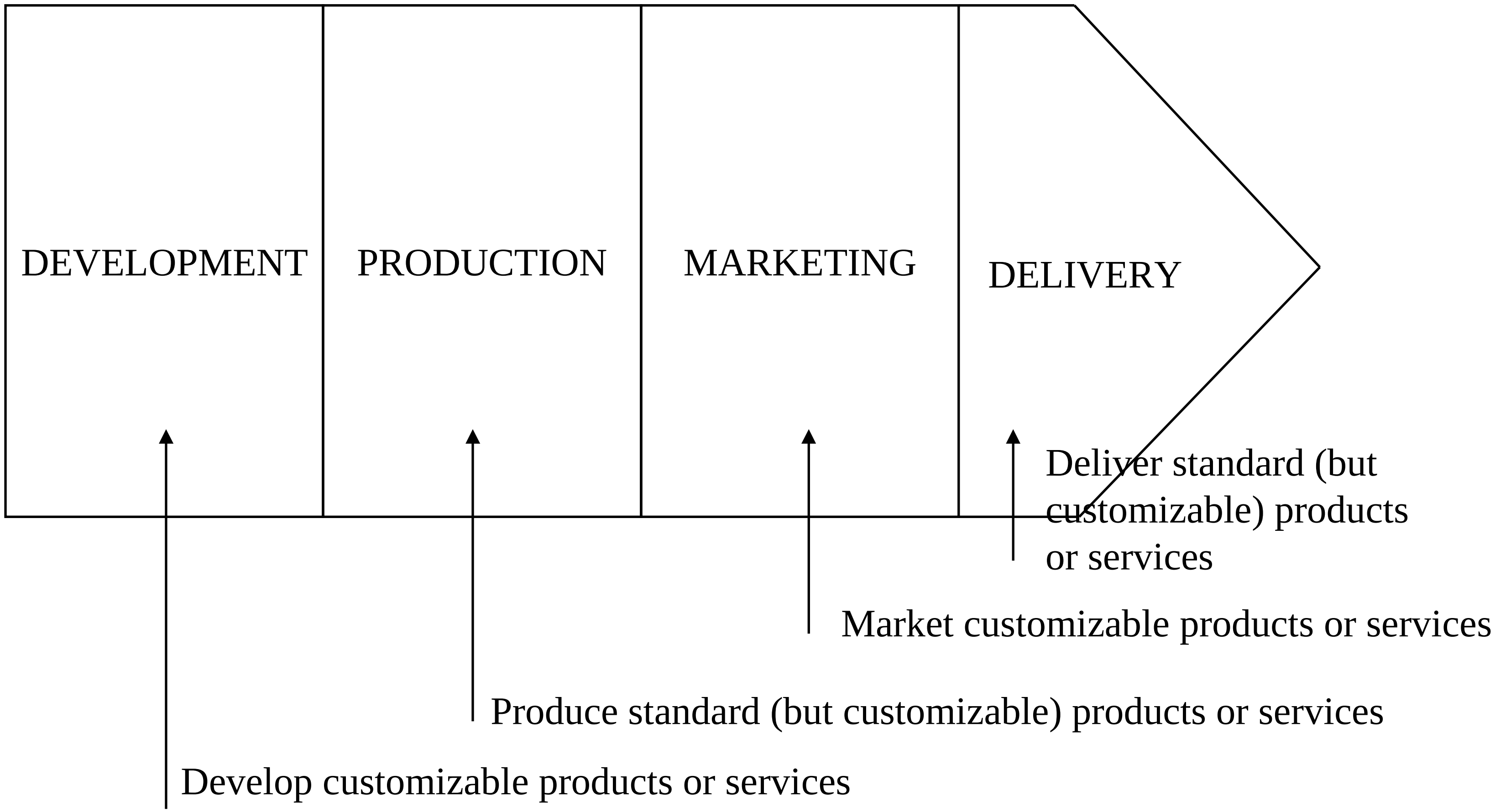
Warranty for contact lenses:

Source: B. Joseph Pine



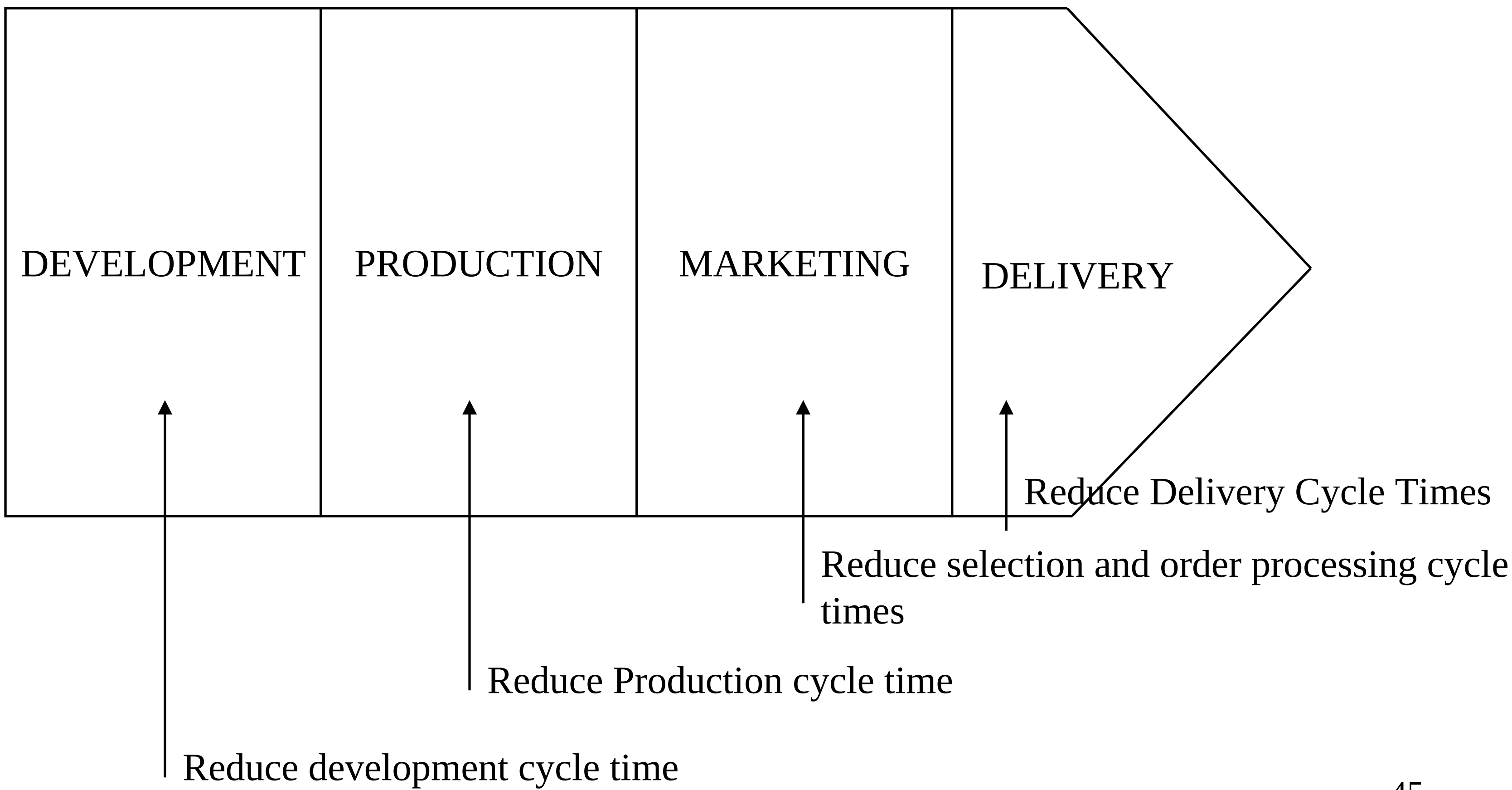
Mass Customization II: Create Customizable Products and Services

Gillette sensor adjusting to the contours of the face
Customizing the look of screen with windows operating system



