The Informational Role of Buyback Contracts

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Manufacturers often offer retailers buyback contracts to reduce retailers' inventory costs by repurchasing unsold inventory at a pre-specified returns price. We examine the signaling role of buyback contracts when the retailer is less informed about either the manufacturer's reliability of honoring the buyback commitment (e.g., for a small/less-established manufacturer) or its product's market potential (e.g., for a national brand manufacturer). We find that these two situations yield contrasting buyback designs: the manufacturer must distort the wholesale and returns prices downward to signal higher reliability but upward to signal higher market potential. Nevertheless, the signaling mechanism in both cases hinges on suitably distorting the manufacturer's returns cost (i.e., the cost of repurchasing retailer's unsold inventory) by influencing the retailer's regular stock (i.e., the portion of inventory carried to meet average demand) and safety stock (i.e., the extra inventory carried to meet potential high demand). Notably, while prior research has highlighted the signaling role of the wholesale price, we show how and why, in a channel with inventory, the returns price plays a relatively more important role. In particular, efficient signaling entails that the returns price is used to distort the manufacturer's returns cost, whereas the wholesale price is used only to mitigate the resulting distortion in the retailer's order quantity. In fact, the returns price emerges as a more efficient signaling instrument and reverses the direction of wholesale price distortion from what is necessary if wholesale price alone is used to signal. We also examine the implications when the two dimensions of manufacturer's private information are correlated.

Key words: buyback contracts, inventory, returns, demand potential, prices, signaling

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1. Introduction

In many product categories, market demand is often stochastic and an upstream manufacturer has to design suitable mechanisms to ensure that the downstream retailer carries sufficient inventory of her product to meet potential demand. One such mechanism is the buyback contract, wherein the manufacturer offers to repurchase any unsold inventory at the end of the selling season at a pre-specified returns price. Careful design of the buyback contract can increase product sales and can thus be crucial for the manufacturer's success and profitability. Accordingly, researchers have studied how the manufacturer can optimally structure the terms of the buyback arrangement, under the assumption that the retailer is equally knowledgeable about demand conditions as the manufacturer and can trust the buyback commitment to be honored (e.g., Marvel and Peck 1995, Padmanabhan and Png 1997, Wang 2004, Gurnani et al. 2010, Tran et al. 2018). Yet, in practice, the retailer may be less informed about market conditions (e.g., compared to large manufacturers who invest considerable resources in proprietary market research), or unsure about the manufacturer's intrinsic reliability of honoring the buyback commitment (e.g., in the case of small or foreign manufacturers, or due to adverse economic environment). Therefore, the implications of designing buyback contracts under these practical situations need to be understood. Indeed, prior literature has highlighted the role of wholesale price contracts in signaling a manufacturer's demand information to the retailer (e.g., Chu 1992, Desai 2000, Gal-Or et al. 2008, Jiang et al. 2016). However, in situations where the retailer must carry inventory because of stochastic demand, it is more natural for the manufacturer to offer a buyback contract. Whether and how the buyback contract can be structured to credibly signal demand conditions or the intrinsic reliability of the buyback commitment, and how such a strategy compares to using the wholesale price alone to signal (as examined in prior research), remain as open questions. In this paper, we aim to shed light on these questions. Our results offer an understanding of the informational role of buyback arrangements, over and above its oft-studied transactional role.

In the context of buyback contracts, a retailer faces two types of inventory-related risks. The first type of risk, which we refer to as *demand risk*, occurs because of the stochastic nature of demand at the time of ordering the product (ahead of the selling season), which can result in the retailer having unsold inventory at the end of the selling season. The buyback arrangement aims to lower the retailer's demand risk by sharing in the retailer's costs of carrying any excess inventory. The buyback arrangement itself, however, leads to a second type of risk, which we refer to as the *returns risk* and occurs because of the buyback commitment not being met at the time of returning the inventory (at the end of the selling season). The returns risk, which is typically inherent to the specific manufacturer, arises because of contingencies or exceptional events that could not be fully anticipated at the time that the buyback contract was offered. For example, as evidenced from multiple lawsuits (e.g., Biddle 2003, Drywall Supply Central, Inc. v. Trex Company 2007, American Suzuki Motor Corporation 2013), a manufacturer may not meet the commitment because of unexpectedly facing bankruptcy or financial distress due to economic downturn or mismanagement; or, in certain cases, because of contractual caveats or legal loopholes that exonerate the manufacturer from fulfilling her buyback commitment.

One type of risk may be more prominent than the other in a particular situation, depending on factors such as the manufacturer's characteristics, nature of the product, and economic conditions. For example, returns risk is likely to be more prominent in the case of small and less-established manufacturers, or foreign manufacturers, or during an economic downturn. Small manufacturers are often liquidity constrained and thus may not be able to repurchase unsold inventory from the retailer. Similarly, less-established foreign manufacturers may lack the operational wherewithal to reliably handle returns; in addition, their proprietary familiarity with their home-country legal systems may also affect the likelihood of fulfilling their contractual obligations.[1] Consequently, retailers are justifiably wary of buyback arrangements offered by such manufacturers, as exemplified by the following comment from a former Vice President of the convenience store chain 7-Eleven:[2]

> *Many retailers have been burned on these buyback guarantees that there is a sense of distrust when it comes to secondary or smaller manufacturers... The retailers are not sure whether the manufacturer will be able to take the product back if the product does not sell.*

While some small or less-established manufacturers may pose lower returns risk than others, for example, because of better financial and operational health, a retailer typically lacks the resources and expertise to investigate the intrinsic risks of individual manufacturers. Consequently, the retailer is less knowledgeable about the returns risk than the manufacturer, and thus skeptical of the manufacturer's buyback commitment. In such situations, can a more reliable manufacturer address the retailer's lack of trust by suitably designing the buyback arrangement? If so, how does the buyback arrangement of a more reliable manufacturer differ from that of a less reliable manufacturer? What is the impact on the manufacturer, the retailer and channel performance? These are some of the questions that we wish to answer in this paper.

In contrast to small or less-established manufacturers, large and well-established manufacturers, who regularly introduce new products, face a different challenge; namely, to convince a retailer about the market potential for their products. In this case, the manufacturer's returns risk, if any, is likely to be minimal and well known to the retailer given past interactions or the manufacturer's reputation in the marketplace. However, the retailer is likely to be less informed than the manufacturer about the market potential, and hence the extent of demand risk, especially for the case of new products. Indeed, even for large manufacturers, not all products are guaranteed to be equally successful (e.g., Schneider and Hall 2011, York 2013). Moreover, a large manufacturer (e.g., P&G, Kraft) often invests considerable resources in proprietary market research and, therefore, has superior information about her product's market potential than the retailer (e.g., ACNielsen 2006, Guo and Iyer 2010). Intuitively, all else being equal, a product with higher demand potential poses lower demand risk for the retailer and warrants carrying higher inventory. However, a retailer that

[1] As pointed out by practitioners (e.g., Rosenfeld 2015), it is an insurmountable challenge for American companies to anticipate all contingencies and include them in upfront contracts when dealing with foreign suppliers.

[2] The comment was made by Paul Pierce, who was VP of Fresh Sales at 7-Eleven, in a private interview by the authors.

is uninformed about the product's market potential requires convincing that the product's market potential is truly high. In such situations, how should a manufacturer, whose product has higher demand potential, structure her buyback arrangement to credibly signal the demand potential? How are the implications similar to or different than those in the case where the manufacturer signals lower returns risk? We address these questions as well.

At this juncture, we should note that while the importance and relevance of studying the strategies of large and established manufacturers may be self-evident, some remarks for the case of small and less-established players may be in order. Small and less-established manufacturers are often an important source of innovation, product variety and competition in the market place. Even though they may not account for a substantial portion of the market at a particular point in time, they may eventually grow to have substantial impact on the entire category; examples include Greek Yogurt maker Chobani (Fast Company Staff 2017), 5-hour Energy producer Living Essentials (Klara 2016), and White Wave, the brand owner of Silk soy and almond milk (Adamy 2005). In this context, a well-designed buyback contract can in fact help small and less-established manufacturers penetrate the market by encouraging the retailer to carry more inventory. At the same time, a challenge for such manufacturers is that a retailer may not trust their buyback commitment. Thus, understanding how a small or less-established manufacturer can leverage the buyback arrangement to her advantage is both important and relevant.

In this paper, we conduct a model-based examination of the signaling role of the buyback contract in two distinct scenarios, namely, where the retailer is uninformed either about the manufacturer's returns risk or the demand risk.[3] To our knowledge, our paper is the first to examine the informational role of the buyback arrangement over and above its oft-studied transactional role. In particular, we introduce the analysis of manufacturer returns risk to the literature, and study how to optimally design the buyback contract to signal returns risk. By studying a channel that faces stochastic demand and hence carries inventory, we uncover a novel signaling mechanism across the two types of manufacturer's private information.

In the presence of stochastic demand and inventory considerations, we find that what distinguishes the manufacturer based on her returns risk or demand risk is the probability of incurring the returns cost under the buyback contract (i.e., the manufacturer's cost of repurchasing the retailer's unsold inventory). We show that, as a result, efficient signaling relies on distorting the manufacturer's returns cost by suitably influencing both the retailer's regular stock (portion of inventory that is carried to meet the average demand) and safety stock (excess inventory carried to meet potential high demand). Interestingly, this signaling mechanism entails that the returns

[3] Later, in Section 6, we study the case when *both* risks are the manufacturer's private information.

price is used to distort the returns cost, whereas the wholesale price is used only to offset the resulting distortion in the retailer's order quantity. In fact, the returns price emerges as a more efficient signaling instrument and reverses the direction of wholesale price distortion from what is necessary if the wholesale price alone is used to distort the returns cost. Thus, while prior research has highlighted the signaling role of the wholesale price, we find that, in a channel with inventory, it is the returns clause of a buyback contract that plays a more important role.

As a result, the two types of manufacturer private information (i.e., returns risk and demand risk) lead to contrasting designs of the buyback contract. Specifically, signaling higher reliability (which corresponds to higher probability of incurring the returns cost) entails that the returns cost is distorted downward, whereas signaling higher market potential (which corresponds to lower probability of incurring the returns cost) entails that the returns cost is distorted upward. We find that a manufacturer must distort both the wholesale price and returns price downward to signal higher reliability but upward to signal higher market potential. We also examine the implications when these two dimensions of private information are correlated and show that our main insights extend to this setting in a natural manner under certain conditions.

The plan for the rest of the paper is as follows. We discuss the relevant literature in the next section and set up the model in Section 3. We dedicate Section 4 to the case of signaling manufacturer's returns risk and Section 5 to the case of signaling demand potential. Section 6 extends the analysis to the case of signaling both types of private information. Section 7 concludes the paper. All the proofs are relegated to the appendices and electronic companion.

2. Literature Review

Research on buyback contracts have mostly studied the transactional role of buyback contracts in facilitating trade in distribution channels. One research stream is based on manufacturers using returns to provide incentive for retailers who face unpredictability in consumer preferences and have to make inventory ordering decisions long before the resolution of demand uncertainty (Marvel and Peck 1995, Padmanabhan and Png 1997, Wang 2004, Gurnani et al. 2010, Tran et al. 2018).[4] In particular, Gurnani et al. (2010) generalize prior work comparing no returns versus full returns policies to explicitly allow for partial returns, and form the building block of our model. The analysis in their study, however, does not include any returns risk or private information about demand potential, which is the focus of our paper. Another research stream has studied the use of return policies to achieve channel coordination (Jeuland and Shugan 1983, Pasternack 1985, Cachon

[4] Wang (2004) showed that Padmanabhan and Png's (1997) conclusions in the retail competition model no longer hold for the case of deterministic demand once the equilibrium is solved by correctly accounting for the retailers' inventory constraints.

2003, Krishnan et al. 2004). The focus of our paper, however, is on studying the informational role of returns policies in communicating upstream proprietary information such as manufacturer risk and demand potential. Relatedly, Arya and Mittendorf (2004) examine the situation where the retailer has private information on market conditions and the manufacturer uses a variety of return policies to elicit that information. We focus instead on the case where the manufacturer has superior demand information.

Researchers have examined the role of channel contracts as signals of the manufacturer's demand information to influence the retailer's decision to carry the manufacturer's product. The common premise therein is the absence of stochastic demand and inventory considerations. Hence, buyback arrangements were not considered. For instance, Chu (1992) finds that the manufacturer with higher demand signals by setting a higher wholesale price and higher advertising. Desai (2000) compares wholesale price, slotting allowance and advertising as signaling instruments (see also the earlier work by Lariviere and Padmanabhan (1997)). He finds that the wholesale price is a more efficient signaling instrument than a slotting allowance. A slotting allowance is used only to compensate the retailer if the stocking costs are high and advertising effectiveness is low. Advertising is also used to signal if the high demand manufacturer also has higher advertising effectiveness (which is unknown to the retailer). In contrast to the upward distortion of the wholesale price in these two papers, we show that in the presence of stochastic demand and buyback considerations, a high-demand potential manufacturer downward distorts its wholesale price, if used alone to signal. Instead, it is the returns price signal that features an upward distortion and plays a more important role than the wholesale price in conveying the manufacturer's demand information. Moreover, the joint use of both price signals reverses the direction of distortion in the wholesale price compared to when it is used alone. Most significantly, we show that, when the retailer must carry inventory to tackle demand risk, the buyback component plays an important role in signaling the manufacturer's private information through a characteristically different mechanism.

With the focus on the manufacturer's information sharing incentives in a distribution channel, researchers (e.g., Gal-Or et al. 2008, Jiang et al. 2016, Dukes et al. 2017) have also examined the wholesale price signal of the manufacturer's demand information (also in the absence of stochastic demand and inventory conditions). This body of literature considers that truthful information sharing can be sustained by the manufacturer's reputational concerns or initial capital investment, and hence information sharing can serve as an alternative to demand signaling. We instead focus on situations where such commitment is impractical or insufficient to sustain truthful information sharing. Moreover, we study the roles of both wholesale and returns prices in signaling demand and returns risk by explicitly accounting for stochastic demand and inventory considerations.

Research on signaling product quality to consumers through prices has also, by and large, found that higher prices are a signal of higher quality (e.g., Bagwell and Riordan 1991, Judd and Riordan 1994, Daughety and Reinganum 1995, Wang and Özkan-Seely 2018), with a few exceptions (e.g., Milgrom and Roberts 1986). In a B2C setting, Moorthy and Srinivasan (1995) show that a full money-back guarantee to consumers can serve as a high-quality signal even if price alone cannot signal. However, price can be a more efficient signal if consumers are heterogeneous in their product valuation, and a higher price can still signal higher quality even with a full money-back guarantee. A full money-back guarantee is analogous to a buyback contract where the returns price is set equal to the wholesale price. In contrast, we study buyback contracts in a B2B setting with inventory concerns. By allowing for a flexible returns price, we find that the returns price is always a more efficient signal than the wholesale price and that the use of the returns price to signal reverses the direction of distortion in the wholesale price.

In addition to price, researchers have also examined the role of other marketing levers to influence consumers' quality inference. For example, Miklós-Thal and Zhang (2013) show that demarketing selling effort can improve product quality image ex post, as consumers attribute good sales to high quality and lower sales to lack of marketing effort. When a manufacturer markets multiple products, Miklós-Thal (2012) shows that umbrella branding can be used to credibly signal positive correlation between the qualities of the products. Guo and Jiang (2016) study the effect of fairness concerns on a firm's signaling strategy when consumers experience some psychological disutility while buying products at unfair prices.

To conclude our literature review, we note that the past literature has neither examined the use of buyback contracts as signaling mechanism, nor explicitly considered manufacturers' returns risk. By demonstrating the contrasting effects between signaling product demand condition versus manufacturer reliability, our work sheds some light on the informational role of buyback contracts in channel management practice.

3. Model

We start with an overview of our model. Consider a manufacturer (hereafter referred to as "she"), who supplies a product to the end market through a retailer (hereafter referred to as "he"). The demand for the product is stochastic, and is not realized prior to the selling season. The retailer, however, must order the product ahead of the selling season. The manufacturer offers the retailer a buyback contract, specifying a wholesale price $w \geq 0$ at which the retailer can order the product prior to the selling season, and a returns price $r \geq 0$ at which the manufacturer promises to repurchase any unsold inventory at the end of the selling season. We set the manufacturer's marginal production cost to zero.[5] As a novel feature in our model, there is an intrinsic risk that the manufacturer may fail to honor the buyback commitment at the end of the selling season, either because she is not able to (e.g., bankruptcy, financial distress) or because she has the opportunity not to without attracting legal sanctions (e.g., contractual caveats, legal loopholes). We model the following decisions. The manufacturer chooses the buyback contract terms to offer. The retailer decides how much of the product to order, and the retail price to sell them. As discussed in the Introduction section, our main interest is to examine the design of the buyback contract in two distinct information scenarios: one, where the manufacturer is better informed (than the retailer) about her intrinsic reliability of honoring the returns, and the other, where the manufacturer is better informed about her product demand potential.

We now describe the product demand. The product demand is given by $d_i = \alpha_i - \beta p_i$, where α_i is the baseline demand, $\beta > 0$ is the price sensitivity, and $p_i \in [0, \alpha_i/\beta]$ is the retail price. The baseline demand α_i is stochastic and can be high (α_h) with probability $\lambda \in (0,1)$, or low (α_l with $\alpha_l < \alpha_h$) with complimentary probability $\lambda^c := 1 - \lambda$. A product with higher λ is more likely to have higher demand. We, therefore, refer to λ as the *demand potential*. For notational convenience, we introduce $\Delta\alpha := \alpha_h - \alpha_l$.

