CORRECTED SOLUTION

EE 3350 Quiz 4A Fall 07 Name:

A tone modulated FM signal is given by

$$\Phi_{FM}(t) = 4\cos(2\pi\, 10^6\, t + 2\sin(2\pi\, 10^3\, t)).$$

(1) What is the bandwidth of the FM signal in kHz?
(2) What is the power of the FM signal?
(3) What is the power of the carrier component?
(4) Sketch the magnitude spectrum of the FM signal (within bandwidth).

$\beta = 2,\quad f_m = 1\text{ kHz} \qquad f_c = 1\text{ MHz} \qquad A = 4$

(1) $BW_{FM} = 2 f_m(\beta+1) = 2\times 1\times(2+1)\text{ kHz} = \boxed{6\text{ kHz}}$

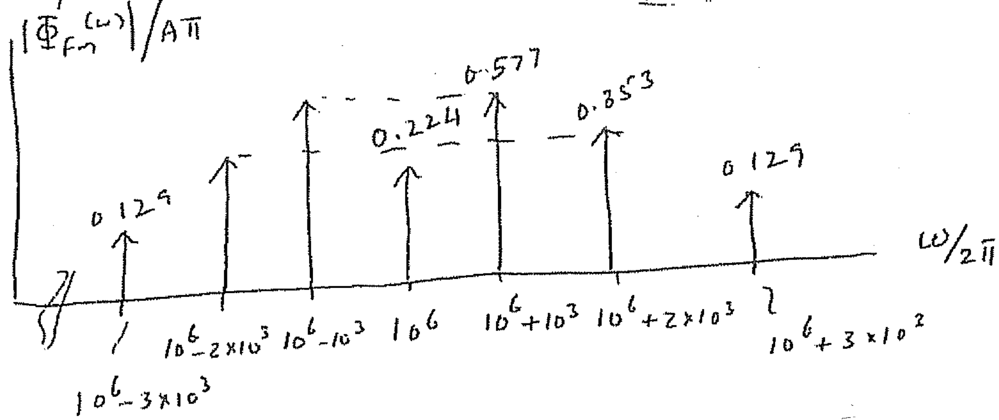
(2) Power of FM $= \dfrac{A^2}{2}$ watts $= \boxed{8\text{ watts}}$

(3) The carrier component is given by

$A \cdot J_0(2) \cdot \cos 2\pi\times 10^6 t$

Its power, $P_{carrier} = \dfrac{A^2}{2} J_0^2(2)$ watts $= 8\times(0.224)^2 = \boxed{0.401\text{ W}}$

(4) only the positive freq. axis is shown $\boxed{A = 4}$



EE 3350 Quiz 4A Fall 07

An angle modulated signal is given by

$$\Phi_{EM}(t) = 4\cos(2\pi 10^6 t + 2\sin(2\pi 10^3 t) + 4\sin(4\pi 10^3 t)).$$

(1) What is the bandwidth of the angle modulated signal? (Carson's rule)
(2) If the angle modulated signal is an FM signal with a frequency deviation constant of $4\pi\ 10^3$ rad/sec/volt, determine the message signal m(t).
(3) What is the value of the frequency deviation ratio (i.e. modulation index)?

$\phi_{Em}(t) = A\cos(\omega_c t + \psi(t))$

$\psi(t) = 2\sin(2\pi\ 10^3 t) + 4\sin(4\pi\ 10^3 t)$

$\omega_i(t) = \omega_c + \frac{d\psi(t)}{dt} = 2\pi\times10^6 + 4\pi\times10^3\cos(2\pi\ 10^3 t) + 16\pi\times10^3\cos(4\pi\ 10^3 t)$ — ①

$\Delta\omega_{Em} = \omega_{i\,max} - \omega_c = 20\pi\times10^3$ or $\Delta f_{Em} = 10$ kHz.

$BW_{m(t)} = 2$ kHz.

