Formulas for closed-book OPRE 6302 Exams

Reminder for Statistics:

• Given a population $\{X_1, X_2, \dots, X_N\}$, **Mean**: $\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$; **Variance**: $\mathrm{Var}(X) = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N}$.

Standard deviation for the population: $\sigma = \sqrt{\mathrm{Var}(X)}$. **Coefficient of Variation** for the population: $CV = \sigma/\bar{X}$.

• **Exponential** distribution fits well to interarrival times to a queue and it has a CV of 1.

$$\textbf{Prob}(\textbf{exponential random variable with mean } \mu \leq t) = 1 - e^{-t/\mu}$$

• For a **normal random variable** $N(\mu, \sigma^2)$, $(N(\mu, \sigma^2) - \mu)/\sigma = N(0,1)$ is the **standard normal random variable**. If we **sum** L **many independent normal random variables** $N(\mu, \sigma^2)$, the sum is a normal random variable $N(L\mu, L\sigma^2)$.

Excel's Normal Probability functions:
$$\begin{cases} normdist(x, mean, stdev, 0): & \text{normal probability density at } x, \\ normdist(x, mean, stdev, 1): & \text{normal cumulative density at } x, \\ nominv(prob, mean, stdev): & \text{inverse of the cumulative density at } prob. \end{cases}$$

Areas under the standard normal curve from $-\infty$ to z and $\mathbf{L(z)} = normdist(z,0,1,0) - z * (1 - normdist(z,0,1,1))$:

z	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Area	0.54	0.58	0.62	0.66	0.69	0.73	0.76	0.79	0.82	0.84	0.86	0.88	0.9	0.919
L(z)	0.35	0.31	0.27	0.23	0.20	0.17	0.14	0.12	0.10	0.08	0.07	0.06	0.05	0.037

z	1.5	1.6	1.65	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6
Area	0.933	0.945	0.950	0.955	0.964	0.971	0.977	0.982	0.986	0.989	0.992	0.994	0.995
L(z)	0.029	0.023	0.020	0.018	0.014	0.011	0.008	0.006	0.005	0.004	0.003	0.002	0.001

OPRE 6302 Formulas:

The following formulas until the diamond sign $\diamond$ are included after students specifically asked for them.

$$\textbf{Inventory turns} = \frac{1}{\text{Flow time}} = \frac{\text{COGS}}{\text{Value of the Inventory}}.$$

$$\textbf{Annual Per Unit Inventory Cost} = \frac{\text{Annual Inventory Cost}}{\text{Number of Inventory Turns Per Annum}}.$$

$$\textbf{Process capacity} = \text{Min}\{\text{Capacity of Res 1}, \dots, \text{Capacity of Res 2}\}.$$

$$\textbf{Thruput} = \text{Min}\{\text{Input rate, Process capacity, Demand rate}\}.$$

$$\textbf{Requested cycle time} = \frac{\text{Operating time per week}}{\text{Demand per week}}.$$

$$\textbf{Designed cycle time} = \frac{1}{\text{Process Capacity}}.$$

$$\textbf{Flow rate} = \frac{1}{\text{Cycle time}}.$$

$$\textbf{Cost of direct labor} = \frac{\text{Total Wages}}{\text{Flow Rate}}.$$

$$\textbf{Idle time for worker at resource } i = \text{Cycle time} \times \text{Number of workers at resource } i - \text{Activity time at resource } i.$$

$$\textbf{Average labor utilization} = \frac{\text{Labor content}}{\text{Labor content + Total idle time}}.$$

$$\textbf{Process Utilization} = \frac{\text{Flow Rate}}{\text{Process Capacity}}.$$

$$\textbf{Implied Utilization} = \frac{\text{Capacity Requested by Demand}}{\text{Available Capacity}}$$

$$\textbf{Time to make X units} = \text{Time through empty system + (X-1)/Process Capacity}$$

$$= \text{Time through empty system + (X-1)Cycle Time.}$$

Replace X-1 with X for continuous production.