We capture the manufacturer's intrinsic reliability of honoring the buyback commitment as follows. There is an exogenous probability $\theta \in [0,1]$ that the manufacturer repurchases the retailer's unsold inventory, if any. We refer to θ as the manufacturer's *returns risk*. In particular, $\theta = 1$ corresponds to a manufacturer who always honors the repurchase clause (as is considered in prior work), whereas $\theta = 0$ corresponds to a manufacturer who never honors the contract, i.e., the returns clause is irrelevant. The situation in practice is likely to be in between these two extremes.

As mentioned before, we focus on two distinct information scenarios that we describe below:

Asymmetric Information About Returns Risk. In this scenario, the retailer is only uncertain about the manufacturer's returns risk (θ), and the demand potential (λ) is common knowledge. The manufacturer is either of a *less risky* type (with higher probability $\theta = \bar{\theta}$ of accepting returns) or of a *riskier* type (with lower probability $\theta = \underline{\theta} < \bar{\theta}$ of accepting returns); we denote $\Delta\theta := \bar{\theta} - \underline{\theta}$. The manufacturer knows her returns risk, while the retailer does not. The retailer only knows the probability that the manufacturer is less risky, denoted as $\mu \in (0,1)$. We analyze this scenario in Section 4.

Asymmetric Information About Demand Potential. In this scenario, the retailer is only uninformed about the manufacturer's demand potential (λ), and the returns risk (θ) is common knowledge. The manufacturer's demand potential is either *high* with $\lambda = \bar{\lambda}$ or *low* with $\lambda = \underline{\lambda} < \bar{\lambda}$;

[5] The qualitative nature of our results is not affected when the marginal production cost becomes positive (see details in Appendix E).

we denote $\Delta\lambda := \overline{\lambda} - \underline{\lambda}$. The manufacturer knows her demand potential, while the retailer does not. The retailer only knows the probability that the demand potential is high, denoted as $\gamma \in (0,1)$. We analyze this scenario in Section 5.

We extend our analysis to the combination of both types of private information in Section 6.

Following the buyback contracting literature (e.g., Padmanabhan and Png 1997, Gurnani et al. 2010, Tran et al. 2018), the sequence of events in both scenarios is as follows (see Figure 1). Prior to the selling season, the baseline demand is uncertain to both the manufacturer and the retailer. The manufacturer offers the retailer a buyback contract (w, r). Based on the contract, the retailer updates his belief about the manufacturer's type, and then decides the order quantity, denoted as $s \geq 0$. During the selling season, the baseline demand α_i ($i \in \{h, l\}$) realizes; accordingly, the retailer sets the retail price p_i and sells $\min\{d_i, s\}$. At the end of the selling season, the retailer returns any unsold inventory to the manufacturer at the returns price r, provided that the manufacturer honors the buyback commitment.[6] Otherwise, the retailer retains the unsold inventory, which is assumed to have no salvage value.

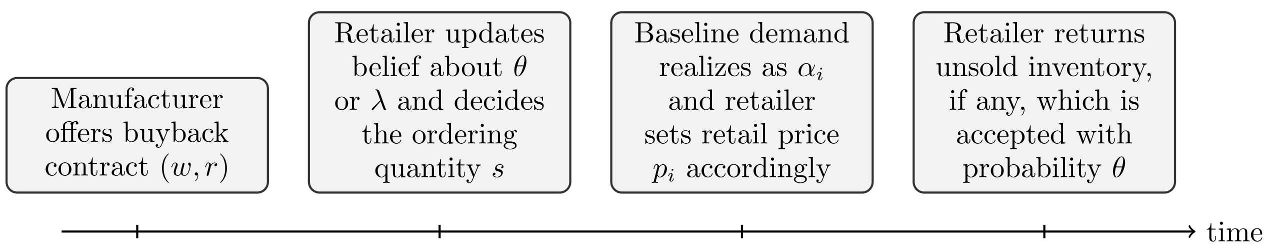


Figure 1 **Sequence of events.**

In both information scenarios, the manufacturer's buyback contract terms offered at the start of the game can "signal" her private information. We use *Perfect Bayesian Equilibrium* (PBE) as our solution concept to analyze the strategic interaction in the channel. Both firms are profit-maximizing and risk neutral. We note that the model analyzed by Gurnani et al. (2010) essentially corresponds to the symmetric information benchmark in our setup, which will be restated in Lemma 2 of Section 4.1 and Lemma 3 of Section 5.1.[7] Before proceeding, it will be useful to consider the retailer's ordering strategy in response to the manufacturer's buyback contract, which is common to both scenarios.

[6] In practice, buyback contracts typically allow the retailer to make returns only after sufficient time has passed in the selling season, for example, to ensure that that the retailer has made sufficient efforts to sell the product before attempting to return unsold units. Consequently, the retailer does not know whether the manufacturer will accept returns till the end of the selling season. The sequence of events in our model captures this situation parsimoniously. We thank an anonymous reviewer for this suggestion.

[7] The results in Lemma 2 can be obtained by replacing the returns price in their results with the expected returns price θr. Similarly, the results in Lemma 3 can be obtained by applying their results to each demand type.

3.1. Retailer's Ordering Strategy

Upon being offered a buyback contract (w,r), the retailer updates his belief that the manufacturer is less risky with probability $\widehat{\mu} := \widehat{\mu}(w,r) = \mathbb{P}\left[\theta = \bar{\theta} \,\middle|\, w,r\right]$, and has a high demand potential with probability $\widehat{\gamma} := \widehat{\gamma}(w,r) = \mathbb{P}\left[\lambda = \bar{\lambda} \,\middle|\, w,r\right]$. Therefore, the manufacturer's returns risk and likelihood of high baseline demand (α_h) perceived by the retailer are $\widehat{\theta} := \widehat{\theta}(w,r) =: \widehat{\mu}\bar{\theta} + (1-\widehat{\mu})\underline{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$ and $\widehat{\lambda} = \widehat{\lambda}(w,r) := \widehat{\gamma}\bar{\lambda} + (1-\widehat{\gamma})\underline{\lambda} \in \left[\underline{\lambda}, \bar{\lambda}\right]$, respectively. Based on his inference, the retailer makes the ordering decision, which is documented in the following lemma. For the rest of the paper, we adopt the convention that $x^+ := \max\{x, 0\}$ for any real number x.

Note that, since $\widehat{\theta}$ is the probability of return perceived by the retailer, $\widehat{\theta}r$ is essentially his perceived expected unit returns price. Thus, $w - \widehat{\theta}r$ is the retailer's perceived cost of ordering an additional unit of unsold inventory. Therefore, we find that if $w - \widehat{\theta}r < 0$, then the retailer would order infinite amount of inventory and result in negative profit for the manufacturer, who thus would never offer such contracts. Consequently, it suffices only to consider contracts with $w - \widehat{\theta}r \geq 0$. Following the analysis as in Gurnani et al. (2010), we characterize below the retailer's ordering strategy in this case.[8]

Lemma 1. *If $w - \widehat{\theta}r \geq 0$, the retailer's optimal order quantity is given by*

$$s^R\left(w, r, \widehat{\theta}, \widehat{\lambda}\right) = \frac{1}{2}\left(\widehat{\lambda}\alpha_h + \widehat{\lambda}^c\alpha_l - \beta w\right) + \frac{\widehat{\lambda}^c}{2}\left[\Delta\alpha - (\beta/\widehat{\lambda})(w - \widehat{\theta}r)\right]^+. \tag{3.1}$$

No unsold inventory is left at the retailer if the baseline demand is high (α_h), whereas inventory of an amount $\frac{1}{2}\left[\Delta\alpha - (\beta/\widehat{\lambda})\left(w - \widehat{\theta}r\right)\right]^+$ is unsold if the baseline demand is low (α_l).

The retailer's order quantity in (3.1) can be thought to consist of two parts: a "regular stock" $\frac{1}{2}(\widehat{\lambda}\alpha_h + \widehat{\lambda}^c\alpha_l - \beta w)$ that is ordered based on the *average* baseline demand $\widehat{\lambda}\alpha_h + \widehat{\lambda}^c\alpha_l$; and a "safety stock" $\frac{\widehat{\lambda}^c}{2}\left[\Delta\alpha - (\beta/\widehat{\lambda})(w - \widehat{\theta}r)\right]^+$ that is carried in anticipation of high baseline demand realization.[9] Note that in the absence of stochastic demand (e.g., $\widehat{\lambda} \in \{0,1\}$ or $\Delta\alpha = 0$), the retailer will not hold safety stock. The retailer orders positive safety stock if and only if his net expected unit cost of unsold inventory is not too high; i.e., $w - \widehat{\theta}r \leq \widehat{\lambda}\Delta\alpha/\beta$. If the baseline demand turns out to be high, then both the regular and safety stocks will be cleared and the retailer is left without any unsold inventory. In contrast, if the baseline demand turns out to be low, which the retailer believes to occur with probability $\widehat{\lambda}^c$, then unsold inventory of an amount $\frac{1}{2}\left[\Delta\alpha - (\beta/\widehat{\lambda})\left(w - \widehat{\theta}r\right)\right]^+$ will result. As such, the safety stock is proportional to the unsold inventory and can be regarded as the

[8] The results in Lemma 1 differ from the analysis in Gurnani et al. (2010) by explicitly incorporating the retailer's beliefs about the manufacturer's type.

[9] This notion of regular and safety stocks is commonly used in the inventory management literature (e.g., Zipkin 2000).

"expected" unsold inventory. As we will see later, the retailer's regular stock and safety stock are instrumental for understanding the informational role of the buyback contract. We also note that the retailer's order quantity $s^R\left(w, r, \widehat{\theta}, \widehat{\lambda}\right)$ marks the volume of trade in the distribution channel, and hence can serve as a measurement of the trade efficiency when comparing different signals, in addition to the profit measurement.

4. Asymmetric Information About Returns Risk

In this section, we study the design of the buyback contract under asymmetric information about the manufacturer's returns risk, i.e., when the retailer is uninformed about the manufacturer's likelihood of honoring the returns commitment $\theta \in \left\{\bar{\theta}, \underline{\theta}\right\}$, but is informed about the demand potential λ of the manufacturer's product.

According to Lemma 1, the retailer orders $s^R\left(w, r, \widehat{\theta}, \lambda\right)$ upfront, and returns unsold inventory of amount $\frac{1}{2}\left[\Delta\alpha - (\beta/\lambda)\left(w - \widehat{\theta}r\right)\right]^+$ only if baseline demand is low, which occurs with probability λ^c. Therefore, given the retailer's belief $\widehat{\theta} \in \left[\underline{\theta}, \bar{\theta}\right]$ about the manufacturer's risk type, the expected profit of a manufacturer of type θ is

$$\Pi\left(w, r \,\middle|\, \widehat{\theta}, \theta\right) := w s^R\left(w, r, \widehat{\theta}, \lambda\right) - \frac{1}{2}\lambda^c \theta r \left[\Delta\alpha - (\beta/\lambda)\left(w - \widehat{\theta}r\right)\right]^+, \tag{4.1}$$

where $\alpha := \lambda\alpha_h + \lambda^c\alpha_l$ denotes the mean baseline demand.

As a benchmark, we first establish the manufacturer's optimal buyback contract and the resulting outcomes under *symmetric information*, i.e., when the manufacturer's returns risk is known to the retailer. We next show how the less risky manufacturer can optimally leverage the buyback contract to credibly signal her lower returns risk when the retailer is uninformed about the manufacturer's returns risk. Finally, we elucidate the signaling mechanism further by comparing the equilibrium outcomes with those in the cases where either the wholesale or returns price alone is used in isolation to signal the manufacturer's returns risk.

4.1. Symmetric Information Benchmark

Under symmetric information, the manufacturer's returns risk θ is known to the retailer; hence, $\widehat{\theta} = \theta$. Thus, the manufacturer of type θ solves the following profit maximization problem:

$$\pi^\circ(\theta) := \max_{w \geq \theta r \geq 0} \Pi(w, r \mid \theta, \theta), \tag{4.2}$$

whose solution is denoted as $(w^\circ(\theta), r^\circ(\theta))$. The following lemma characterizes the symmetric information outcomes.

Lemma 2. *In the symmetric information benchmark, the manufacturer with returns risk θ offers a wholesale price $w^\circ(\theta) \equiv w^\circ := \alpha/(2\beta)$, independent of θ, and a returns price $r^\circ(\theta) = \alpha_l/(2\beta\theta)$, earning an expected profit of $\pi^\circ = \left[\lambda^c\lambda(\Delta\alpha)^2 + \alpha^2\right]/(8\beta)$. The retailer orders $s^\circ := \alpha_h/4$, and is left with unsold inventory $q^\circ := \Delta\alpha/4$ only if the baseline demand is low.*

We observe from Lemma 2 that when the manufacturer's returns risk is known to the retailer, the less risky manufacturer offers the same wholesale price (w°) as the riskier manufacturer does, and a lower returns price than the riskier manufacturer does, i.e., $\overline{r}^\circ := r^\circ(\overline{\theta}) < \underline{r}^\circ := r^\circ(\underline{\theta})$. Essentially, since the retailer is risk neutral, his ordering and returns decisions depend on the manufacturer's risk type θ only through the expected returns price θr. Therefore, it suffices for the manufacturer, who is also risk neutral, to optimize her contract effectively in terms of the wholesale price w and the expected returns price θr. Consequently, both manufacturer types offer the same wholesale price w° and expected returns price $\theta r^\circ(\theta)$ in the optimum (which necessitates a lower returns price for the less risky manufacturer $\overline{r}^\circ = r^\circ(\overline{\theta}) < \underline{r}^\circ = r^\circ(\underline{\theta})$ because $\bar{\theta} > \underline{\theta}$), resulting in the same retailer's order quantity s°, unsold inventory q° and the same manufacturer's expected profit π° for both manufacturer types.

4.2. Design of Buyback Contract to Signal Returns Risk

We now turn to the case where the manufacturer is privately informed about her returns risk. In this case, the manufacturer's contract terms (i.e., the wholesale and returns prices) can convey information about her risk type and subsequently the retailer can update his belief ($\widehat{\theta}$) about the manufacturer's type before making ordering and returns decisions.

We first note that if the two manufacturer types offered their respective symmetric information contracts, it is the riskier manufacturer who has the incentive to mimic the less risky manufacturer. Under the symmetric information contracts, the less risky and riskier manufacturers offer $(w^\circ, \overline{r}^\circ)$ and $(w^\circ, \underline{r}^\circ)$, respectively, with $\overline{r}^\circ < \underline{r}^\circ$. Consequently, the retailer would believe that the manufacturer is less risky (resp. riskier) if the returns price is lower (resp. higher). However, the risker manufacturer would then have an incentive to mimic the less risky manufacturer's lower returns price $\overline{r}^\circ$, because doing so will induce the retailer to order the same quantity (s°) and leave the same unsold inventory (q°) as under her own symmetric information contract but with a lower expected returns price $\underline{\theta}\overline{r}^\circ < \underline{\theta}\underline{r}^\circ$. By the same argument, the less risky manufacturer would have no incentive to mimic the riskier manufacturer's higher returns price $\underline{r}^\circ$.

Therefore, it is the less risky manufacturer who has to bear the signaling burden of distinguishing herself from the riskier type. To understand how the buyback contract should be structured to signal lower returns risk, we solve for the *most efficient separating equilibrium* (e.g., Lariviere and Padmanabhan 1997, Kalra et al. 2003, Guda and Subramanian 2019), namely the separating equilibrium that maximizes the less risky manufacturer's profit. We also show in Lemma B.2 in Appendix B that the most efficient separating equilibrium is the unique PBE that survives Cho and Kreps's (1987) intuitive criterion. In this equilibrium, the riskier manufacturer offers her symmetric information contract $(w^\circ, \underline{r}^\circ)$, while the less risky manufacturer's contract, denoted as $(\overline{w}^\star, \overline{r}^\star)$, deviates from that under symmetric information. The retailer updates his belief to $\widehat{\theta} = \bar{\theta}$ upon being offered the contract $(\overline{w}^\star, \overline{r}^\star)$ and to $\widehat{\theta} = \underline{\theta}$ otherwise. To determine her most profitable separation, the less risky manufacturer solves the following problem:

$$\begin{aligned} \overline{\pi}^\star := & \max_{\overline{w} \geq \overline{\theta}\overline{r} \geq 0} \Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\theta}, \overline{\theta}\right) \\ & \text{subject to } \Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\theta}, \underline{\theta}\right) \leq \pi^\circ \text{ and } \Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\theta}, \overline{\theta}\right) \geq \Pi\left(w^\circ, \underline{r}^\circ \,\middle|\, \underline{\theta}, \overline{\theta}\right). \end{aligned} \tag{4.3}$$

The two constraints in (4.3) ensure that mimicry is not profitable for the riskier and the less risky manufacturer, respectively. (Recall that π°, given by Lemma 2, is the riskier's manufacturer's profit of offering the symmetric information contract.) As is common in signaling games, only the first constraint (i.e., the non-mimicry condition for the riskier type) will be binding at the optimum. To establish the existence of such a separating equilibrium, we also need to show that the less risky manufacturer does not have an incentive to deviate to any other off-equilibrium buyback contract (following which the retailer updates his belief to $\widehat{\theta} = \underline{\theta}$). The following proposition establishes that the most efficient separating equilibrium always exists, and characterizes the less risky manufacturer's buyback contract $(\overline{w}^\star, \overline{r}^\star)$ and retailer's quantity decisions in this equilibrium.