(1) $BW_{Em} = 2(\Delta f_{Em} + BW_{m(t)})$ Hz $= 2\times(10+2)$ kHz $= \boxed{24 \text{ kHz}}$

(2) For FM signal, $\omega_i(t) = \omega_c + k_f \cdot m(t)$ with $k_f = 4\pi\times10^3$ rad/sec/volt,

$\omega_i(t) = 2\pi\times10^6 + 4\pi\times10^3\, m(t)$ — ②

Comparing ① and ②, we get

$\boxed{m(t) = \cos(2\pi\times10^3 t) + 4\cos(2\pi\times2\times10^3 t)}$

(3) Frequency Dev. Ratio, $\beta = \frac{\Delta f}{BW_{m(t)}} = \frac{10}{2} = \boxed{5}$

EE 3350 Quiz 4B Fall 07 Name:

- In a tone modulated FM system, the message signal is given by $m(t) = 2\cos(2\pi\, 10^3\, t)$, the frequency deviation constant is $2\pi\, 10^3$ rad/sec/volt, the carrier frequency is 100 MHz, and the FM signal amplitude is 10 volts.
  (1) Write an expression for $\Phi_{FM}(t)$ without any integral.
  (2) What is the power of the FM signal?
  (3) What is the power of the first upper side band component?
  (4) Sketch the magnitude spectrum of the FM signal (within the bandwidth).

$A = 10 \qquad \omega_c = 2\pi \times 10^6, \qquad m(t) = 2\cos(2\pi \times 10^3 t)$

$k_f = 2\pi \times 10^3$ rad/volt/sec.

$$\beta = \frac{k_f\, d}{\omega_m} = \frac{2\pi \times 10^3 \times 2}{2\pi \times 10^3} = \boxed{2}$$

(1) $\phi_{FM\text{-}tone}(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$

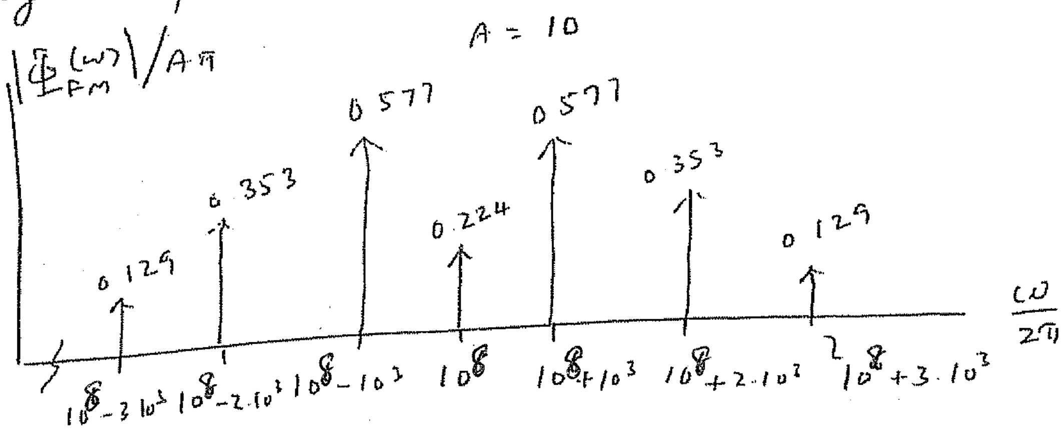
$$\boxed{\phi_{FM}(t) = 10\cos(2\pi 10^8 t + 2\sin(2\pi 10^3 t))}$$

(2) Power of FM signal $= \frac{A^2}{2}$ watts $= \frac{10^2}{2}$ W $= \boxed{50\text{ W}}$

(3) Power of the first upper side band component is given by $\frac{A^2}{2} \cdot J_1^2(2) = 50 \times (0.577)^2 \cong \boxed{16.65\text{ W}}$

(4) Only the positive freq. axis is shown

$A = 10$



EE 3350 Quiz 4B Fall 07

- An angle modulated signal is given by

$$\Phi_{EM}(t) = 10\cos(2\pi 10^6 t + \underbrace{\boxed{2\sin(2\pi 10^4 t) + 4\sin(4\pi 10^4 t)}}_{\psi(t)}).$$

(1) What is the bandwidth of the angle modulated signal? (Carson's rule)
(2) If the angle modulated signal is a PM signal with a phase deviation constant of 4 rad/volt, determine the message signal m(t).
(3) What is the value of the frequency deviation ratio (i.e. modulation index)?