$$\textbf{Capacity given Batch Size} = \frac{\text{Batch Size}}{\text{Set-up time + Batch-size*Time per unit}}.$$

$$\textbf{Recommended Batch Size} = \frac{\text{Flow Rate * Setup Time}}{\text{1-Flow Rate*Time per Unit}}.$$

$$\textbf{Return on invested capital} = \frac{\text{Return}}{\text{Invested Capital}} = \frac{\text{Return}}{\text{Revenue}}\frac{\text{Revenue}}{\text{Invested Capital}};$$

$$\frac{\text{Return}}{\text{Revenue}} = \frac{\text{Revenue - Fixed costs - Flow rate*Variable costs}}{\text{Revenue}},$$

$$\frac{\text{Revenue}}{\text{Invested Capital}} = \frac{\text{Flow Rate*Price}}{\text{Invested Capital}}. \diamond$$

Little's Law: Average Inventory = Average Flow rate * Average Flow time.

Economic Production/Order Quantity model to find lot size Q:

R: Demand rate per time. P: Production rate per time. K: Fixed (Setup) cost. h: Holding cost rate per time per unit.
Average Inventory=$Q/2$. **Length of an Inventory Cycle**=Q/R.

$$\textbf{Total EPQ cost per time} = C(Q;P) = \underbrace{\frac{1}{2}\frac{Q}{P}(P-R)h}_{\textit{Inventory holding cost per time}} + \underbrace{\frac{KR}{Q}}_{\textit{Set up cost per time}}$$

$$\textbf{EPQ}(P) = \sqrt{\frac{2KR}{(1-R/P)h}}$$

Set $P=\infty$ to obtain EOQ. Specifically,

$$\textbf{Total EOQ cost per time} = C(Q;P=\infty) = \underbrace{\frac{1}{2}Qh}_{\textit{Inventory holding cost per time}} + \underbrace{\frac{KR}{Q}}_{\textit{Set up cost per time}}$$

$$\textbf{EOQ} = \sqrt{\frac{2KR}{h}} = \lim_{P\to\infty}\sqrt{\frac{2KR}{(1-R/P)h}} = EPQ(P=\infty)$$

With $P=\infty$ and $Q=EOQ$, the total cost $C(Q)$ per time becomes $C(Q=EOQ;P=\infty)=\sqrt{2KRh}$.

Level production plan:

The regular production is kept constant (level) over periods and demand fluctuations are satisfied with overtime production. If the regular production has a capacity then, we may be forced to produce at that capacity. In general,

$$\textbf{Regular production quantity} = \min\left\{\frac{\text{Sum of the demands}}{\text{Number of periods}}, \text{Regular production capacity}\right\}$$

Here is an example, suppose the demand for the next two weeks are 50 and 100 and the regular production capacity is 70. We can produce only 140 units in two weeks on regular time. The remaining 10 (=150-140) units are produced in overtime. It is better to do the overtime in the second week to save on inventory holding. Then the regular production is 70 and 70 in the first and the second week while the overtime production is 0 and 10 units in the first and the second week.

Queues

CV_a = St.Dev.of interarrival time/Aver. interarrival time. CV_p = St.Dev.of service time/Aver. service time. CV_a and CV_p are CV of interarrival and service times.
If a queue with m servers has average interarrival times a and activity times p, its utilization u and the approximate expected waiting time T_q are:

$$\mathbf{u} = \frac{p}{a\,m}. \qquad \mathbf{T_q} = \left(\frac{p}{m}\right)\left(\frac{u^{\sqrt{2(m+1)}-1}}{1-u}\right)\left(\frac{CV_a^2 + CV_p^2}{2}\right).$$