Proposition 1. *The most efficient separating equilibrium of the returns risk signaling game exists. In this equilibrium,*

i) the riskier manufacturer offers her symmetric information contract $(w^\circ, \underline{r}^\circ)$*;*

ii) the less risky manufacturer offers contract $(\overline{w}^\star, \overline{r}^\star)$ *with both wholesale and returns prices lower than their respective symmetric information counterparts, i.e.,* $\overline{w}^\star < w^\circ$ *and* $\overline{r}^\star < \overline{r}^\circ < \underline{r}^\circ$*.*

iii) Under contract $(\overline{w}^\star, \overline{r}^\star)$*, the retailer's order quantity* $\overline{s}^\star$ *and unsold inventory* $\overline{q}^\star$ *in case of low baseline demand are both lower than their symmetric information counterparts, respectively, i.e.,* $\overline{s}^\star < s^\circ$ *and* $\overline{q}^\star < q^\circ$*; no unsold inventory results from high baseline demand realization.*

Proposition 1 shows that the optimal buyback contract to credibly signal low returns risk involves distorting both the wholesale and returns prices *downward* relative to those in the symmetric information contract. As can be seen from (4.1), given the retailer's belief $\widehat{\theta}$, the manufacturer's expected profit from selling to the retailer depends on her actual risk type θ only through the second term, which is her *expected* returns cost $\theta\lambda^c \cdot rc$, where $rc := r \cdot \frac{1}{2}\left[\Delta\alpha - (\beta/\lambda)\left(w - \widehat{\theta}r\right)\right]^+$ is the *returns cost* (i.e., the cost of repurchasing the retailer's unsold inventory $\frac{1}{2}\left[\Delta\alpha - (\beta/\lambda)\left(w - \widehat{\theta}r\right)\right]^+$) and $\theta\lambda^c$ is the probability that the manufacturer incurs this cost (i.e., returns occur and are honored). Thus, because $\underline{\theta} < \bar{\theta}$, lowering the returns cost generates lower expected cost savings for the risker manufacturer than for the less risky manufacturer. Consequently, to deter the riskier manufacturer's mimicry, the less risky manufacturer offers a buyback contract that distorts the returns cost downward from its symmetric information level. Proposition 1 shows that this distortion in returns cost is achieved most efficiently by distorting both the wholesale and returns prices downward. Determining the exact magnitude of these distortions requires solving a two-dimensional optimization problem, whose first-order conditions reduce to a system of bivariate quadratic equations that do not admit closed form solution in general. Nonetheless, Proposition 1 completely characterizes the qualitative nature of the equilibrium distortions.

To understand further how the most efficient separation distorts the contract terms, it is useful to express the less risky manufacturer's problem (4.3) in terms of the retailer's induced quantity decision, i.e., his regular and safety stocks.[10] According to (3.1), a less risky manufacturer's contract $(\overline{w}, \overline{r})$ induces the retailer to order:

$$\text{regular stock} \quad s_r(\overline{w}) := \frac{1}{2}\left(\alpha - \beta\overline{w}\right), \quad \text{and} \tag{4.4}$$

$$\text{safety stock} \quad \bar{s}_s(\overline{w}, \overline{r}) := \frac{\lambda^c}{2}\left[\Delta\alpha - (\beta/\lambda)\left(\overline{w} - \bar{\theta}\overline{r}\right)\right]. \tag{4.5}$$

We also note that the symmetric information regular stock and safety stock are given by

$$s_r^\circ := s_r(w^\circ) = \frac{\alpha}{4} \quad \text{and} \quad s_s^\circ := \bar{s}_s(w^\circ, \overline{r}^\circ) = \frac{\lambda^c \Delta\alpha}{4}, \quad \text{respectively.}$$

Now, using (4.1), (4.4) and (4.5), we can express the less risky manufacturer's profit in terms of the deviations in the retailer's quantity decisions from their symmetric information levels as:

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \bar{\theta}, \bar{\theta}\right) = \pi^\circ - \underbrace{\frac{2}{\beta\lambda^c}\left\{\lambda^c\left[s_r^\circ - s_r(\overline{w})\right]^2 + \lambda\left[s_s^\circ - \bar{s}_s(\overline{w}, \overline{r})\right]^2\right\}}_{\text{signaling cost}}, \tag{4.6}$$

where π° is the symmetric information profit level and the second term is essentially the less risky manufacturer's *signaling cost*, because it captures the profit reduction from π°.

Similarly, the riskier manufacturer's *gain from mimicry* (i.e., the difference between the two sides of the first constraint in (4.3)) can be rewritten as

$$\begin{aligned}
\Pi\left(\overline{w}, \overline{r} \,\middle|\, \bar{\theta}, \underline{\theta}\right) - \pi^\circ = & \left\{\Pi\left(\overline{w}, \overline{r} \,\middle|\, \bar{\theta}, \underline{\theta}\right) - \Pi\left(\overline{w}, \overline{r} \,\middle|\, \bar{\theta}, \bar{\theta}\right)\right\} + \left\{\Pi\left(\overline{w}, \overline{r} \,\middle|\, \bar{\theta}, \bar{\theta}\right) - \pi^\circ\right\} \\
= & \lambda^c \Delta\theta \cdot \underbrace{\frac{2}{\beta(\lambda^c)^2\bar{\theta}}\left[\frac{\lambda^c \alpha_l}{2} - \lambda^c s_r(\overline{w}) + \lambda \bar{s}_s(\overline{w}, \overline{r})\right]\bar{s}_s(\overline{w}, \overline{r})}_{\text{returns cost}} \\
& - \underbrace{\frac{2}{\beta\lambda^c}\left\{\lambda^c\left[s_r^\circ - s_r(\overline{w})\right]^2 + \lambda\left[s_s^\circ - \bar{s}_s(\overline{w}, \overline{r})\right]^2\right\}}_{\text{signaling cost}},
\end{aligned} \tag{4.7}$$

where we express the returns cost in terms of the retailer's quantity decisions in (4.4) and (4.5).

[10] We thank the associate editor for suggesting this approach.

From (4.7), we note that for a given level of the less risky manufacturer's signaling cost, the riskier manufacturer's gain from mimicry can be minimized by minimizing the returns cost. This goal can be achieved by increasing the regular stock $s_r(\overline{w})$ and decreasing the safety stock $\bar{s}_s(\overline{w},\overline{r})$. We note from (4.6) that the combinations of regular stock and safety stock that lead to a given level of the less risky manufacturer's signaling cost form an ellipse with the symmetric information stock levels (s_r°, s_s°) as the center, as illustrated in Figure 2. Consequently, the regular stock needs to be distorted upward relative to the symmetric information level, i.e., $s_r(\overline{w}^\star) > s_r^\circ$, while the safety stock needs to be distorted downward, i.e., $\bar{s}_s(\overline{w}^\star,\overline{r}^\star) < s_s^\circ$; see Figure 2.

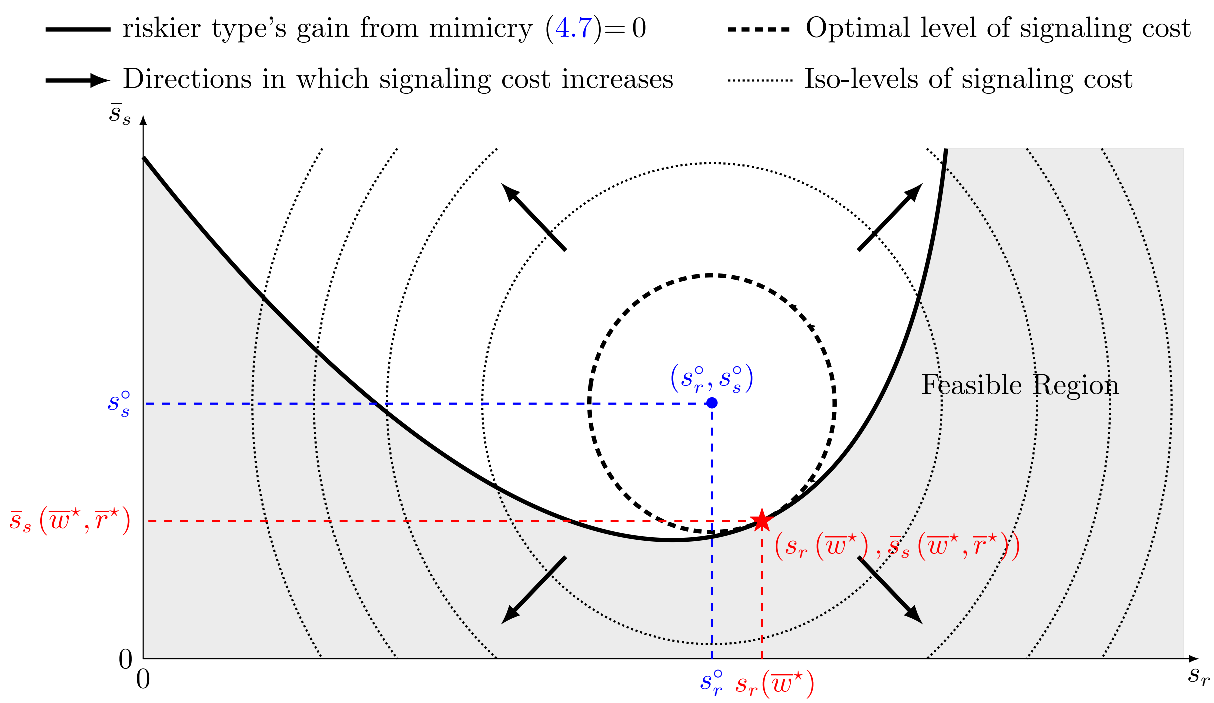


Figure 2 **The retailer's regular and safety stocks under the less risky manufacturer's most efficient separating contract** $(\overline{w}^\star,\overline{r}^\star)$ **(**$\alpha_h = 10$, $\alpha_l = 4$, $\lambda = 0.5$, $\beta = 1$, $\bar{\theta} = 1$ **and** $\underline{\theta} = 0.3$**).**

These distortions in quantities in turn uniquely determine how the wholesale and return prices are distorted. As can be seen from (4.4) and (4.5), the regular stock $s_r(\overline{w})$ is decreasing in the wholesale price and independent of the returns price, while the safety stock $\bar{s}_s(\overline{w},\overline{r})$ is also decreasing in the wholesale price but increasing in the returns price. Hence, to increase the regular stock, the wholesale price needs to be distorted downward; and to lower the safety stock, the returns price needs to be distorted downward (given that the wholesale price has been distorted downward).

Note that the decrease in the wholesale price counteracts the desired distortion in the safety stock. Thus, changing the wholesale price creates opposing effects on the regular and safety stocks relative to how they need to be distorted for efficient separation, whereas the returns price only affects the safety stock and hence does not create such opposing effects. In fact, the decrease in safety stock dominates the increase in regular stock, resulting in a net downward distortion of the retailer's order quantity (i.e., $s_r(\overline{w}^\star) + \bar{s}_s(\overline{w}^\star, \bar{r}^\star) = \bar{s}^\star < s^\circ$). Essentially, the returns price is used to lower the safety stock, while the wholesale price is used to mitigate the resulting downward distortion in the retailer's overall order quantity (by increasing the regular stock). These observations suggest that the returns price is a relatively more efficient signaling instrument than the wholesale price, a point we will elaborate in the next subsection.

4.3. Importance of Returns Price to Signal Returns Risk

Prior literature has largely focused on the informational role of the wholesale price. Our analysis shows how the buyback arrangement, and in particular the returns price, can play an important role in signaling manufacturer's returns risk. In fact, as discussed at the end of Section 4.2, the returns price may even be the relatively more efficient signaling instrument than the wholesale price. To isolate and further elucidate the individual role of the wholesale price and the returns price, we now examine two (partial) signaling benchmarks, in which either only the wholesale price or only the returns price can be distorted from their respective symmetric information levels to signal the manufacturer's returns risk. We determine the efficient separating equilibrium in each of the two benchmarks and compare them with the most efficient separation.[11]

Signaling returns risk only through wholesale price. In this benchmark, we fix the less risky manufacturer's returns price at her symmetric information level $\bar{r}^\circ$ and only allow her to set her wholesale price to signal her lower returns risk in the most profitable manner. Thus, the less risky manufacturer's most efficient wholesale price, denoted as $\overline{w}^\ddagger$, is determined as the solution to

$$\begin{aligned} \bar{\pi}^\ddagger := \max_{\overline{w} \geq \bar{\theta}\bar{r}^\circ} \;& \Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right) \\ \text{subject to } \;& \Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \underline{\theta}\right) \leq \pi^\circ \text{ and } \Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right) \geq \Pi\left(w^\circ, \underline{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right). \end{aligned} \tag{4.8}$$

The retailer's order quantity under contract $\left(\overline{w}^\ddagger, \bar{r}^\circ\right)$ is denoted as $\bar{s}^\ddagger$. Proposition B.1 in Appendix B characterizes the equilibrium outcome in this case.

Signaling returns risk only through returns price. In this benchmark, we fix the less risky manufacturer's wholesale price at the symmetric information level w°, which is incidentally the same as that offered by the riskier manufacturer, and only allow her to set her returns price to signal her lower returns risk in the most profitable manner. Thus, the less risky manufacturer's most efficient returns price, denoted as $\bar{r}^\dagger$, is determined as the solution to

$$\begin{aligned} \bar{\pi}^\dagger := \max_{w^\circ \geq \bar{\theta}\bar{r} \geq 0} \;& \Pi\left(w^\circ, \bar{r} \,\middle|\, \bar{\theta}, \bar{\theta}\right) \\ \text{subject to } \;& \Pi\left(w^\circ, \bar{r} \,\middle|\, \bar{\theta}, \underline{\theta}\right) \leq \pi^\circ \text{ and } \Pi\left(w^\circ, \bar{r} \,\middle|\, \bar{\theta}, \bar{\theta}\right) \geq \Pi\left(w^\circ, \underline{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right). \end{aligned} \tag{4.9}$$

[11] We note that any separating equilibrium within these two benchmarks is also a separating equilibrium of the original signaling game supported by suitable off-equilibrium beliefs. Thus they are well-defined equilibria of the original game, albeit (by construction) not the most efficient ones.

The retailer's order quantity under contract $(w^{\circ}, \bar{r}^{\dagger})$ is denoted as $\bar{s}^{\dagger}$. Proposition B.2 in Appendix B characterizes the equilibrium outcome in this case.

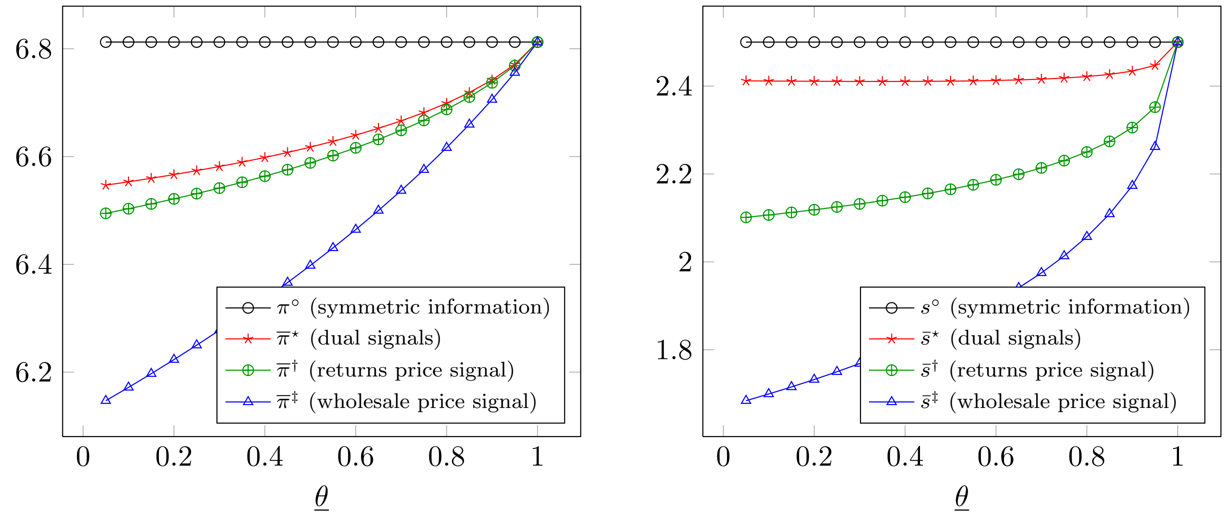
Comparison of equilibria outcomes. The following proposition summarizes our findings.

PROPOSITION 2. *For the returns risk signaling game, the equilibria outcomes under symmetric information, the most efficient separation, and the partial signaling benchmarks are ranked as follows:*

$$\overline{w}^{\star} < w^{\circ} < \overline{w}^{\ddagger}, \quad \overline{r}^{\star} < \overline{r}^{\dagger} < \overline{r}^{\circ} < \underline{r}^{\circ}, \quad \overline{s}^{\ddagger} < \overline{s}^{\dagger} < \overline{s}^{\star} < s^{\circ}, \quad \textit{and} \quad \overline{\pi}^{\ddagger} < \overline{\pi}^{\dagger} < \overline{\pi}^{\star} < \pi^{\circ}. \tag{4.10}$$

Proposition 2 demonstrates the returns price as a more efficient signal of the returns risk than the wholesale price in three dimensions. First, signaling through the returns price alone generates *higher* profit (and hence closer to the profit under the most efficient separation) for the less risky manufacturer than signaling through the wholesale price alone does, i.e., $\overline{\pi}^{\ddagger} < \overline{\pi}^{\dagger} < \overline{\pi}^{\star}$. Second, signaling through the returns price alone also yields *smaller* distortion to the retailer's order quantity (and hence closer to the order quantity under the most efficient separation), i.e., $\overline{s}^{\ddagger} < \overline{s}^{\dagger} < \overline{s}^{\star}$. Finally, signaling through the returns price alone is always achievable (shown by Proposition B.2), whereas signaling through the wholesale price alone is not always feasible. As identified by Proposition B.1, the parameter range for which separation through the wholesale price alone is feasible corresponds to situations when the information asymmetry about the returns risk is not too severe (i.e., $\Delta\theta/\bar{\theta} \le (1+\sqrt{\lambda^c})\Delta\alpha/\alpha_l$).