$\phi_{Em}(t) = A\cos(\omega_c t + \psi(t))$

$\omega_i(t) = \omega_c + \frac{d\psi(t)}{dt} = 2\pi \times 10^6 + 4\pi \times 10^4 \cos(2\pi \cdot 10^4 t) + 16\pi \times 10^4 \cos(4\pi \cdot 10^4 t)$ — ①

$\Delta\omega_{Em} = \omega_{i\,max} - \omega_c = 20\pi \times 10^4$ rad/sec or $\boxed{\Delta f_{Em} = 100\text{ kHz}}$

$BW_{m(t)} = 20$ kHz

(1) $BW_{Em} = 2(\Delta f_{Em} + BW_{m(t)}) = 2(100+20)\text{ kHz} = \boxed{240\text{ kHz}}$

(2) For a phase modulated signal,

$\phi_{pm}(t) = A\cos(\omega_c t + k_p m(t))$ For our case, $k_p = 4$ rad/volt

or $\psi(t) = k_p m(t)$

$\boxed{m(t) = \frac{1}{2}\sin(2\pi \cdot 10^4 t) + \sin(4\pi \cdot 10^4 t)}$

(3) Freq Dev Ratio, $\beta = \frac{\Delta f}{BW_{m(t)}} = \frac{100}{20} = \boxed{5}$.

EE 3350 Quiz 4C Fall 07 Name:

In a tone modulated FM system, the message signal is given by $m(t) = 2\cos(2\pi\, 10^3\, t)$, the frequency deviation constant is $2\pi\, 10^3$ rad/sec/volt, the carrier frequency is 100 MHz, and the FM signal amplitude is 10 volts.

(1) Write an expression for $\Phi_{FM}(t)$ <u>without</u> any integral.
(2) What is the power of the FM signal?
(3) What is the power of the first upper side band component?
(4) Sketch the magnitude spectrum of the FM signal (within the bandwidth).

$A = 10$ $\quad \omega_c = 2\pi \times 10^8$, $\quad m(t) = 2\cos(2\pi \times 10^3 t)$

$k_f = 2\pi \times 10^3$ rad/volt/sec,

$$\beta = \frac{k_f A}{\omega_m} = \frac{\cancel{2\pi} \times \cancel{10^3} \times 2}{\cancel{2\pi \times 10^3}} = \boxed{2}$$

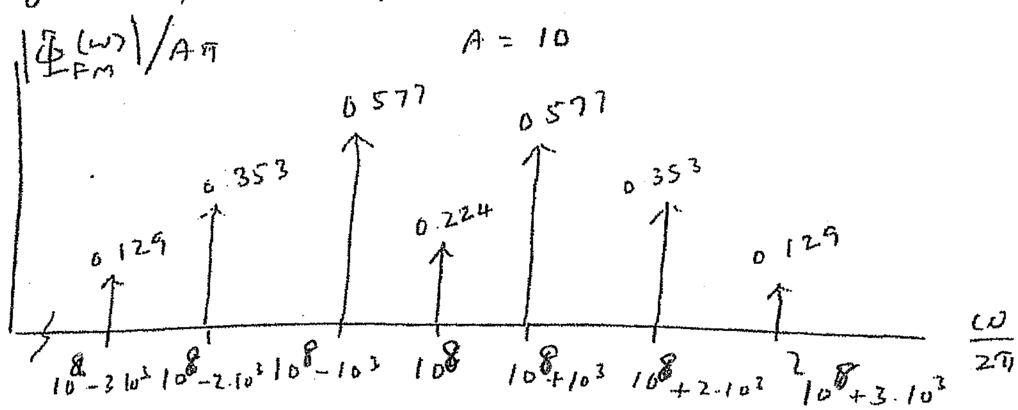
(1) $\phi_{FM\text{-}tone}(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$

$$\boxed{\phi_{FM}(t) = 10\cos(2\pi\, 10^8 t + 2\sin(2\pi 10^3 t))}$$

(2) Power of FM signal $= \frac{A^2}{2}$ watts $= \frac{10^2}{2}$ W $= \boxed{50\text{ W}}$