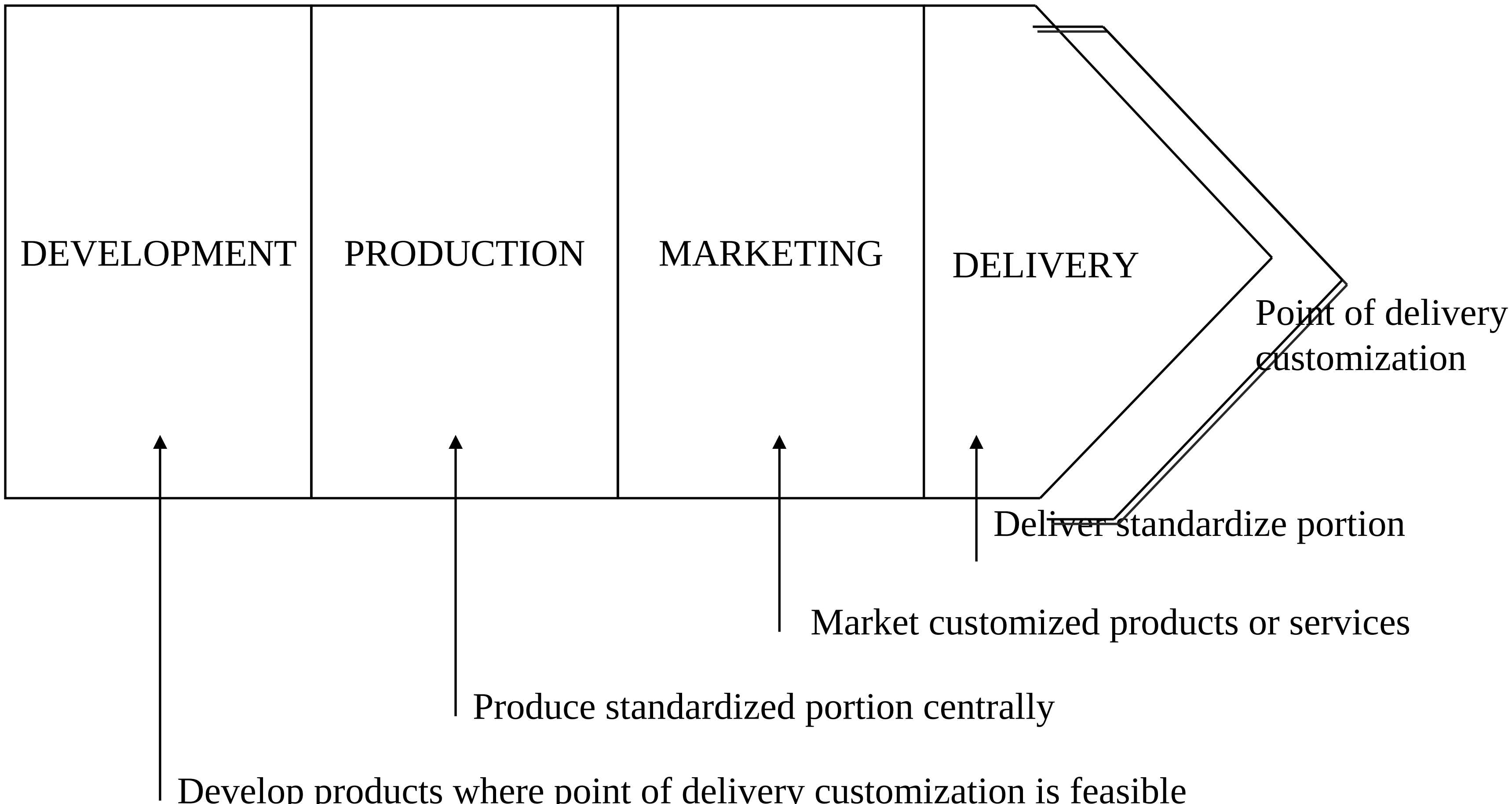
Mass Customization III: Provide Quick Response Throughout Value Chain

Skiing parkas manufactured abroad:



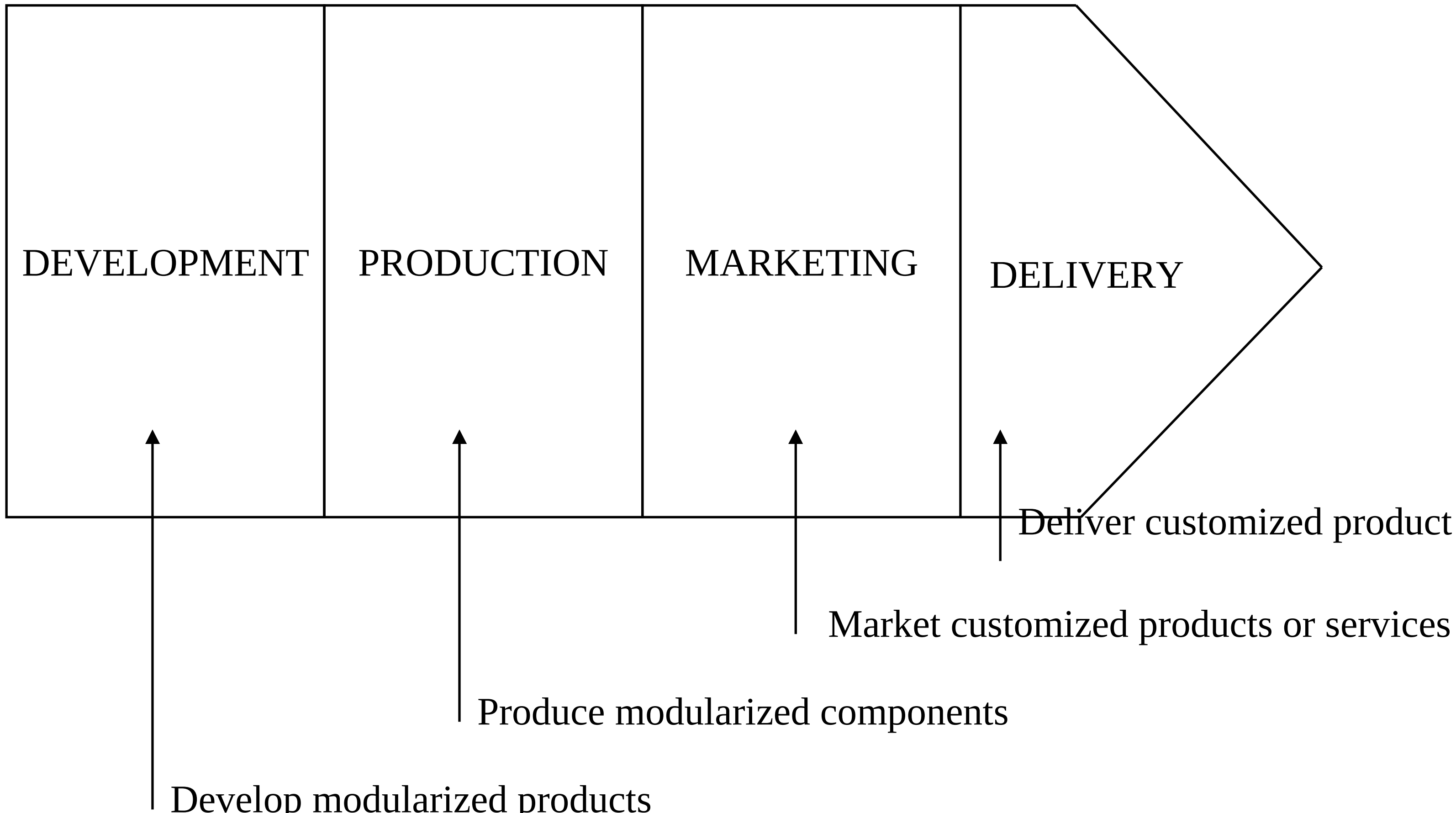
Mass Customization IV: Provide Point of Delivery Customization

Paint mixing
Lenscrafters for glasses.



Mass Customization V: Modularize Components to Customize End Products

Computer industry, Dell computers:



Modular Design

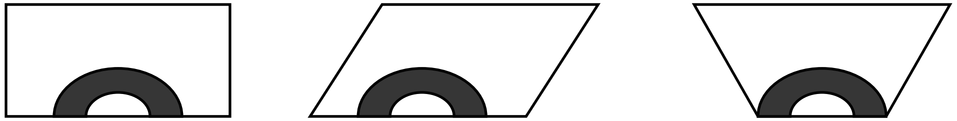
Modular design is a form of standardization in which component parts are subdivided into modules that are easily replaced or interchanged.