Then, we have: $\mathbf{T} = T_q + p.$ $\mathbf{I_p} = m \cdot u.$ $\mathbf{I} = I_q + I_p.$ $\mathbf{I_q} = (1/a)T_q.$ $\mathbf{I} = (1/a)(T_q + p).$

$$\textbf{Cost of Direct Labor} = \frac{\text{Total wages per time}}{\text{Flow rate per time}} = \frac{m * \text{Wages per time}}{1/a} = \frac{p * \text{Wages per time}}{u}$$

If a queue with m servers has average interarrival times a and activity times p, let $r := p/a$, then

$$\textbf{Prob(All servers are busy)} = \frac{(r^m)/(m!)}{\sum_{i=0}^{m}(r^i)/(i!)} \quad \text{or use Erlang loss table.}$$

$$\textbf{The rate of served output} = \frac{1}{a}\text{Prob(Not all servers are busy).}$$

Quality

- If the sandard deviation of a population is σ, then the standard deviation of the sample means of this population is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

where n is the size of each sample.

- Control charts:

Mean Chart: **UCL=Average of Sample Means** $+ z \cdot$ **StDev of Sample Means**

LCL=Average of Sample Means $- z \cdot$ **StDev of Sample Means**

where we choose z so that Type I error probability is α. One can also use

LCL=norminv(α/2,mean,stdev), **UCL**=norminv(1-α/2,mean,stdev).

One can also use

LCL=Average of Sample Means $- A_2 \cdot \bar{R}$, **UCL=Average of Sample Means** $+ A_2 \cdot \bar{R}$.

Range Chart: **LCL**=$D_3\bar{R}$ and **UCL**=$D_4\bar{R}$. Standard deviation can be estimated by $\sigma_{\bar{X}} = \bar{R}/d_2$.

Table for A_2, D_4, D_3 and d_2 under 99.74% or $3\sigma_{\bar{X}}$ confidence:

Sample size	A_2	D_3	D_4	d_2	Sample size	A_2	D_3	D_4	d_2
2	1.88	0	3.27	1.12	6	0.48	0	2.00	2.53
3	1.02	0	2.57	1.69	7	0.42	0.08	1.92	2.70
4	0.73	0	2.28	2.06	8	0.37	0.14	1.86	2.85
5	0.58	0	2.11	2.33	9	0.34	0.18	1.82	2.97

For **p-chart** with sample size n:

$$\textbf{StDev of Sample Mean} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

• Process capability:

$$\mathbf{C_p} = \textbf{Process capability ratio} = \frac{\text{Upper specification level} - \text{Lower specification level}}{6\sigma_{\bar{X}}}$$

Inventory

• The newsvendor optimal order quantity formula:

$$\textbf{In-stock Probability} = \textbf{Prob}(\textbf{Demand} \leq \textbf{Q}) = \frac{C_u}{C_u + C_o}$$

• Expected (lost sales=shortage) in a season = E(max$\{$Demand in a season $- Q, 0\}$). When the demand is normally distributed with mean μ and standard deviation σ, the **expected lost sales** is $\sigma \times \mathbf{L}(\mathbf{z})$, where

$$\mathbf{z} = \frac{Q-\mu}{\sigma} \text{ and } \mathbf{L}(\mathbf{z}) = normdist(z,0,1,0) - z * (1 - normdist(z,0,1,1))$$

• Demand=Sales+Lost Sales and Inventory=Sales+Left Over Inventory.

Expected demand = μ = Expected sales + Expected lost sales.

Inventory = Q = Expected sales + Expected left over inventory.

• Service measures:

Instock probability = Prob(Demand $\leq Q$)

$$\textbf{Fill rate} = \frac{\text{Expected sales}}{\text{Expected demand}} = 1 - \frac{\text{Expected lost sales}}{\text{Expected demand}}$$

• **Inventory level**=(On-hand inventory)-(Backorder) and Inventory position=(Inventory level)+(On-order inventory)

• In the basestock policy, we keep inventory position at **S to cover (lead time + 1) periods' demand**. Basestock is just another name for order-up-to level.