(3) Power of the first upper side band component is given by $\frac{A^2}{2} \cdot J_1^2(2) = 50 \times (0.577)^2 \simeq \boxed{16.65\text{ W}}$

(4) Only the positive freq. axis is shown



EE 3350 Quiz 4C Fall 07 **Name:**

A signal m(t) frequency modulates a 100 kHz carrier to produce the following narrowband FM signal:

$$\varphi_{NBFM}(t) = 10\cos(2\pi\, 10^5 t + 0.0050 \sin 2\pi\, 10^4 t).$$

Generate (block diagram design) the wideband FM signal $\varphi_{WBFM}(t)$ with a carrier frequency of 125 MHz and a (peak) frequency deviation of 100 kHz. Assume that the following are available for the design:

- Frequency Multipliers of any (integer) value
- A local oscillator whose frequency can be tuned to any value between 50 MHz to 150 MHz
- An ideal band pass filter with tunable center frequency and bandwidth.

Your block diagram design must clearly specify the carrier frequencies and frequency deviations at all logical points, as well as the center frequency and bandwidth of the band pass filter.

$\beta = 0.005 \quad f_m = 10\,kHz \Rightarrow \Delta f_0 = \beta f_m = 50\,Hz$

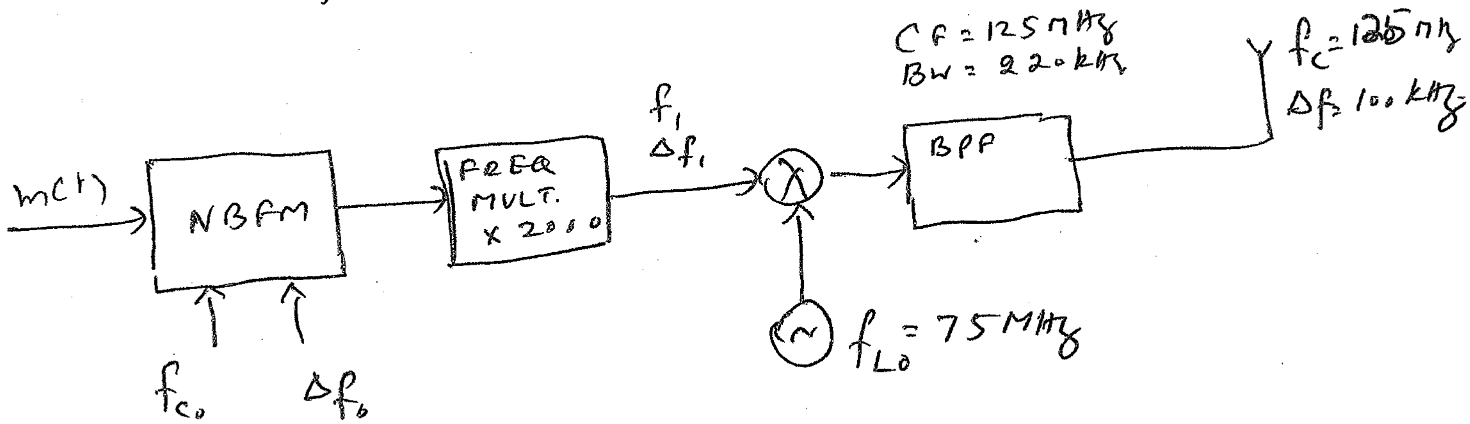
Target $\Delta f = 100\,kHz \Rightarrow N = \frac{\Delta f}{\Delta f_0} = \frac{\cancel{100}^{2}\times 10^{4}}{\cancel{50}} = \boxed{2000}$

$f_{c_0} = 100\,kHz \Rightarrow f_1 = 100\,kHz \times 2000 = 200\,MHz$

$f_c = 125\,MHz = |f_1 - f_{LO}| \Rightarrow \boxed{f_{LO} = 75\,MHz}$

BPF CF $= f_c = \boxed{125\,MHz}$

BW of BPF $= 2(\Delta f + B) = 2(100+10)\,kHz = \boxed{220\,kHz}$



EE 3350 Quiz 4D Fall 07 **Name:**

A tone modulated FM signal is given by

$$\Phi_{FM}(t) = 10 \cos [2\pi\, 10^6\, t + 2 \sin (10\pi\, 10^4\, t)]$$

(1) What is the bandwidth of the FM signal in kHz? (2) What is the power of the FM signal? (3) What is the power of the carrier component in the FM signal? (4) Sketch the magnitude spectrum of the FM signal (within the bandwidth).