Furthermore, Proposition 2 shows that ignoring the informational role of the returns component in the buyback contract reverses the direction of distortion in the wholesale price, leading to qualitatively different insights regarding the design of buyback contracts when signaling only through the wholesale price. More specifically, when the wholesale price is used alone, it is distorted *upward* (i.e., $\overline{w}^{\ddagger} > w^{\circ}$), which is opposite to what we found when it is used in conjunction with the returns price in the most efficient separation. We note that in the most efficient separation as well as the partial signaling benchmarks, the distortions of equilibrium prices are driven by the less risky manufacturer's desire to reduce her returns cost. When signaling through the wholesale price alone (and fixing the returns price), the less risky manufacturer can only distort the safety stock (and hence the returns quantity) downward, which in turn necessitates an *upward* distortion of the wholesale price. When signaling through the returns price alone (and fixing the wholesale price), the less risky manufacturer distorts the returns price *downward* (i.e., $\overline{r}^{\star} < \overline{r}^{\dagger} < \overline{r}^{\circ}$) as in the most efficient separation (albeit with a smaller extent), because this reduces her returns cost by lowering both the returns price itself as well as the safety stock (and hence the returns quantity).



(a) Manufacturer's expected profit.

(b) Retailer's order quantity.

Figure 3 Equilibria performances for less risky manufacturer ($\alpha_h = 10$**,** $\alpha_l = 3$**,** $\lambda = 0.5$**,** $\beta = 1$**, and** $\bar{\theta} = 1$**).**

The superior efficiency of the returns price signal can also be illustrated by a numerical example depicted in Figure 3. As seen from Figure 3(a), the less risky manufacturer can in fact signal only using the returns price and appropriate most of her profit that she would earn in the most efficient separating equilibrium using both price intruments, whereas the profit from using only the wholesale price signal is considerably lower. From the order quantity perspective (see Figure 3(b)), the retailer's order quantity induced by the returns price signal, rather than by the wholesale price signal, is also closer to that of the most efficient equilibrium. In both figures, the gaps between each pair of equilibria profits or quantities shrink, as the severity of information asymmetry diminishes (i.e., as $\underline{\theta}$ increases to $\bar{\theta} = 1$).

Our results, taken together, shed light on the optimal design of the buyback contract in practice when the retailer is uninformed about the manufacturer's returns risk. Buyback arrangements have been shown to be an effective means for a manufacturer to encourage a retailer to carry sufficient inventory of her product. However, a challenge for a small or less-established manufacturer is that a retailer may not adequately trust the manufacturer's buyback commitment. By explicitly incorporating the manufacturer's returns risk in the analysis of buyback arrangements, we are able to address this issue. Indeed, we find that if the retailer is unsure of the manufacturer's returns risk, then the riskier manufacturer has an incentive to masquerade as the less risky manufacturer. We further find that to credibly communicate her lower returns risk, the less risky manufacturer should offer a more competitive wholesale price and a lower returns price. Doing so gains the retailer's trust at the expense of some trade-efficiency (by distorting the order quantity downwards). The retailer, on the other hand, should exercise caution against being lured by a manufacturer who offers an attractive returns price to offset a high upfront wholesale price. Adding to the price signaling literature that has mostly recognized high prices as a signal of superior quality or demand (see Section 2), our results show that lower wholesale and returns prices can signal lower returns risk. In particular, we find the returns price to be a more efficient signal than the wholesale price.

5. Asymmetric Information About Demand Potential

In this section, we study the design of the buyback contract under asymmetric information about the manufacturer's demand potential, i.e., when the retailer is uninformed about the likelihood of the manufacturer's high baseline demand $\lambda \in \{\bar{\lambda}, \underline{\lambda}\}$ with $\Delta\lambda := \bar{\lambda} - \underline{\lambda}$. For expositional simplicity, we refer to $\bar{\lambda}$ as *high-demand* type and $\underline{\lambda}$ as *low-demand* type. Their expected baseline demands are denoted as $\bar{\alpha} := \bar{\lambda}\alpha_h + \bar{\lambda}^c\alpha_l$ and $\underline{\alpha} := \underline{\lambda}\alpha_h + \underline{\lambda}^c\alpha_l$, respectively. To focus on the asymmetric information about demand potential, we assume that the manufacturer is free of returns risk (i.e., $\theta \equiv 1$).[12]

Similar to (4.1), we can formulate the manufacturer's profit function according to the retailer's ordering strategy in Lemma 1. Given the retailer's belief $\widehat{\lambda} \in [\underline{\lambda}, \bar{\lambda}]$ about the manufacturer's demand potential, the manufacturer of type $\lambda \in \{\bar{\lambda}, \underline{\lambda}\}$, who offers a buyback contract (w, r), earns an expected profit of

$$\Pi\left(w, r \,\middle|\, \widehat{\lambda}, \lambda\right) := ws^R\left(w, r, 1, \widehat{\lambda}\right) - \frac{1}{2}\lambda^c r\left[\Delta\alpha - (\beta/\widehat{\lambda})(w - r)\right]^+. \tag{5.1}$$

Similar to Section 4, we first establish the symmetric information benchmark, in which the manufacturer's demand potential is known to the retailer. We then characterize the equilibrium contract and outcomes when the retailer is uninformed about the manufacturer's demand potential. Finally, we further elucidate the signaling mechanism by comparing the equilibrium outcomes with those in the cases where each individual price alone, wholesale or returns, is used to signal the manufacturer's demand potential.

5.1. Symmetric Information Benchmark

Under symmetric information, the manufacturer's demand potential λ is known to the retailer (i.e., $\widehat{\lambda} = \lambda$). The following lemma characterizes the manufacturer's optimal contract offer in this benchmark case.

LEMMA 3. *In the symmetric information benchmark, the low-demand and high-demand manufacturer types offer the same returns price* $r^\circ = \alpha_l/(2\beta)$*, and offer wholesale prices*

$$\underline{w}^\circ := \frac{\underline{\alpha}}{2\beta} < \overline{w}^\circ := \frac{\bar{\alpha}}{2\beta}, \quad \textit{respectively,} \tag{5.2}$$

[12] This is without loss of generality because, due to the risk neutrality of the manufacturer and the retailer, θ simply appears as a scaling factor for the returns price r in all subsequent analysis. We further note that large and well-established manufacturers who typically have the resources and means to acquire proprietary demand information, also tend to have enough financial assets or reputation at stake that they will honor the buyback returns for sure.

earning expected profits of

$$\underline{\pi}^\circ := \frac{\underline{\lambda}^c\underline{\lambda}(\Delta\alpha)^2 + \underline{\alpha}^2}{8\beta} < \overline{\pi}^\circ := \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + \bar{\alpha}^2}{8\beta}, \quad \textit{respectively.} \tag{5.3}$$

In response, the retailer orders $s^\circ := \alpha_h/4$, his unsold inventory in case of low baseline demand is $q^\circ := \Delta\alpha/4$, and no unsold inventory results from high baseline demand realization.

When the manufacturer's demand potential is known to the retailer, the manufacturer essentially optimizes the prices w and $w - r$ charged to the retailer for the regular stock and safety stock, respectively. We find that high-demand manufacturer charges a higher wholesale price than the low-demand manufacturer does (i.e., $\overline{w}^\circ > \underline{w}^\circ$), while both types of manufacturer offer the same returns price r°. The retailer's order quantity s° and the unsold inventory in the case of low baseline demand q° are the same for both manufacturer types. Consequently, the high-demand manufacturer generates higher revenue because of the higher wholesale price (i.e., $\overline{w}^\circ s^\circ > \underline{w}^\circ s^\circ$) but incurs lower expected returns cost (i.e., $\bar{\lambda}^c r^\circ q^\circ < \underline{\lambda}^c r^\circ q^\circ$), thus earning higher expected profit than the low-demand manufacturer (i.e., $\overline{\pi}^\circ > \underline{\pi}^\circ$).

5.2. Design of Buyback Contract to Signal Demand Potential

We next turn to the focal case where the manufacturer's demand potential is her private information and hence her buyback contract may signal this information. We find that it is the high-demand manufacturer that must bear the signaling burden in this case; in particular, under the symmetric information contracts, the higher wholesale price enjoyed by the high-demand manufacturer creates an incentive for the low-demand manufacturer to mimic, as the induced order quantity and size of unsold inventory from the retailer would then remain unchanged (see Lemma 3). We again solve for the most efficient separating equilibrium, in which the low-demand manufacturer offers her symmetric information contract $(\underline{w}^\circ, r^\circ)$ and the high-demand manufacturer deviates from her symmetric information contract $(\overline{w}^\circ, r^\circ)$. We also show in Lemma C.1 in Appendix C that the most efficient separating equilibrium is the unique PBE that survives the intuitive criterion. Let $(\overline{w}^{\star\star}, \overline{r}^{\star\star})$ denote the high-demand manufacturer's contract. Then the retailer updates his belief to $\widehat{\lambda} = \bar{\lambda}$ upon being offered the contract $(\overline{w}^{\star\star}, \overline{r}^{\star\star})$ and to $\widehat{\lambda} = \underline{\lambda}$ otherwise. Therefore, the high-demand manufacturer's contract $(\overline{w}^{\star\star}, \overline{r}^{\star\star})$ is determined as the solution to

$$\begin{aligned} \overline{\pi}^{\star\star} := \max_{\overline{w} \geq \overline{r} \geq 0} \quad & \Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) \\ \text{subject to} \quad & \Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) \leq \underline{\pi}^\circ \text{ and } \Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) \geq \Pi\left(\underline{w}^\circ, r^\circ \,\middle|\, \underline{\lambda}, \overline{\lambda}\right), \end{aligned} \tag{5.4}$$

where the non-mimicry constraints act to deter either demand type of manufacturer from pretending to be of the other type. We establish the feasibility of the most efficient separation and characterize it in the next proposition.

PROPOSITION 3. *The most efficient separating equilibrium of the demand potential signaling game exists. In this equilibrium,*

i) the low-demand manufacturer offers her symmetric information contract $(\underline{w}^\circ, r^\circ)$*;*

ii) the high-demand manufacturer offers contract $(\overline{w}^{\star\star}, \overline{r}^{\star\star})$ *with both wholesale and returns prices higher than their respective symmetric information counterparts, i.e.,* $\overline{w}^{\star\star} > \overline{w}^\circ > \underline{w}^\circ$ *and* $\overline{r}^{\star\star} > r^\circ$*.*

iii) Under contract $(\overline{w}^{\star\star}, \overline{r}^{\star\star})$*, the retailer's order quantity* $\overline{s}^{\star\star}$ *and unsold inventory* $\bar{q}^{\star\star}$ *in case of low baseline demand are both higher than their symmetric information counterparts, i.e.,* $\overline{s}^{\star\star} > s^\circ$ *and* $\bar{q}^{\star\star} > q^\circ$*; no unsold inventory results from high baseline demand realization.*

In contrast to the returns risk case (see Proposition 1), Proposition 3 shows that the optimal buyback contract to credibly signal high demand potential distorts both the wholesale and returns prices *upward* above their symmetric information levels. Similar to the way how the returns risk type enters the manufacturer's profit function, the manufacturer's demand type λ affects her profit only through her *expected* returns cost $\lambda^c \cdot rc$, where λ^c is the likelihood of low baseline demand realization (which results in the returns) and $rc := r \cdot \frac{1}{2}\left[\Delta\alpha - (\beta/\widehat{\lambda})\,(w-r)\right]^+$ is again her *returns cost* (i.e., the cost of repurchasing the retailer's unsold inventory given that it occurs); see (5.1). However, different from the returns risk case, because $\bar{\lambda}^c < \underline{\lambda}^c$, lowering the returns cost now generates lower benefit for the high-demand manufacturer than for the low-demand manufacturer. Consequently, a contract that induces a higher returns cost is now less attractive for the low-demand manufacturer to mimic, calling for the high-demand manufacturer to distort her returns cost *upward* from her symmetric information level. Proposition 3 finds that the most efficient way of doing so is to distort both the wholesale and returns prices upward.

Similar to the returns risk case, we can uncover the mechanism behind the above-mentioned distortions by expressing the high-demand manufacturer's problem (5.4) via the retailer's quantity decision. According to (3.1), the retailer's order under the high-demand manufacturer's contract $(\overline{w}, \overline{r})$ consists of

$$\text{regular stock} \quad \bar{s}_r(\overline{w}) := \frac{1}{2}\,(\bar{\alpha} - \beta\overline{w})\,, \quad \text{and}$$
$$\text{safety stock} \quad \bar{s}_s(\overline{w}, \overline{r}) := \frac{\bar{\lambda}^c}{2}\left[\Delta\alpha - (\beta/\bar{\lambda})\,(\overline{w} - \overline{r})\right],$$

with the symmetric information regular stock and safety stock given by

$$\bar{s}_r^\circ := \bar{s}_r(\overline{w}^\circ) = \frac{\bar{\alpha}}{4} \quad \text{and} \quad \bar{s}_s^\circ := \bar{s}_s(\overline{w}^\circ, r^\circ) = \frac{\bar{\lambda}^c \Delta\alpha}{4}, \quad \text{respectively.}$$

According to (5.1), the high-demand manufacturer's profit from offering contract $(\overline{w}, \overline{r})$ can be expressed as

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) = \overline{\pi}^{\circ} - \underbrace{\frac{2}{\beta \bar{\lambda}^c} \left\{ \bar{\lambda}^c \left[\bar{s}_r^{\circ} - \bar{s}_r(\overline{w})\right]^2 + \bar{\lambda} \left[\bar{s}_s^{\circ} - \bar{s}_s(\overline{w}, \overline{r})\right]^2 \right\}}_{\text{signaling cost}}, \tag{5.5}$$

where the second term above again is the high-demand manufacturer's *signaling cost*.

Now, the low-demand manufacturer's gain from mimicry (i.e., the difference between the two sides of the first constraint in (5.4)) can be similarly expressed as

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) - \underline{\pi}^{\circ} = \overline{\pi}^{\circ} - \underline{\pi}^{\circ} - \underbrace{\frac{2}{\beta \bar{\lambda}^c} \left\{ \bar{\lambda}^c \left[\bar{s}_r^{\circ} - \bar{s}_r(\overline{w})\right]^2 + \bar{\lambda} \left[\bar{s}_s^{\circ} - \bar{s}_s(\overline{w}, \overline{r})\right]^2 \right\}}_{\text{signaling cost}}$$
$$- \Delta\lambda \cdot \underbrace{\frac{2}{\beta (\bar{\lambda}^c)^2} \left[\frac{\bar{\lambda}^c \alpha_l}{2} - \bar{\lambda}^c \bar{s}_r(\overline{w}) + \bar{\lambda} \bar{s}_s(\overline{w}, \overline{r}) \right] \bar{s}_s(\overline{w}, \overline{r})}_{\text{returns cost under contract } (\overline{w}, \overline{r})}. \tag{5.6}$$

Now, for a given level of the high-demand manufacturer's signaling cost, reducing the low-demand manufacturer's gain from mimicry (so as to lower the signaling cost in turn) calls for raising the returns cost (i.e., decreasing the last term in (5.6)). Thus, the efficient deterrence of mimicry is achieved by distorting the regular stock downward (i.e., $\bar{s}_r(\overline{w}^{\star\star}) < \bar{s}_r^{\circ}$) and the safety stock upward (i.e., $\bar{s}_s(\overline{w}^{\star\star}, \overline{r}^{\star\star}) > \bar{s}_s^{\circ}$). This is opposite to the returns risk case and consequently results in upward distortions of both the wholesale and returns prices when signaling the high demand potential. Nonetheless, akin to the returns risk case, the distortion in the wholesale prices exerts counteracting forces against the desired directions of distortion in the regular and safety stocks, whereas the returns price is able to focus on distorting the safety stock without affecting the regular stock. Again, the wholesale price is used to mitigate the distortion in the retailer's overall order quantity inflicted by the returns price. Therefore, the returns price should be a more efficient signal than the wholesale price, as will be verified in the next subsection.

5.3. Importance of Returns Price to Signal Demand Potential

Parallel to our analysis in the returns risk case, we demonstrate the higher efficiency of the returns price signal relative to the wholesale price signal by examining the following two partial signaling benchmarks and comparing their equilibrium outcomes with those in the most efficient separation.

Signaling demand potential only through wholesale price. In this benchmark, we fix the high-demand manufacturer's returns price at her symmetric information level r° and only allow her to set her wholesale price to signal her high demand potential in the most profitable manner.

Thus, the high-demand manufacturer's most efficient wholesale price, denoted as $\overline{w}^\sharp$, is determined as the solution to

$$\begin{aligned}\overline{\pi}^\sharp := \max_{\overline{w}\geq r^\circ} \ & \Pi\left(\overline{w}, r^\circ \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) \\ \text{subject to} \ & \Pi\left(\overline{w}, r^\circ \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) \leq \underline{\pi}^\circ \text{ and } \Pi\left(\overline{w}, r^\circ \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) \geq \Pi\left(\underline{w}^\circ, r^\circ \,\middle|\, \underline{\lambda}, \overline{\lambda}\right).\end{aligned} \tag{5.7}$$

The retailer's order quantity under contract $\left(\overline{w}^\sharp, r^\circ\right)$ is denoted as $\bar{s}^\sharp$. Proposition C.1 in Appendix C characterizes the equilibrium outcome in this case.