- A bad example: Earlier Ford SUVs shared the lower body with Ford cars

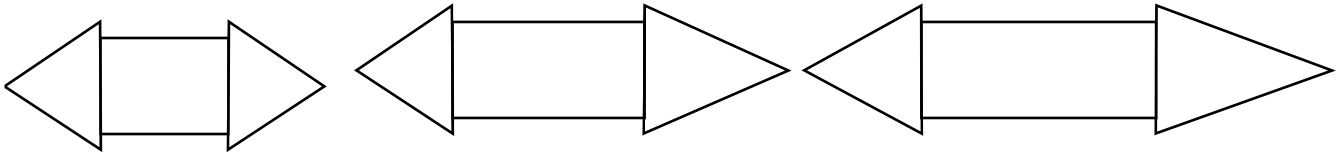
It allows:

- easier diagnosis and remedy of failures
- easier repair and replacement
- simplification of manufacturing and assembly

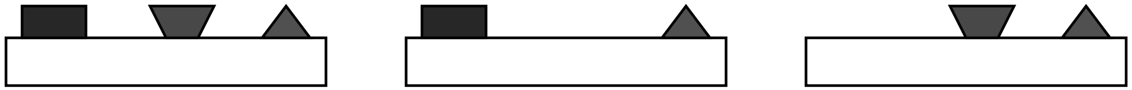
Types of Modularity for Mass Customization



Component Sharing Modularity, Dell



Cut-to-Fit Modularity, Gutters that do not require seams



Bus Modularity, E-books

Mix Modularity, Paints

Sectional Modularity, LEGO

Inventory–Transportation Costs: Eastern Electric Corporation: p.427

- **Major appliance manufacturer, buys motors from Westview motors in Dallas**
- **Annual demand = 120,000 motors**
- **Cost per motor = $120; Weight per motor 10 lbs.**
- **Current order size = 3,000 motors**
- **Lead time=1 + days in transit**
- **Safety stock carried = 50% of demand during delivery lead time**
- **Holding cost = 25%**
- **Evaluating the mode of transportation under all unit discounting based on shipment weight**
  - **cwt=cent weight=100pounds**

Inventory–Transportation trade off: Eastern Electric Corporation, see p.426-8 for details

Alternative (Lot size)	Transport Cost	Cycle Inventory	Safety Inventory	Transit Inventory	Inventory Cost	Total Cost
AM Rail (2,000)	$78,000 120000(0.65)	1,000	986	1,644 120000(5/365)	$108,900	$186,900
Northeast Trucking (1,000)	$90,000	500	658	986	$64,320	$154,320
Golden (500)	$96,000 120000(0.80)	250	658	986 120000(3/365)	$56,820	$152,820
Golden (2,500)	$86,400	1,250	658	986	$86,820	$173,220
Golden (3,000)	$78,000	1,500	658	986	$94,320	$172,320
Golden (4,000)	$67,500	2,000	658	986	$109,320	$176,820

If fast transportation not justified cost-wise, need to consider rapid response

Physical Inventory Aggregation: Inventory vs. Transportation cost: p.428

- HighMed Inc. producer of medical equipment sold directly to doctors
- Located in Wisconsin serves 24 regions in USA
- As a result of physical aggregation
  - Inventory costs decrease
  - Inbound transportation cost decreases
    - Inbound lots are larger
  - Outbound transportation cost increases

Inventory Aggregation at HighMed

Highval (\$200, .1 lbs/unit) demand in each of 24 territories

– $\mu_H = 2$, $\sigma_H = 5$

Lowval (\$30/unit, 0.04 lbs/unit) demand in each territory

– $\mu_L = 20$, $\sigma_L = 5$

UPS rate: \$0.66 + 0.26*x* {for replenishments}

FedEx rate: \$5.53 + 0.53*x* {for customer shipping}

Customers order 1 H + 10 L

Inventory Aggregation at HighMed

	Current Scenario	***Option A***	***Option B***
# Locations	24	24	1
Reorder Interval	4 weeks	1 week	1 week
Inventory Cost	$54,366	$29,795	$8,474
Shipment Size	8 H + 80 L	2 H + 20 L	1 H + 10 L
Transport Cost	$530	$1,148	$14,464
Total Cost	$54,896	$30,943	$22,938

If shipment size to customer is 0.5H + 5L, total cost of option B increases to $36,729.

Summary of Cycle and Safety Inventory