$\beta = 2$, $f_m = 50$ kHz $f_c = 1$ MHz $A = 10$

(1) $BW_{FM} = 2 f_m (\beta + 1) = 2 \times 50 \times (2+1)$ kHz $= \boxed{300\text{kHz}}$

(2) Power of FM $= \frac{A^2}{2}$ watts $= \boxed{50 \text{ watts}}$

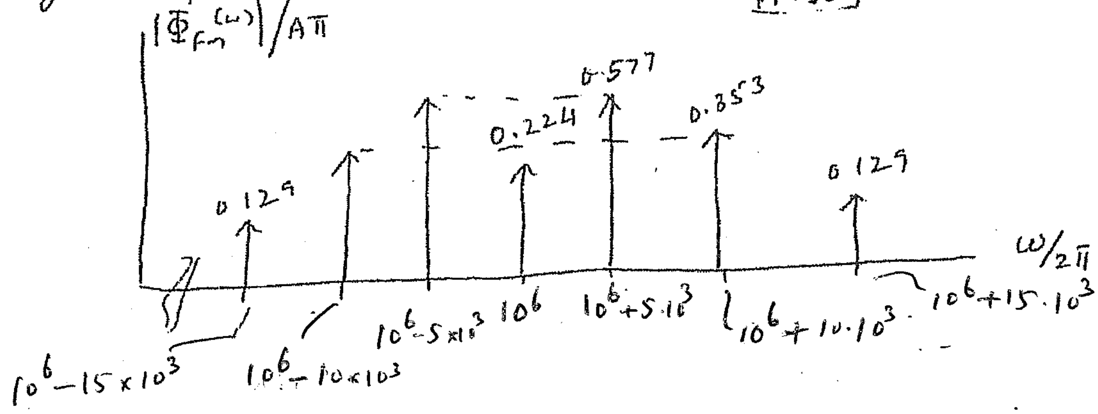
(3) The carrier component is given by

$A \cdot J_0(2) \cdot \cos 2\pi \times 10^6 t$

Its power, $P_{carrier} = \frac{A^2}{2} J_0^2(2)$ watts $= 50 \times (0.224)^2 = \boxed{2.5 \text{ W}}$

(4) only the positive freq. axis is shown

$\boxed{A = 10}$



EE 3350 Quiz 4D Fall 07 **Name:**

A signal m(t) frequency modulates a 100 kHz carrier to produce the following narrowband FM signal:

$$\varphi_{NBFM}(t) = 5\cos(2\pi\, 10^5 t + 0.0050 \sin 2\pi\, 10^4 t).$$

Generate (block diagram design) the wideband FM signal $\varphi_{WBFM}(t)$ with a carrier frequency of 75 MHz and a (peak) frequency deviation of 75 kHz. Assume that the following are available for the design:

- Frequency Multipliers of any (integer) value
- A local oscillator whose frequency can be tuned to any value between 50 MHz to 150 MHz
- An ideal band pass filter with tunable center frequency and bandwidth.

Your block diagram design must clearly specify the carrier frequencies and frequency deviations at all logical points, as well as the center frequency and bandwidth of the band pass filter.

$\beta = 0.005 \quad f_m = 10\text{ kHz} \Rightarrow \Delta f_0 = \beta f_m = \boxed{50\text{ Hz}}$

$f_{c_0} = 100\text{ kHz}.$

$N = \dfrac{\Delta f}{\Delta f_0} = \dfrac{75 \times 10^3}{50} = \boxed{1500}$

$f_1 = N \cdot f_{c_0} = 1500 \times 100\text{ kHz} = 150\text{ MHz}.$

$f_c = |f_1 - f_{LO}| \Rightarrow \boxed{f_{LO} = 75\text{ MHz}}$

CF of BPF = $\boxed{75\text{ MHz}}$ BW of BPF = $2(\Delta f + B) = 2(75+10)\text{ kHz}$

$= \boxed{170\text{ kHz}}$