Signaling demand potential only through returns price. In this benchmark, we fix the high-demand manufacturer's wholesale price at the symmetric information level $\overline{w}^\circ$, and only allow her to set her returns price to signal her high demand potential in the most profitable manner. Thus, the high-demand manufacturer's most efficient returns price, denoted as $\overline{r}^\flat$, is determined as the solution to

$$\begin{aligned}\overline{\pi}^\flat := \max_{\overline{w}^\circ\geq\overline{r}\geq 0} \ & \Pi\left(\overline{w}^\circ, \overline{r} \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) \\ \text{subject to} \ & \Pi\left(\overline{w}^\circ, \overline{r} \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) \leq \underline{\pi}^\circ \text{ and } \Pi\left(\overline{w}^\circ, \overline{r} \,\middle|\, \overline{\lambda}, \overline{\lambda}\right) \geq \Pi\left(\underline{w}^\circ, r^\circ \,\middle|\, \underline{\lambda}, \overline{\lambda}\right).\end{aligned} \tag{5.8}$$

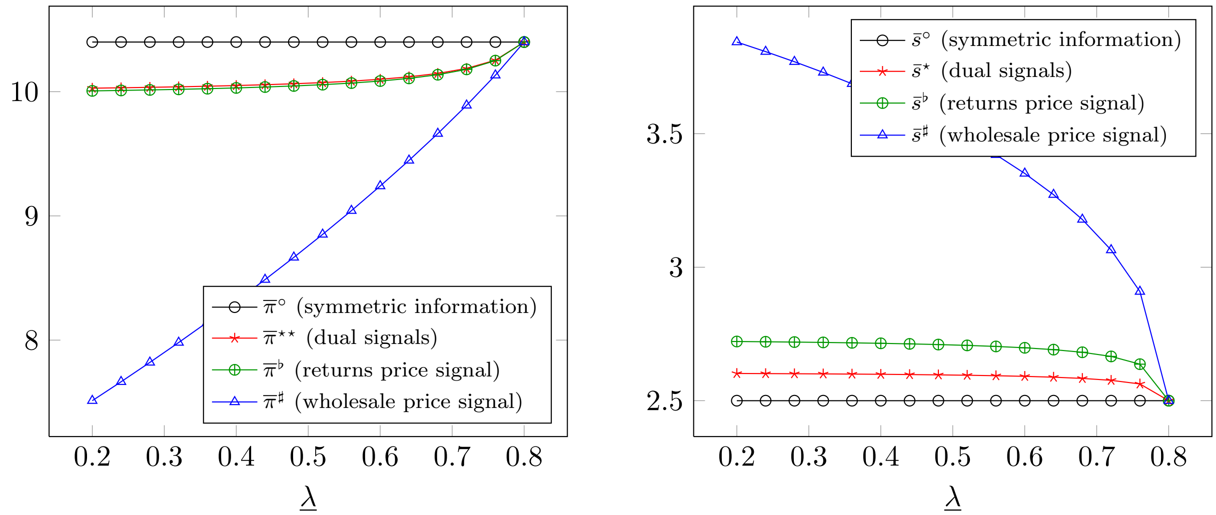
The retailer's order quantity under contract $\left(\overline{w}^\circ, \overline{r}^\flat\right)$ is denoted as $\bar{s}^\flat$. Proposition C.2 in Appendix C characterizes the equilibrium outcome in this case.

Comparison of equilibria outcomes. We now compare outcomes in different equilibria.

PROPOSITION 4. *For the demand potential signaling game, the equilibria outcomes under symmetric information, the most efficient separation, and the partial signaling benchmarks are ranked as follows:*

$$\overline{w}^{\star\star} > \overline{w}^\circ > \underline{w}^\circ > \overline{w}^\sharp, \quad \overline{r}^{\star\star} > \overline{r}^\flat > r^\circ, \quad \bar{s}^\sharp > \bar{s}^\flat > \bar{s}^{\star\star} > s^\circ, \quad \textit{and} \quad \overline{\pi}^\sharp < \overline{\pi}^\flat < \overline{\pi}^{\star\star} < \overline{\pi}^\circ. \tag{5.9}$$

Proposition 4 demonstrates the returns price as a more efficient signal of the demand potential than the wholesale price: relative to signaling through the wholesale price alone, signaling through the returns price alone generates *higher* profit (and hence closer to the profit under most efficient separation) and *smaller* distortion to the retailer's order quantity (and hence closer to the order quantity under the most efficient separation), i.e., $\overline{\pi}^\sharp < \overline{\pi}^\flat < \overline{\pi}^{\star\star}$ and $\bar{s}^\sharp > \bar{s}^\flat > \bar{s}^{\star\star}$. Furthermore, we find that the returns price alone can always signal the high demand potential (as shown by Proposition C.2), whereas the wholesale price alone can do so for certain parameter range (specifically, for $\Delta\lambda/\underline{\lambda}\left[1+\bar{\lambda}/(4\underline{\lambda})\right] > 4\alpha_h\Delta\alpha/\alpha_l^2$; see Proposition C.1). Finally, by enabling both the wholesale and returns price signals, the most efficient separation reverses the direction of distortion in the wholesale price from when the wholesale price alone is used to signal (i.e., $\overline{w}^{\star\star} > \overline{w}^\circ > \overline{w}^\sharp$) and enlarges the magnitude of distortion in the returns price when it acts alone (i.e., $\overline{r}^{\star\star} > \overline{r}^\flat > r^\circ$), albeit with the directions opposite to that in those in the returns risk case. Again, these distortions



(a) Manufacturer's expected profit.

(b) Retailer's order quantity.

Figure 4 Equilibria performances for high-demand manufacturer ($\alpha_h = 10$, $\alpha_l = 4$, $\beta = 1$, $\bar{\lambda} = 0.8$).

are driven by the high-demand manufacturer's desire to distort the retailer's induced regular and safety stocks in the directions that lead to a higher overall returns cost as required by mimicry deterrence.

Now, we illustrate in Figure 4 the superior efficiency of the returns price in conveying the manufacturer's demand information through a numerical example. Most notably, the use of returns price alone allows the manufacturer to capture most of the gains from the most efficient separation using both price instruments in terms of the manufacturer's profit (Figure 4(a)) as well as the retailer's order quantity (Figure 4(b)). As the information asymmetry diminishes (i.e., $\underline{\lambda}$ approaches to $\bar{\lambda}$), the profitability gap and trade inefficiency go down.

Our findings from Proposition 3 can be particularly relevant for large and established manufacturers who face the challenge of convincing retailers about the demand potential for their products. Prior research has examined how a manufacturer can structure the contract terms to credibly convey this information under deterministic demand (see Section 2). In the presence of stochastic demand and inventory considerations, however, the manufacturer may leverage a buyback arrangement to convey her demand information. Specifically, a high-demand manufacturer should offer a more generous returns price and a higher wholesale price than she would if her demand potential were known to the retailer. The retailer, on the other hand, should not be tempted by a lower wholesale price, and should instead pay more attention to the returns price. Thus, signaling higher demand potential requires the manufacturer to design her buyback contract in the direction opposite to that when signaling lower returns risk. Nonetheless, a consistent finding is that the returns price constitutes a more efficient signal than the wholesale price.

6. Asymmetric Information About both Returns Risk and Demand Potential

Our analysis in the previous sections shed light on the optimal design and information role of buyback contracts in situations where one type of manufacturer's inventory-related risk, either her returns risk or her demand risk, is the predominant source of unobservable information for the retailer. We find that the optimal buyback contracts to signal low returns risk and to signal high demand potential both involve distorting the manufacturer's corresponding returns cost, albeit in opposing directions. A natural question to ask then is how the buyback contract should be structured to signal the manufacturer's private information in the case where the less risky manufacturer also has high demand potential, and the retailer is uninformed about both types of inventory-related risks.[13] For example, this may represent situations where a manufacturer having higher demand potential is also likely to be in better financial health, and hence poses lower returns risk, compared to other manufacturers who struggle with their sales and are thus plagued by liquidity problems.

To answer this question, we examine the most efficient separating contract when the manufacturer can be one of two types: a *low-risk high-demand* type with probability $\bar{\theta}$ of repurchasing the retailer's unsold inventory and probability $\bar{\lambda}$ of high baseline demand realization, or a *high-risk low-demand* type with probability $\underline{\theta}$ of repurchasing the retailer's unsold inventory and probability $\underline{\lambda}$ of high baseline demand realization. That is, the manufacturer's private information consists of two dimensions that are perfectly correlated. The sequence of events is the same as specified in Section 3.

We start by examining the symmetric information benchmark, where the retailer is informed about the manufacturer's type. In this case, the low-risk high-demand manufacturer offers contract

$$\overline{w}^\circ = \frac{\bar{\alpha}}{2\beta} \quad \text{and} \quad \overline{r}^\circ = \frac{\alpha_l}{2\beta\bar{\theta}}, \quad \text{earning profit } \overline{\pi}^\circ = \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + \bar{\alpha}^2}{8\beta}, \tag{6.1}$$

while the high-risk low-demand manufacturer offers contract

$$\underline{w}^\circ = \frac{\underline{\alpha}}{2\beta} \quad \text{and} \quad \underline{r}^\circ = \frac{\alpha_l}{2\beta\underline{\theta}}, \quad \text{earning profit } \underline{\pi}^\circ = \frac{\underline{\lambda}^c\underline{\lambda}(\Delta\alpha)^2 + \underline{\alpha}^2}{8\beta}. \tag{6.2}$$

Under symmetric information, the low-risk high-demand manufacturer enjoys a higher wholesale price (i.e., $\overline{w}^\circ > \underline{w}^\circ$) and a higher expected profit (i.e., $\overline{\pi}^\circ > \underline{\pi}^\circ$) while offering a lower returns price (i.e., $\overline{r}^\circ < \underline{r}^\circ$) than the high-risk low-demand manufacturer. Consequently, under asymmetric information, the high-risk low-demand manufacturer (namely, the *bad* type) would have an incentive to mimic the low-risk high-demand manufacturer (namely, the *good* type). Hence, the good type needs to distort her buyback contract from that under symmetric information so as to separate herself from the bad type. We now characterize the most-efficient separating equilibrium.

[13] We thank the department editor, the associate editor, and an anonymous reviewer for suggesting this investigation.

Proposition 5. *In the most efficient separating equilibrium of the returns risk and demand potential signaling game, if it exists, the high-risk low-demand manufacturer* $(\underline{\theta}, \underline{\lambda})$ *offers her symmetric information contract* $(\underline{w}^\circ, \underline{r}^\circ)$*, while the equilibrium contract offered by the low-risk high-demand manufacturer* $(\bar{\theta}, \bar{\lambda})$*, denoted as* $(\overline{w}^{\star\star\star}, \overline{r}^{\star\star\star})$*, demonstrates the following characteristics:*

1. *If* $\underline{\theta\lambda}^c < \bar{\theta}\bar{\lambda}^c$*, then* $(\overline{w}^{\star\star\star}, \overline{r}^{\star\star\star})$ *either i) satisfies* $\overline{w}^{\star\star\star} < \overline{w}^\circ$ *and* $0 \leq \overline{r}^{\star\star\star} < \overline{r}^\circ$ *or ii) induces the retailer to carry no safety stock (and hence make no returns).*
2. *If* $\underline{\theta\lambda}^c > \bar{\theta}\bar{\lambda}^c$*, then* $\overline{w}^{\star\star\star} > \overline{w}^\circ$ *and* $\overline{r}^{\star\star\star} > \overline{r}^\circ$*.*
3. *If* $\underline{\theta\lambda}^c = \bar{\theta}\bar{\lambda}^c$*, then there exist at least one contract such that* $\overline{w}^{\star\star\star} < \overline{w}^\circ$ *and* $\overline{r}^{\star\star\star} < \overline{r}^\circ$ *as well as one such that* $\overline{w}^{\star\star\star} > \overline{w}^\circ$ *and* $\overline{r}^{\star\star\star} > \overline{r}^\circ$*.*

As before we find that efficient separation requires that the manufacturer distort the returns cost. For a manufacturer of type (θ, λ), the net probability of incurring the returns cost is given by $\theta\lambda^c$, which is the joint probability of a low baseline demand realization (which results in unsold inventory) and the manufacturer's acceptance of the retailer's returns. Proposition 5 shows that the direction of distortion depends on whether the net probability of incurring the returns cost is higher or lower for the good type than low type, leading to the following three cases.

- When $\underline{\theta\lambda}^c < \bar{\theta}\bar{\lambda}^c$, the effect of the returns risk dominates that of the demand potential; hence, the good type should design her optimal buyback contract to distort the returns cost downward relative to the symmetric information benchmark, as in the returns risk case. As a result, the safety stock is distorted downward. If the equilibrium safety stock is positive, then both the wholesale and returns prices are distorted *downward* (i.e., $\overline{w}^{\star\star\star} < \overline{w}^\circ$ and $\overline{r}^{\star\star\star} < \overline{r}^\circ$), the same as in the returns risk case. In particular, this case reduces to the returns risk case if $\underline{\lambda} = \bar{\lambda}$ (see section 4). However, it is possible for the equilibrium safety stock to be distorted to zero, in which case the direction of distortion in the equilibrium prices is not uniquely determined.[14]
- Instead, when $\underline{\theta\lambda}^c > \bar{\theta}\bar{\lambda}^c$, the effect of the demand potential dominates that of the returns risk; hence, the good type should design her optimal buyback contract to distort the returns cost upward. Subsequently, both the wholesale and returns prices are distorted *upward* (i.e., $\overline{w}^{\star\star\star} > \overline{w}^\circ$ and $\overline{r}^{\star\star\star} > \overline{r}^\circ$), the same as in the demand potential case. Indeed, this case reduces to the demand potential case if $\underline{\theta} = \bar{\theta} = 1$ (see section 5).
- Lastly, when $\underline{\theta\lambda}^c = \bar{\theta}\bar{\lambda}^c$, the effects of the returns risk and the demand potential balance each other. Then, we find that the direction of distortion in the wholesale and returns prices is not uniquely determined. In particular, the wholesale and returns prices can both be distorted downward (i.e., $\overline{w}^{\star\star\star} < \overline{w}^\circ$ and $\overline{r}^{\star\star\star} < \overline{r}^\circ$) as in the returns risk case, or both be distorted upward (i.e., $\overline{w}^{\star\star\star} > \overline{w}^\circ$ and $\overline{r}^{\star\star\star} > \overline{r}^\circ$) as in the demand potential case.

[14] This situation may emerge only for sufficiently large $\Delta\lambda$ (i.e., $\Delta\lambda \geq \bar{\lambda}^c \min\left\{\alpha_l^2, \bar{\lambda}(\Delta\alpha)^2\right\} / [\Delta\alpha(\alpha_l + \alpha_h)]$) as shown by Lemma D.5.

Thus, Proposition 5 generalizes our previous findings and allows us to identify a unified signaling mechanism across the different settings. This mechanism can be again uncovered by expressing the bad type's gain from mimicry in terms of the retailer's quantity decisions:

$$\begin{aligned}\text{bad type's gain from mimicry} = &\overline{\pi}^\circ - \underline{\pi}^\circ - \underbrace{\frac{2}{\beta\bar{\lambda}^c}\left\{\bar{\lambda}^c\left[\bar{s}_r^\circ - \bar{s}_r(\overline{w})\right]^2 + \bar{\lambda}\left[\bar{s}_s^\circ - \bar{s}_s(\overline{w},\overline{r})\right]^2\right\}}_{\text{good type's signaling cost}} \\ &- \left(\underline{\theta\lambda}^c - \bar{\theta}\bar{\lambda}^c\right)\cdot \underbrace{\frac{2}{\beta\bar{\theta}(\bar{\lambda}^c)^2}\left[\frac{\bar{\lambda}^c\alpha_l}{2} - \bar{\lambda}^c\bar{s}_r(\overline{w}) + \bar{\lambda}\bar{s}_s(\overline{w},\overline{r})\right]\bar{s}_s(\overline{w},\overline{r})}_{\text{returns cost under contract }(\overline{w},\overline{r})},\end{aligned}$$

where $\bar{s}_r(\overline{w}) := \frac{1}{2}\left(\bar{\alpha} - \beta\overline{w}\right)$ and $\bar{s}_s(\overline{w},\overline{r}) := \frac{\bar{\lambda}^c}{2}\left[\Delta\alpha - (\beta/\bar{\lambda})\left(\overline{w} - \bar{\theta}\overline{r}\right)\right]$ are the retailer's regular and safety stocks, respectively, with $\bar{s}_r^\circ$ and $\bar{s}_s^\circ$ being their symmetric information levels. As before, for a given level of signaling cost, efficient separation aims to reduce the bad type's gain from mimicry by distorting the returns cost in the last term above. The direction of distortion, however, critically depends on the sign of the difference in the net probability of incurring the returns cost, $\underline{\theta\lambda}^c - \bar{\theta}\bar{\lambda}^c$, between the bad and good types, as characterized by Proposition 5.

We find that the most efficient separating equilibrium may not always exist because, under two-dimensional private information, mimicking the good type entails the benefit of being perceived to be of both low returns risk and high demand potential, resulting in higher gain from mimicry for the bad type than that under a single-dimensional case. Establishing the exact condition for the existence of the most-efficient separating equilibrium is analytically intractable. The following corollary provides sufficient conditions under which we are able to establish the existence of the most efficient separating equilibrium analytically.

Corollary 1. *The most efficient separating equilibrium of the returns risk and demand potential signaling game always exists under any one of the following conditions:*

1. $\Delta\theta/\bar{\theta} \geq 1 - \frac{\bar{\lambda}^c}{\left(1+\sqrt{\underline{\lambda}}\right)^2}$ *or* $\Delta\theta/\bar{\theta} > \Delta\lambda/\underline{\lambda}^c \geq \frac{\bar{\lambda}^c \min\left\{\alpha_l^2,\, \bar{\lambda}(\Delta\alpha)^2\right\}}{\underline{\lambda}^c\Delta\alpha(\alpha_l+\alpha_h)}$*, which both imply* $\underline{\theta\lambda}^c < \bar{\theta}\bar{\lambda}^c$.
2. $\Delta\theta/\bar{\theta} \leq \min\left\{\frac{\left(\underline{\lambda}^c\bar{\lambda} - \bar{\lambda}^c\Delta\lambda\right)\Delta\alpha}{2\underline{\lambda}^c\bar{\alpha}},\, 1 - \frac{\bar{\lambda}^c}{\left(1-\sqrt{\underline{\lambda}}\right)^2}\right\}$*, which implies* $\underline{\theta\lambda}^c > \bar{\theta}\bar{\lambda}^c$.
3. $\underline{\theta\lambda}^c = \bar{\theta}\bar{\lambda}^c$.

Numerical analysis shows that the separating equilibrium can exist even beyond the above sufficient conditions. In cases where the separating equilibrium does not exist, the manufacturer types may pool on the buyback contract (i.e., offer the same contract), eliminating the informational role of the buyback contract.

7. Conclusion

Retailers often face the challenge of managing their inventory to match supply with uncertain demand. Past research has extensively examined the use of buyback arrangements by manufacturers to share inventory risk with their retailers, under the assumption that the buyback commitment will be honored and that both manufacturers and retailers are equally informed about the product's demand potential. In practice, however, not all manufacturers may be able to honor their buyback commitment, thus making retailers wary of buyback offers. Or, retailers may be less informed about market conditions than the manufacturers, and thus be unconvinced about a product's demand potential, leading to lower order quantities. We seek to shed light on the use and design of buyback arrangements in such situations.

Overall, our findings highlight the strategic and informational role of buyback contracts, over and above their oft-studied transactional role in the literature. In the presence of stochastic demand and inventory considerations, efficient signaling of manufacturer's returns risk or demand potential necessitates distorting its *returns cost* (i.e., the cost of repurchasing the retailer's unsold inventory) away from the symmetric-information level. The direction of distortion depends on whether the separating type has a higher or lower *net probability of incurring the returns cost* (i.e., probability that returns occur and is honored by the manufacturer) than the mimicking manufacturer type. The signaling distortion in returns cost uniquely determines how the retailer's induced regular stock and safety stock are distorted and, in turn, how the underlying contractual prices are adjusted. In fact, changing the wholesale price creates opposing effects on the regular and safety stocks relative to how they need to be distorted for efficient separation, whereas the returns price only affects the safety stock without creating such opposing effects. As a result, we find the returns price to be a relatively more efficient signaling instrument than the wholesale price. In particular, the returns price reverses the direction of distortion in the wholesale price from what is necessary for the wholesale price alone to distort the returns cost, and the wholesale price is used to mitigate the signaling distortion in the retailer's overall order quantity.

This novel signaling mechanism results in contrasting design of the buyback contracts between signaling the manufacturer's returns risk and signaling her demand potential. Efficient signaling of low returns risk entails *downward* distortion of both the wholesale and returns prices below their symmetric-information counterparts, whereas efficient signaling of high demand potential entails *upward* distortion of both prices. If the manufacturer needs to signal both low returns risk and high demand potential, then the direction of the price distortions depends on the manufacturer's net probability of incurring the returns cost. When this net probability is higher for the "good" manufacturer (who is better on both dimensions), then the direction of distortion is the same as that in the case of signaling low returns risk alone. Instead, when the net probability is higher for the "bad" manufacturer (who is worse on both dimensions), then the direction of distortion is the same as that in the case of signaling high demand potential alone.

Our research speaks to manufacturers who manage their distribution channels plagued with asymmetric information about inventory-related risks. In a market with small and less-established manufacturers, a more competitive wholesale price together with a lower returns price can help the less risky manufacturers to distinguish themselves and assure the downstream retailers of their reliability to fulfill their returns commitment. For markets dominated by large and well-established manufacturers (e.g., national brands), returns risk may not be an issue. Yet, manufacturers typically have the incentive to acquire proprietary information about the downstream demand potential. In this situation, a higher wholesale price together with a more generous returns price can signal to the downstream retailers the confidence of manufacturers with high demand potential.

We regard our work as a first attempt to examine the informational role of buyback arrangements with a number of directions for future exploration. For instance, we assumed no salvage value for retailer's unsold inventory. A positive salvage value should not affect our results if it is lower than the equilibrium returns price. We further expect that our results will continue to hold qualitatively if the salvage value is not too high. However, if the salvage value is sufficiently high, it lowers the retailer's reliance on the manufacturer's returns price and it becomes more onerous for the manufacturer to signal through the returns price. We defer the analysis for this case as future research. Another interesting avenue is to examine the role of manufacturer's trade credit.[15] Trade credit refers to short-term financing offered by a supplier to a downstream buyer to facilitate the purchase of supplies without immediate payment. While offering trade credit can potentially mitigate the retailer's returns risk, it poses significant costs for the manufacturer (e.g., Woodruff 2009). In particular, small and less-established manufacturers who typically pose returns risk for the retailer may not be able to offer trade credit. Moreover, trade credit may also be used by manufacturers to screen buyers with private default risk (e.g., Smith 1987). This context is different from ours as manufacturers in our setting have private information. It would be interesting for future research to study the interaction and tradeoff between offering trade credit and signaling returns risk through the buyback contract. Finally, we assume that both manufacturer and retailer are risk neutral in our setting. Future research could examine the implication of relaxing the risk-neutrality of one or both channel members (e.g., Jiang et al. 2016).

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[15] We thank an anonymous reviewer for this suggestion.

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Electronic Companion for "The Informational Role of Buyback Contracts"

Appendix A: Proofs in Section 3

To derive the retailer's response, we first examine the retailer's pricing decision after the uncertain baseline demand has realized during the selling season. Conditional on the contract (w, r), the retailer's perceived return risk $\widehat{\theta}$, the realized baseline demand $\alpha_i \in \{\alpha_l, \alpha_h\}$ and the retailer's stocking quantity s, the retailer sets his retail price p_i by maximizing his posterior profit:[16]

$$\Pi_i^R\left(s \,\middle|\, w, r, \widehat{\theta}\right) := \max_{\substack{0 \le p_i \le \alpha_i/\beta \\ d_i = \alpha_i - \beta p_i}} p_i \min\{d_i, s\} + \widehat{\theta} r\left[s - \min\{d_i, s\}\right] - ws, \tag{A.1}$$

whose solution, the retailer's price decision, is denoted as $p_i^R = p_i^R\left(s \,\middle|\, w, r, \widehat{\theta}\right)$ for $i = h, l$. We characterize the retailer's optimal pricing decision in the following lemma.

Lemma A.1. *For $i = l, h$, (1) if $s \le \frac{1}{2}\left(\alpha_i - \beta\widehat{\theta}r\right)$, then the retailer sets the price $p_i^R = (\alpha_i - s)/\beta$ to clear the stock; (2) if $s \ge \frac{1}{2}\left(\alpha_i - \beta\widehat{\theta}r\right)$, then the retailer sets the price $p_i^R = (\alpha_i + \beta\widehat{\theta}r)/(2\beta)$ and the unsold inventory at the end of the selling season is $s - \frac{1}{2}\left(\alpha_i - \beta\widehat{\theta}r\right)$. The retailer's optimal posterior profit is given by*

$$\Pi_i^R\left(s \,\middle|\, w, r, \widehat{\theta}\right) = \begin{cases} \left(\frac{\alpha_i - s}{\beta} - w\right)s, & \text{if} \quad s \le \frac{\alpha_i - \beta\widehat{\theta}r}{2}, \\ \frac{\left(\alpha_i - \beta\widehat{\theta}r\right)^2}{4\beta} - \left(w - \widehat{\theta}r\right)s, & \text{if} \quad s \ge \frac{\alpha_i - \beta\widehat{\theta}r}{2}. \end{cases} \tag{A.2}$$

Proof of Lemma A.1. We now rewrite (A.1) as

$$\Pi_i^R\left(s \,\middle|\, w, r, \widehat{\theta}\right) = \max_{0 \le p_i \le \frac{\alpha_i}{\beta}} \left(p_i - \widehat{\theta}r\right)\min\{\alpha_i - \beta p_i, s\} - \left(w - \widehat{\theta}r\right)s,$$

where

$$\left(p_i - \widehat{\theta}r\right)\min\{\alpha_i - \beta p_i, s\} = \begin{cases} \left(p_i - \widehat{\theta}r\right)s, & \text{if} \quad p_i \le \frac{\alpha_i - s}{\beta}, \\ \left(p_i - \widehat{\theta}r\right)(\alpha_i - \beta p_i), & \text{if} \quad p_i \ge \frac{\alpha_i - s}{\beta}. \end{cases} \tag{A.3}$$

We note that $\left(p_i - \widehat{\theta}r\right)(\alpha_i - \beta p_i)$ is a quadratic function of p_i that achieves its (unconstrained) maximum at $p_i = (\alpha_i + \beta\widehat{\theta}r)/(2\beta)$. Therefore,

(1) if $\frac{\alpha_i + \beta\widehat{\theta}r}{2\beta} \le \frac{\alpha_i - s}{\beta}$, i.e., $s \le \frac{1}{2}\left(\alpha_i - \beta\widehat{\theta}r\right)$, (A.3) achieves its maximum at $p_i^R = (\alpha_i - s)/\beta$;

(2) if, instead, $\frac{\alpha_i + \beta\widehat{\theta}r}{2\beta} \ge \frac{\alpha_i - s}{\beta}$, i.e., $s \ge \frac{1}{2}\left(\alpha_i - \beta\widehat{\theta}r\right)$, (A.3) achieves its maximum at $p_i^R = (\alpha_i + \beta\widehat{\theta}r)/(2\beta)$.

Substituting the optimal price p_i^R into (A.3) immediately yields (A.2). □

Given his posterior profit function $\Pi_i^R\left(s \,\middle|\, w, r, \widehat{\theta}\right)$, the retailer maximizes the following *ex ante* expected profit by choosing the inventory stocking quantity at the beginning of the selling season:

$$\max_{s \ge 0} \ \widehat{\lambda}\Pi_h^R\left(s \,\middle|\, w, r, \widehat{\theta}\right) + \widehat{\lambda}^c\Pi_l^R\left(s \,\middle|\, w, r, \widehat{\theta}\right). \tag{A.4}$$

Proof of Lemma 1. For notational efficiency, we denote in this proof that $\widehat{\alpha} := \widehat{\lambda}\alpha_h + \widehat{\lambda}^c\alpha_l$. By Lemma A.1, the objective function in (A.4) reduces to

$$\begin{cases} \left(\frac{\widehat{\alpha} - s}{\beta} - w\right)s, & \text{if} \quad s \le \frac{\alpha_l - \beta\widehat{\theta}r}{2}, \\ \widehat{\lambda}\frac{(\alpha_h - s)s}{\beta} - \left(w - \widehat{\lambda}^c\widehat{\theta}r\right)s + \widehat{\lambda}^c\frac{\left(\alpha_l - \beta\widehat{\theta}r\right)^2}{4\beta}, & \text{if} \quad \frac{\alpha_l - \beta\widehat{\theta}r}{2} \le s \le \frac{\alpha_h - \beta\widehat{\theta}r}{2}, \\ -\left(w - \widehat{\theta}r\right)s + \widehat{\lambda}^c\frac{\left(\alpha_l - \beta\widehat{\theta}r\right)^2}{4\beta} + \widehat{\lambda}\frac{\left(\alpha_h - \beta\widehat{\theta}r\right)^2}{4\beta}, & \text{if} \quad s \ge \frac{\alpha_h - \beta\widehat{\theta}r}{2}, \end{cases} \tag{A.5}$$

[16] By "posterior", we refer to "after the realization of uncertain baseline demand".

where $\left(\frac{\widehat{\alpha}-s}{\beta}-w\right)s$ is a quadratic function achieving its unconstrained maximum at $s_1 := \frac{\widehat{\alpha}-\beta w}{2}$; $\widehat{\lambda}\frac{(\alpha_h-s)s}{\beta} - \left(w-\widehat{\lambda}^c\widehat{\theta}r\right)s + \widehat{\lambda}^c\frac{\left(\alpha_l-\beta\widehat{\theta}r\right)^2}{4\beta}$ is also a quadratic function achieving its unconstrained maximum at $s_2 := \frac{\widehat{\lambda}^c\beta\widehat{\theta}r+\widehat{\lambda}\alpha_h-\beta w}{2\widehat{\lambda}}$; and $-\left(w-\widehat{\theta}r\right)s+\widehat{\lambda}^c\frac{\left(\alpha_l-\beta\widehat{\theta}r\right)^2}{4\beta}+\widehat{\lambda}\frac{\left(\alpha_h-\beta\widehat{\theta}r\right)^2}{4\beta}$ is a linear function of s.

- If $w-\widehat{\theta}r<0$, obviously, the retailer would stock infinite inventory and earn infinite expected profit. As such, the manufacturer would never offer such a contract.
- If $0\le w-\widehat{\theta}r\le\frac{\widehat{\lambda}\Delta\alpha}{\beta}$, then it is straightforward to verify that

$$\frac{\alpha_l-\beta\widehat{\theta}r}{2}\le s_1\le s_2\le\frac{\alpha_h-\beta\widehat{\theta}r}{2}. \tag{A.6}$$

Therefore, $s^R=s_2$. By Lemma A.1, (A.6) implies that all inventory is sold out in the case of high baseline demand α_h realization while there is an excess of inventory $s_2-\frac{\alpha_l-\beta\widehat{\theta}r}{2}=\frac{1}{2}\left[\Delta\alpha-\frac{\beta}{\widehat{\lambda}}\left(w-\widehat{\theta}r\right)\right]$ in the case of low baseline demand α_l realization.

- If $w-\widehat{\theta}r\ge\frac{\widehat{\lambda}\Delta\alpha}{\beta}$, then it is straightforward to verify that

$$s_2\le s_1\le\frac{\alpha_l-\beta\widehat{\theta}r}{2}\le\frac{\alpha_h-\beta\widehat{\theta}r}{2}. \tag{A.7}$$

Therefore, $s^R=s_1$. By Lemma A.1, (A.7) implies that all inventory will be sold out whether the baseline demand is α_h or α_l. ☐

Appendix B: Proofs in Section 4

LEMMA B.1. *Given $\theta\in\{\underline{\theta},\overline{\theta}\}$, any contract (w,r) such that $w-\theta r<0$ is weakly dominated by a contract (w,r') such that $w-\theta r'\ge 0$.*

Proof of Lemma B.1. If $w-\theta r<0$, because $w\ge0$, there must exists $r'\in[0,r]$ such that $w-\theta r'\ge0$. Then, we must have

$$\begin{aligned}\Pi\left(w,r'\,\middle|\,\widehat{\theta}(w,r'),\theta\right)=&\frac{\lambda^c\beta}{2\lambda}\underbrace{(w-\theta r')}_{\ge0}\left[\frac{\lambda\Delta\alpha}{\beta}-w+\widehat{\theta}(w,r')r'\right]^+ +\frac{1}{2}\left[-\beta w^2+\alpha w\right]\\ \ge&\frac{1}{2}\left(-\beta w^2+\alpha w\right)\\ \ge&\frac{\lambda^c\beta}{2\lambda}\underbrace{(w-\theta r)}_{<0}\left[\frac{\lambda\Delta\alpha}{\beta}-w+\widehat{\theta}(w,r)r\right]^+ +\frac{1}{2}\left(-\beta w^2+\alpha w\right)=\Pi\left(w,r\,\middle|\,\widehat{\theta}(w,r),\theta\right),\end{aligned}$$

which demonstrates the result. ☐

Proof of Lemma 2. After simple algebraic manipulation, it is straightforward to verify that the manufacturer's expected profit in (4.1) can be rewritten as

$$\Pi\left(w,r\,\middle|\,\widehat{\theta},\theta\right)=H\left(w,w-\theta r,w-\widehat{\theta}r\right), \tag{B.1}$$

where

$$H\left(w,u,\widehat{u}\right):=\frac{\lambda^c\beta}{2\lambda}u\left(\frac{\lambda\Delta\alpha}{\beta}-\widehat{u}\right)^+ +\frac{1}{2}\left(-\beta w^2+\alpha w\right). \tag{B.2}$$

Let (w°,u°) be the solution to the following optimization problem:

$$\max_{w\ge0,0\le u\le w}H\left(w,u,u\right). \tag{B.3}$$

Then, $w^\circ(\theta) = w^\circ$ and $r^\circ(\theta) = (w^\circ - u^\circ)/\theta$ solves (4.2). Our general strategy to identify the solution (w°, u°) to (B.3) is to first optimize $H(w,u,u)$ over $u \in [0,w]$, and then optimize the resulting function that contains only w.

We first note that function $u(\lambda\Delta\alpha/\beta - u)^+$ is increasing in $u \in [0, \lambda\Delta\alpha/(2\beta)]$, decreasing in $u \in [\lambda\Delta\alpha/(2\beta), \lambda\Delta\alpha/\beta]$, and remains a constant zero for $u \geq \lambda\Delta\alpha/\beta$. Therefore, we examine the following two cases.

1. For any $w \leq \lambda\Delta\alpha/(2\beta)$, the maximizing u in (B.3) must equal w, suggesting

$$\max_{0\leq u\leq w} H(w,u,u) = H(w,w,w) = \frac{1}{2\lambda}\left[-\beta w^2 + \lambda\alpha_h w\right],$$

which is increasing $w \leq \lambda\Delta\alpha/(2\beta) < \alpha/(2\beta)$. As such, we only need to restrict to $w \geq \lambda\Delta\alpha/(2\beta)$.

2. For any $w \geq \lambda\Delta\alpha/(2\beta)$, we have $\max_{0\leq u\leq w} H(w,u,u)$ is achieved at $u^\circ = \lambda\Delta\alpha/(2\beta)$, suggesting

$$\max_{0\leq u\leq w} H(w,u,u) = \frac{\lambda^c(\lambda\Delta\alpha - \beta c)^2}{8\beta\lambda} + \frac{1}{2}\left[-\beta w^2 + \alpha w\right],$$

which is quadratic in w and achieves its maximum $\Pi^\circ(\theta) = \frac{\lambda^c\lambda(\Delta\alpha)^2 + \alpha^2}{8\beta}$ at $w^\circ = \alpha/(2\beta)$. Thus, $r^\circ(\theta) = (w^\circ - u^\circ)/\theta = \alpha_l/(2\beta\theta)$. □

Since $w^\circ - \theta r^\circ(\theta) = \frac{\alpha}{2\beta} - \frac{\alpha_l}{2\beta} = \frac{\lambda\Delta\alpha}{2\beta} \leq \frac{\lambda\Delta\alpha}{\beta}$, which, by (3.1) of Lemma 1, suggests that the retailer orders

$$s^\circ = s^R(w^\circ, r^\circ(\theta), \theta, \lambda) = \frac{\lambda^c\beta\theta r^\circ(\theta) + \lambda\alpha_h - \beta w^\circ}{2\lambda} = \frac{\alpha_h}{4}$$

and that inventory is sold out if the baseline demand is high, but that unsold inventory of an amount $\frac{1}{2}\left[\Delta\alpha - \frac{\beta}{\lambda}(w^\circ - \theta r^\circ(\theta))\right] = \frac{\Delta\alpha}{4}$ will be requested by the retailer to return. □

Lemma B.2. *In the returns risk signaling game, the unique equilibrium that survives the intuitive criterion is the most efficient separating equilibrium. In this equilibrium, the riskier manufacturer offers her symmetric information contract* $(w^\circ, \underline{r}^\circ)$.

Proof of Lemma B.2. We first note that $\Pi\left(w, r \,\middle|\, \widehat{\theta}, \theta\right)$ is increasing in $\widehat{\theta}$, as direct calculation reveals that

$$\Pi_{\widehat{\theta}}\left(w, r \,\middle|\, \widehat{\theta}, \theta\right) = \begin{cases} \frac{\lambda^c\beta}{2\lambda}(w - \theta r)r \geq 0, & \text{if} \quad \beta(w - \widehat{\theta}r) \geq \lambda\Delta\alpha,, \\ 0, & \text{if} \quad \beta(w - \widehat{\theta}r) \leq \lambda\Delta\alpha. \end{cases}$$

By the weakened condition of Cho and Sobel (1990) in Engers (1987), it suffices to show that the marginal rate of substitution (MRS) of *one* of signals (i.e., w or r) for the belief $\widehat{\theta}$ is monotonic in θ. Direct calculation reveals that the MRS of r for $\widehat{\theta}$ is

$$-\frac{\Pi_r\left(w, r \,\middle|\, \widehat{\theta}, \theta\right)}{\Pi_{\widehat{\theta}}\left(w, r \,\middle|\, \widehat{\theta}, \theta\right)} = \frac{\lambda\Delta\alpha - \beta(w - \widehat{\theta}r)}{\beta r\left[w/\theta - r\right]} - \frac{\widehat{\theta}}{r}, \quad \text{for } \beta(w - \widehat{\theta}r) \leq \lambda\Delta\alpha,$$

which is monotonically increasing in θ. (For $\beta(w - \widehat{\theta}r) \geq \lambda\Delta\alpha$, $\Pi\left(w, r \,\middle|\, \widehat{\theta}, \theta\right)$ is independent of r, θ, and $\widehat{\theta}$, and hence is irrelevant.)

We now show that the riskier manufacturer must offer the symmetric-information contract terms in any separating equilibrium. By way of contradiction, suppose the manufacturer of type $\underline{\theta}$ offers another contract

$(\underline{w},\underline{r}) \neq (w^\circ, \underline{r}^\circ)$ in a separating equilibrium and, hence, earns an expected profit of $\Pi(\underline{w},\underline{r} \mid \underline{\theta},\underline{\theta})$. If manufacturer $\underline{\theta}$ deviates to $(w^\circ,\underline{r}^\circ)$, let $\widehat{\underline{\theta}} \geq \underline{\theta}$ be the retailer's perceived manufacturer's return risk such that $w^\circ - \widehat{\underline{\theta}}\underline{r}^\circ \geq 0$. By the unique optimality of $(w^\circ,\underline{r}^\circ)$ (Lemma 2), we immediately have

$$\Pi(\underline{w},\underline{r} \mid \underline{\theta},\underline{\theta}) < \pi^\circ = \Pi(w^\circ,\underline{r}^\circ \mid \underline{\theta},\underline{\theta}) \leq \Pi\left(w^\circ,\underline{r}^\circ \,\middle|\, \widehat{\underline{\theta}},\underline{\theta}\right),$$

where the last inequality follows from $\Pi_{\widehat{\theta}}\left(w^\circ,\underline{r}^\circ \,\middle|\, \widehat{\underline{\theta}},\underline{\theta}\right)$ is of the sign $w^\circ - \underline{\theta}\underline{r}^\circ = \lambda\Delta\alpha/(2\beta) > 0$ and Lemma 2). Therefore, the manufacturer $\underline{\theta}$ can be strictly better off by deviating to $(w^\circ,\underline{r}^\circ)$, which proves that $(\underline{w},\underline{r})$ cannot be played in that separating equilibrium. □

Road map of remaining proofs. The rest of this section is to identify the separating contracts for the less risky manufacturer and establish their properties. As standard in the literature, our general strategy is to first recognize a separating contract as the solution to a constrained optimization problem that maximizes the less risky manufacturer's profit (or equivalently minimize her signaling cost) subject to the riskier manufacturer's non-mimicry incentive constraint (the less risky manufacturer's non-mimicry constraint is always non-binding). Then, we verify that such separating contract can be supported by the most pessimistic off-equilibrium belief (i.e., any deviation away from the separating contract would lead the retailer to believe that the manufacturer is of higher returns risk). More specifically, we first transform the less risky manufacturer's optimization problem in terms of the price decisions to one in terms of the retailer's induced quantity decisions through a change of variable that is essentially equivalent to (4.4) and (4.5) but re-centers them at zero stocks (see Lemma B.3). Propositions B.1 and C.1 establish the partial signaling benchmarks formulated in (4.8) and (4.9), respectively. In particular, these two problems involves a single decision variable and the optimal solutions can be obtained in closed form, so the supporting off-equilibrium belief can be directly verified. The most efficient separating contract formulated in (4.3) does not admit closed-form characterization. Despite this challenge, Lemma B.6 identifies its direction of distortion through Lagrangian, and Lemma B.7 shows that the less risky manufacturer has no incentive to deviate from the most efficient separating contract under the pessimistic off-equilibrium belief. All these results culminate in the proofs of Propositions 1 and 2.

LEMMA B.3 **(Change of Variable)**. *For any $(\overline{w},\overline{r})$ feasible to (4.3), let $\widetilde{w} := \overline{w} - w^\circ = \overline{w} - \frac{\alpha}{2\beta}$ and $\widetilde{u} := \overline{w} - w^\circ - \overline{\theta}(\overline{r} - \overline{r}^\circ) = \overline{w} - \overline{\theta}\overline{r} - \frac{\lambda\Delta\alpha}{2\beta}$. Then, $\overline{w} = w^\circ + \widetilde{w}$, $\overline{r} = \overline{r}^\circ + (\widetilde{w} - \widetilde{u})/\overline{\theta}$, and it is without loss of generality to restrict to*

$$\widetilde{w} + \frac{\alpha_l}{2\beta} \geq \widetilde{u} \geq -\frac{\lambda\Delta\alpha}{2\beta}, \quad \text{and} \quad \widetilde{u} \leq \frac{\lambda\Delta\alpha}{2\beta}. \tag{B.4}$$

Furthermore, the objective function of (4.3) can be written as

$$\Pi\left(\overline{w},\overline{r} \,\middle|\, \overline{\theta},\overline{\theta}\right) = \pi^\circ - \frac{\beta}{2\lambda}\left(\lambda\widetilde{w}^2 + \lambda^c\widetilde{u}^2\right), \tag{B.5}$$

the first constraint in (4.3) is equivalent to

$$\lambda\widetilde{w}^2 + \lambda^c\widetilde{u}^2 + \frac{\lambda^c\Delta\theta}{\overline{\theta}}\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\widetilde{u} - \frac{\lambda\Delta\alpha}{2\beta}\right) \geq 0, \tag{B.6}$$

and the second constraint in (4.3) is equivalent to

$$\lambda\widetilde{w}^2 + \lambda^c\widetilde{u}^2 \leq \frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\underline{\theta}}. \tag{B.7}$$

Proof. The first constraint in (B.4) follows from the fact that $\overline{w} \geq \bar{\theta}\bar{r} \geq 0$ by Lemma B.1. By (B.1), it is straightforward to verify that

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\theta},\bar{\theta}\right) = \begin{cases} \frac{\alpha^2}{8\beta} - \frac{\beta}{2}\widetilde{w}^2, & \text{if } \widetilde{u} \geq \frac{\lambda\Delta\alpha}{2\beta}, \\ \pi^\circ - \frac{\beta}{2\lambda}\left(\lambda\widetilde{w}^2 + \lambda^c\widetilde{u}^2\right), & \text{if } \widetilde{u} \leq \frac{\lambda\Delta\alpha}{2\beta}; \end{cases}$$

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\theta},\underline{\theta}\right) = \begin{cases} \frac{\alpha^2}{8\beta} - \frac{\beta}{2}\widetilde{w}^2, & \text{if } \widetilde{u} \geq \frac{\lambda\Delta\alpha}{2\beta}, \\ \pi^\circ - \frac{\beta}{2\lambda}\left[\lambda\widetilde{w}^2 + \lambda^c\widetilde{u}^2 + \frac{\lambda^c\Delta\theta}{\bar{\theta}}\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\widetilde{u} - \frac{\lambda\Delta\alpha}{2\beta}\right)\right], & \text{if } \widetilde{u} \leq \frac{\lambda\Delta\alpha}{2\beta}. \end{cases}$$

Also, direct calculation yields

$$\begin{aligned}\Pi\left(w^\circ,\underline{r}^\circ\,\middle|\,\underline{\theta},\bar{\theta}\right) =& H\left(w^\circ, w^\circ - \bar{\theta}\underline{r}^\circ, w^\circ - \underline{\theta}\underline{r}^\circ\right) \\ =& \frac{\lambda^c\beta}{2\lambda}\left(w^\circ - \bar{\theta}\underline{r}^\circ\right)\left(\frac{\lambda\Delta\alpha}{\beta} - \left(w^\circ - \underline{\theta}\underline{r}^\circ\right)\right) + \frac{1}{2}\left[-\beta\left(w^\circ\right)^2 + \alpha w^\circ\right] \\ =& \frac{\lambda^c\beta}{2\lambda}\left(w^\circ - \underline{\theta}\underline{r}^\circ\right)\left(\frac{\lambda\Delta\alpha}{\beta} - \left(w^\circ - \underline{\theta}\underline{r}^\circ\right)\right) + \frac{1}{2}\left[-\beta\left(w^\circ\right)^2 + \alpha w^\circ\right] - \frac{\lambda^c\beta}{2\lambda}\Delta\theta\underline{r}^\circ\left(\frac{\lambda\Delta\alpha}{\beta} - \left(w^\circ - \underline{\theta}\underline{r}^\circ\right)\right) \\ =& \pi^\circ - \frac{\lambda^c\Delta\theta\alpha_l\Delta\alpha}{8\beta\underline{\theta}}.\end{aligned}$$

We claim that we can restrict the search for the optimal solution of (4.3) within $\widetilde{u} \leq \frac{\lambda\Delta\alpha}{2\beta}$, under which the objective function in (4.3) is equivalent to (B.5) while the two constraints in (4.3)reduce to (B.6) and (B.7), respectively. Indeed, for $\widetilde{u} \geq \frac{\lambda\Delta\alpha}{2\beta}$, the first constraint in (4.3)automatically holds, while the second one reduces to $\widetilde{w}^2 \leq \frac{\lambda^c\Delta\alpha\left(\bar{\theta}\alpha_l - \underline{\theta}\alpha\right)}{4\beta^2\underline{\theta}}$, reducing (4.3) to

$$\min_{\widetilde{w}+\frac{\alpha_l}{2\beta}\geq\widetilde{u}\geq\frac{\lambda\Delta\alpha}{2\beta}} \widetilde{w}^2, \quad \text{subject to} \quad \widetilde{w}^2 \leq \frac{\lambda^c\Delta\alpha\left(\bar{\theta}\alpha_l - \underline{\theta}\alpha\right)}{4\beta^2\underline{\theta}}.$$

As the decision variable $\widetilde{u}$ is absent from the objective function as well as the other constraint, it can without loss of generality be taken as $\widetilde{u} = \frac{\lambda\Delta\alpha}{2\beta}$, allowing us to focus on $\widetilde{u} \leq \frac{\lambda\Delta\alpha}{2\beta}$. □

PROPOSITION B.1. *The solution to (4.8) is given by*

$$\overline{w}^\ddagger = \frac{\alpha}{2\beta} + \frac{1}{4\beta\bar{\theta}}\left\{\sqrt{\left(\lambda^c\alpha_l\Delta\theta\right)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta} - \lambda^c\alpha_l\Delta\theta\right\} > w^\circ. \tag{B.8}$$

Contract $(\overline{w}^\ddagger, \bar{r}^\circ)$ can be sustained as a separating equilibrium of the returns risk signaling game if and only if $\Delta\theta/\bar{\theta} \leq (1+\sqrt{\lambda^c})\Delta\alpha/\alpha_l$. In this equilibrium, the retailer's order quantity and unsold inventory in case of low baseline demand are given by

$$\bar{s}^\ddagger = \frac{\alpha_h}{4} + \frac{1}{8\lambda\bar{\theta}}\left\{\lambda^c\alpha_l\Delta\theta - \sqrt{(\lambda^c\alpha_l\Delta\theta)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta}\right\} < s^\circ, \text{ and} \tag{B.9}$$

$$\bar{q}^\ddagger = \frac{\Delta\alpha}{4} + \frac{1}{8\lambda\bar{\theta}}\left\{\lambda^c\alpha_l\Delta\theta - \sqrt{(\lambda^c\alpha_l\Delta\theta)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta}\right\} < q^\circ, \text{ respectively;} \tag{B.10}$$

no unsold inventory results from high baseline demand realization.

Proof. We solve (4.8) by first ignoring the second constraint $\Pi\left(\overline{w},\bar{r}^\circ\,\middle|\,\bar{\theta},\bar{\theta}\right) \geq \Pi\left(w^\circ,\underline{r}^\circ\,\middle|\,\underline{\theta},\bar{\theta}\right)$ and then verifying that it will be satisfied by the solution to the relaxed problem. Using the change of variable in Lemma B.3, we have the solution to the relaxed problem $\overline{w}^\ddagger = w^\circ + \widetilde{w}^\ddagger$, where $\widetilde{w}^\ddagger$ is the solution to

$$\min_{-\frac{\lambda\Delta\alpha}{2\beta}\leq\widetilde{w}\leq\frac{\lambda\Delta\alpha}{2\beta}} \widetilde{w}^2, \quad \text{subject to } \widetilde{w}^2 + \frac{\lambda^c\alpha_l\Delta\theta}{2\beta\bar{\theta}}\left(\widetilde{w} - \frac{\lambda\Delta\alpha}{2\beta}\right) \geq 0. \tag{B.11}$$

As the quadratic function on the left-hand side of the constraint in (B.11) achieves its minimum at $\widetilde{w} = -\frac{\lambda^c\alpha_l\Delta\theta}{4\beta\bar{\theta}} < 0$, the optimal solution to (B.11) is thus given by its larger (and positive) root

$$\widetilde{w}^{\ddagger} = -\frac{\lambda^c\alpha_l\Delta\theta}{4\beta\bar{\theta}} + \frac{1}{4\beta\bar{\theta}}\sqrt{(\lambda^c\alpha_l\Delta\theta)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta} > 0, \tag{B.12}$$

from which (B.8) follows immediately. In particular, we note that

$$\begin{aligned}\widetilde{w}^{\ddagger} < \frac{\lambda\Delta\alpha}{2\beta} \quad &\Leftrightarrow \quad \frac{4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta}{4\beta\bar{\theta}\left[\lambda^c\alpha_l\Delta\theta + \sqrt{(\lambda^c\alpha_l\Delta\theta)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta}\right]} < \frac{\lambda\Delta\alpha}{2\beta} \\ &\Leftrightarrow \quad 2\lambda^c\alpha_l\Delta\theta < \lambda^c\alpha_l\Delta\theta + \sqrt{(\lambda^c\alpha_l\Delta\theta)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta},\end{aligned}$$

which obviously holds.

To verify the ignored constraint $\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right) \geq \Pi\left(w^\circ, \underline{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right)$, it suffices to show, by (B.7), that $(\widetilde{w}^{\ddagger})^2 \leq \frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\underline{\theta}}$. As $\widetilde{w}^{\ddagger}$ binds the constraint in (B.11), this is equivalent to

$$\left(\widetilde{w}^{\ddagger}\right)^2 = \frac{\lambda^c\alpha_l\Delta\theta}{2\beta\bar{\theta}}\left(\frac{\lambda\Delta\alpha}{2\beta} - \widetilde{w}^{\ddagger}\right) \leq \frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\underline{\theta}} \quad \Leftrightarrow \quad \widetilde{w}^{\ddagger} \geq \frac{\lambda\Delta\alpha}{2\beta}\left(1 - \bar{\theta}/\underline{\theta}\right),$$

which obviously holds by (B.12).

For contract $(\overline{w}^{\ddagger}, \bar{r}^\circ)$ to be sustained by some equilibrium belief, we need to show that neither the less risky nor the riskier manufacturer has an incentive to deviate to any off-equilibrium strategy under that belief.

- For the riskier manufacturer, we have shown that her profit of deviating to any $(\overline{w}, \bar{r}^\circ)$ with $\overline{w} \geq \overline{w}^{\ddagger}$ and hence being mistaken as a less risky type is dominated by her equilibrium profit, i.e., $\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \underline{\theta}\right) \leq \pi^\circ$. Since all other $(\underline{w}, \underline{r})$ induces a belief that she is the riskier type, the symmetric-information $(w^\circ, \underline{r}^\circ)$ maximizes her profit $\Pi(\underline{w}, \underline{r} \,|\, \underline{\theta}, \underline{\theta})$ to π°. Therefore, the riskier manufacturer indeed has no incentive to deviate from her symmetric-information $(w^\circ, \underline{r}^\circ)$.

- For the less risky manufacturer whose return price is restricted to $\bar{r}^\circ$, it suffices to show that she has no incentive to deviate her wholesale price to any $\overline{w} \neq \overline{w}^{\ddagger}$ and thus to be mistaken as a riskier type $\underline{\theta}$, i.e., $\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right) \leq \Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right)$ for all $\overline{w}$. Indeed, if this condition fails, no other deviation belief $\widehat{\theta}$ can support $\overline{w}^{\ddagger}$, because $\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \widehat{\theta}, \bar{\theta}\right)$ is non-decreasing in $\widehat{\theta}$ as pointed out in the proof of Lemma B.2 and hence $\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \widehat{\theta}, \bar{\theta}\right) \geq \Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right) > \Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right)$ for all $\widehat{\theta} \geq \underline{\theta}$. In light of the fact that $\Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \underline{\theta}\right) = \pi^\circ$, it is equivalent to show that

$$\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right) - \pi^\circ \leq \Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right) - \Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \underline{\theta}\right). \tag{B.13}$$

Direct calculation reveals

$$\begin{aligned}\Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \bar{\theta}\right) - \Pi\left(\overline{w}^{\ddagger}, \bar{r}^\circ \,\middle|\, \bar{\theta}, \underline{\theta}\right) &= -\frac{\lambda^c\beta}{2\lambda}\Delta\theta\bar{r}^\circ\left(\frac{\lambda\Delta\alpha}{\beta} - \overline{w}^{\ddagger} + \bar{\theta}\bar{r}^\circ\right) \\ &= -\frac{\lambda^c\alpha_l\Delta\theta}{8\beta\lambda\bar{\theta}}\left[\lambda\Delta\alpha + \frac{\lambda^c\alpha_l\Delta\theta}{2\bar{\theta}} - \sqrt{\frac{(\lambda^c\alpha_l\Delta\theta)^2}{4\bar{\theta}^2} + \frac{\lambda\lambda^c\alpha_l\Delta\alpha\Delta\theta}{\bar{\theta}}}\right],\end{aligned} \tag{B.14}$$

and

$$\Pi\left(\overline{w}, \bar{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right) - \pi^\circ = \frac{\lambda^c\beta}{2\lambda}\left(\overline{w} - \bar{\theta}\bar{r}^\circ\right)\left(\frac{\lambda\Delta\alpha}{\beta} - \overline{w} + \underline{\theta}\bar{r}^\circ\right)^+ - \frac{\beta}{2}\left(\overline{w} - \frac{\alpha}{2\beta}\right)^2 - \frac{\lambda^c\lambda(\Delta\alpha)^2}{8\beta}$$

$$=\frac{\lambda^c\beta}{2\lambda}\left(\widetilde{w}+\frac{\lambda\Delta\alpha}{2\beta}\right)\left(\frac{\lambda\Delta\alpha}{2\beta}-\frac{\alpha_l\Delta\theta}{2\beta\bar{\theta}}-\widetilde{w}\right)^+-\frac{\beta}{2}\widetilde{w}^2-\frac{\lambda^c\lambda(\Delta\alpha)^2}{8\beta}. \quad \text{(B.15)}$$

Therefore, (B.13) is equivalent to showing

$$\begin{aligned} b(\widetilde{w}) &:= \lambda^c\left(\widetilde{w}+\tfrac{\lambda\Delta\alpha}{2\beta}\right)\left(\tfrac{\lambda\Delta\alpha}{2\beta}-\tfrac{\alpha_l\Delta\theta}{2\beta\bar{\theta}}-\widetilde{w}\right)^+-\lambda\widetilde{w}^2 \\ &\le \tfrac{\lambda^c(\lambda\Delta\alpha)^2}{4\beta^2}-\tfrac{\lambda^c\alpha_l\Delta\theta}{4\beta^2\bar{\theta}}\left[\lambda\Delta\alpha+\tfrac{\lambda^c\alpha_l\Delta\theta}{2\bar{\theta}}-\sqrt{\tfrac{(\lambda^c\alpha_l\Delta\theta)^2}{4\bar{\theta}^2}+\tfrac{\lambda\lambda^c\alpha_l\Delta\alpha\Delta\theta}{\bar{\theta}}}\right], \quad \forall\widetilde{w}\ge-\tfrac{\alpha}{2\beta}. \end{aligned} \quad \text{(B.16)}$$

We note that the piecewise quadratic equation

$$b(\widetilde{w})=\begin{cases} -\lambda\widetilde{w}^2, & \text{if } \widetilde{w}\ge\frac{1}{2\beta}\left(\lambda\Delta\alpha-\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right), \\ -\widetilde{w}^2-\frac{\lambda^c\alpha_l\Delta\theta}{2\beta\bar{\theta}}\widetilde{w}+\frac{\lambda^c\lambda\Delta\alpha}{4\beta^2}\left(\lambda\Delta\alpha-\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right), & \text{if } \widetilde{w}\le\frac{1}{2\beta}\left(\lambda\Delta\alpha-\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right), \end{cases} \quad \text{(B.17)}$$

achieves its maximum

$$\max b(\widetilde{w})=\begin{cases} \left(\frac{\lambda^c\alpha_l\Delta\theta}{4\beta\bar{\theta}}\right)^2+\frac{\lambda^c\lambda\Delta\alpha}{4\beta^2}\left(\lambda\Delta\alpha-\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right), & \text{if } \lambda\Delta\alpha\ge\frac{1+\sqrt{\lambda}}{2}\frac{\alpha_l\Delta\theta}{\bar{\theta}}, \\ 0, & \text{if } \lambda\Delta\alpha\le\frac{1+\sqrt{\lambda}}{2}\frac{\alpha_l\Delta\theta}{\bar{\theta}}, \end{cases} \quad \text{(B.18)}$$

at $\widetilde{w}=-\frac{\lambda^c\alpha_l\Delta\theta}{4\beta\bar{\theta}}>-\frac{\alpha}{2\beta}$ and $\widetilde{w}=0>-\frac{\alpha}{2\beta}$, respectively.

—For $\lambda\Delta\alpha\ge\frac{1+\sqrt{\lambda}}{2}\frac{\alpha_l\Delta\theta}{\bar{\theta}}$, we have

$$\left(\frac{\lambda^c\alpha_l\Delta\theta}{4\beta\bar{\theta}}\right)^2+\frac{\lambda^c\lambda\Delta\alpha}{4\beta^2}\left(\lambda\Delta\alpha-\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right)\le\frac{\lambda^c(\lambda\Delta\alpha)^2}{4\beta^2}-\frac{\lambda^c\alpha_l\Delta\theta}{4\beta^2\bar{\theta}}\left[\lambda\Delta\alpha+\frac{\lambda^c\alpha_l\Delta\theta}{2\bar{\theta}}-\sqrt{\frac{(\lambda^c\alpha_l\Delta\theta)^2}{4\bar{\theta}^2}+\frac{\lambda\lambda^c\alpha_l\Delta\alpha\Delta\theta}{\bar{\theta}}}\right]$$

$$\Leftrightarrow\quad \frac{3}{4}\frac{\lambda^c\alpha_l\Delta\theta}{\bar{\theta}}\le\sqrt{\frac{(\lambda^c\alpha_l\Delta\theta)^2}{4\bar{\theta}^2}+\frac{\lambda\lambda^c\alpha_l\Delta\alpha\Delta\theta}{\bar{\theta}}} \quad\Leftrightarrow\quad \frac{5}{16}\frac{\lambda^c\alpha_l\Delta\theta}{\bar{\theta}}\le\lambda\Delta\alpha,$$

which holds when $\lambda\Delta\alpha\ge\frac{1+\sqrt{\lambda}}{2}\frac{\alpha_l\Delta\theta}{\bar{\theta}}$.

—For $\lambda\Delta\alpha\le\frac{1+\sqrt{\lambda}}{2}\frac{\alpha_l\Delta\theta}{\bar{\theta}}$, we have

$$\begin{aligned} &\frac{\lambda^c(\lambda\Delta\alpha)^2}{4\beta^2}-\frac{\lambda^c\alpha_l\Delta\theta}{4\beta^2\bar{\theta}}\left[\lambda\Delta\alpha+\frac{\lambda^c\alpha_l\Delta\theta}{2\bar{\theta}}-\sqrt{\frac{(\lambda^c\alpha_l\Delta\theta)^2}{4\bar{\theta}^2}+\frac{\lambda\lambda^c\alpha_l\Delta\alpha\Delta\theta}{\bar{\theta}}}\right]\ge 0 \\ \Leftrightarrow\quad &(\lambda\Delta\alpha)^2-2(\lambda\Delta\alpha)\left(\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right)+\lambda\left(\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right)^2\le 0, \end{aligned}$$

which holds for $\left(1-\sqrt{\lambda^c}\right)\left(\frac{\alpha_l\Delta\theta}{\bar{\theta}}\right)\le\lambda\Delta\alpha\le\frac{1+\sqrt{\lambda}}{2}\frac{\alpha_l\Delta\theta}{\bar{\theta}}$, i.e., if and only if $\Delta\theta/\bar{\theta}\le(1+\sqrt{\lambda^c})\Delta\alpha/\alpha_l$.

Finally, we determine the retailer's order quantity as well as unsold inventory. By Lemma 1, since we have $\overline{w}^\ddagger-\bar{\theta}\bar{r}^\circ=w^\circ+\widetilde{w}^\ddagger-\frac{\alpha_l}{2\beta}=\widetilde{w}^\ddagger+\frac{\lambda\Delta\alpha}{2\beta}\le\frac{\lambda\Delta\alpha}{\beta}$ because $\widetilde{w}^\ddagger<\frac{\lambda\Delta\alpha}{2\beta}$, all inventory is sold out in the case of high baseline demand realization and, in particular, (3.1) implies that the retailer's stocking quantity is

$$\begin{aligned} \bar{s}^\ddagger=\frac{\lambda^c\beta\bar{\theta}\bar{r}^\circ+\lambda\alpha_h-\beta\overline{w}^\ddagger}{2\lambda} &=\frac{\lambda^c\beta\bar{\theta}\bar{r}^\circ+\lambda\alpha_h-\beta w^\circ}{2\lambda}+\frac{\beta}{2\lambda}\left(w^\circ-\overline{w}^\ddagger\right) \\ &=\frac{\alpha_h}{4}+\frac{1}{8\lambda\bar{\theta}}\left\{\lambda^c\alpha_l\Delta\theta-\sqrt{(\lambda^c\alpha_l\Delta\theta)^2+4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta}\right\}, \end{aligned}$$

obtaining (B.9). By Lemma 1, again, the unsold inventory in the case of low baseline demand realization is

$$\bar{q}^\ddagger=\frac{1}{2}\left[\Delta\alpha-\beta/\lambda\left(\overline{w}^\ddagger-\bar{\theta}\bar{r}^\circ\right)\right]=\frac{1}{2}\left[\Delta\alpha/2-\beta/\lambda\widetilde{w}^\ddagger\right],$$

from which (B.10) follows by (B.12). □

PROPOSITION B.2. *The solution to (4.9) is given by*

$$\bar{r}^\dagger = \frac{\alpha_l}{2\beta\bar{\theta}} + \frac{1}{4\beta\underline{\theta}\bar{\theta}}\left\{\alpha\Delta\theta - \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}\right\} < \bar{r}^\circ < \underline{r}^\circ. \tag{B.19}$$

Contract $(w^\circ, \bar{r}^\dagger)$ can always be sustained as a separating equilibrium of the returns risk signaling game, in which the retailer's order quantity and unsold inventory in case of low baseline demand are given by

$$\bar{s}^\dagger = \frac{\alpha_h}{4} + \frac{\lambda^c}{8\lambda\underline{\theta}}\left\{\alpha\Delta\theta - \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}\right\} < s^\circ, \text{ and} \tag{B.20}$$

$$\bar{q}^\dagger = \frac{\Delta\alpha}{4} + \frac{1}{8\lambda\underline{\theta}}\left\{\alpha\Delta\theta - \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}\right\} < q^\circ, \text{ respectively;} \tag{B.21}$$

no unsold inventory results from high baseline demand realization.

Proof of Proposition B.2. We solve (4.9) by first ignoring the second constraint $\Pi\left(w^\circ, \bar{r} \,\middle|\, \bar{\theta}, \bar{\theta}\right) \geq \Pi\left(w^\circ, \underline{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right)$ and then verifying that it will be satisfied by the solution to the relaxed problem. Using the change of variable in Lemma B.3, we have the solution to the relaxed problem $\bar{r}^\dagger = \bar{r}^\circ - \widetilde{u}^\dagger/\bar{\theta}$, where $\widetilde{u}^\dagger$ is the solution to

$$\min_{\widetilde{u}\in\left[-\frac{\lambda\Delta\alpha}{2\beta}, \frac{\lambda\Delta\alpha}{2\beta}\wedge\frac{\alpha_l}{2\beta}\right]} \widetilde{u}^2, \quad \text{subject to } \widetilde{u}^2 + \frac{\Delta\theta}{\bar{\theta}}\left(\frac{\alpha_l}{2\beta} - \widetilde{u}\right)\left(\widetilde{u} - \frac{\lambda\Delta\alpha}{2\beta}\right) \geq 0. \tag{B.22}$$

Straightforward algebra reduces the constraint in (B.22) to

$$\widetilde{u}^2 + \frac{\alpha\Delta\theta}{2\beta\underline{\theta}}\widetilde{u} - \frac{\lambda\Delta\alpha\alpha_l\Delta\theta}{4\beta^2\underline{\theta}} \geq 0, \tag{B.23}$$

where the quadratic function on the left-hand side is minimized at $\widetilde{u} = -\frac{\alpha\Delta\theta}{4\beta\underline{\theta}} < 0$. Therefore, (B.22) is minimized at its larger (and positive) root

$$\widetilde{u}^\dagger = -\frac{\alpha\Delta\theta}{4\beta\underline{\theta}} + \frac{1}{4\beta\underline{\theta}}\sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta} > 0, \tag{B.24}$$

from which (B.19) follows. In particular, we notice that

$$\begin{aligned}\widetilde{u}^\dagger < \frac{\lambda\Delta\alpha}{2\beta} \quad &\Leftrightarrow \quad \frac{4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}{4\beta\underline{\theta}\left[\alpha\Delta\theta + \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}\right]} < \frac{\lambda\Delta\alpha}{2\beta} \\ &\Leftrightarrow \quad 2\alpha_l\Delta\theta < (2\alpha\Delta\theta <)\alpha\Delta\theta + \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta},\end{aligned}$$

which obviously holds; and that

$$\begin{aligned}\widetilde{u}^\dagger < \frac{\alpha_l}{2\beta} \quad &\Leftrightarrow \quad \frac{4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}{4\beta\underline{\theta}\left[\alpha\Delta\theta + \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}\right]} < \frac{\alpha_l}{2\beta} \\ &\Leftrightarrow \quad 2\lambda\Delta\alpha\Delta\theta < (2\alpha\Delta\theta <)\alpha\Delta\theta + \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\Delta\alpha\alpha_l\underline{\theta}\Delta\theta},\end{aligned}$$

which again obviously holds.

To verify that $\Pi\left(w^\circ, \bar{r}^\dagger \,\middle|\, \bar{\theta}, \bar{\theta}\right) \geq \Pi\left(w^\circ, \underline{r}^\circ \,\middle|\, \underline{\theta}, \bar{\theta}\right)$, it suffices to show, by (B.7), that $\left(\widetilde{u}^\dagger\right)^2 \leq \frac{\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\underline{\theta}}$. As $\widetilde{u}^\dagger$ binds (B.23), this is equivalent to

$$\left(\widetilde{u}^\dagger\right)^2 = \frac{\lambda\Delta\alpha\alpha_l\Delta\theta}{4\beta^2\underline{\theta}} - \frac{\alpha\Delta\theta}{2\beta\underline{\theta}}\widetilde{u}^\dagger \leq \frac{\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\underline{\theta}} \quad \Leftrightarrow \quad \widetilde{u}^\dagger \geq 0,$$

which in fact follows from (B.24).

We claim that contract $(w^\circ, \bar{r}^\dagger)$ can be sustained by the retailer's posterior belief that the manufacturer offering such a contract is less risky and is otherwise riskier. To that end, we need to show that neither the less risky nor the riskier manufacturer has incentive to deviate to the off-equilibrium strategies under such a posterior belief.

- For the riskier manufacturer, we have shown that her profit of deviating her return price to $\bar{r}^\dagger$ and hence being mistaken as a less risky type is dominated by her equilibrium profit, i.e., $\Pi\left(w^\circ, \bar{r}^\dagger \,\middle|\, \bar{\theta}, \underline{\theta}\right) \le \pi^\circ$. As any other contract $(\underline{w}, \underline{r})$ induces a belief that she is a riskier type, the symmetric-information $(w^\circ, \underline{r}^\circ)$ maximizes her profit $\Pi(\underline{w}, \underline{r} \,|\, \underline{\theta}, \underline{\theta})$ to π°. Therefore, the riskier manufacturer indeed has no incentive to deviate from her symmetric-information contract $(w^\circ, \underline{r}^\circ)$.
- For the less risky manufacturer whose wholesale price is restricted to w°, it suffices to show that she has no incentive to deviate her return price to any $\bar{r} \ne \bar{r}^\dagger$ and thus to be mistaken as a riskier type, i.e., $\Pi\left(w^\circ, \bar{r} \,\middle|\, \underline{\theta}, \bar{\theta}\right) \le \Pi\left(w^\circ, \bar{r}^\dagger \,\middle|\, \bar{\theta}, \bar{\theta}\right)$ for all $\bar{r}$, which is, by (B.1), equivalent to

$$h(\bar{r}) := (w^\circ - \bar{\theta}\bar{r})\left(\frac{\lambda\Delta\alpha}{\beta} - w^\circ + \underline{\theta}\bar{r}\right)^+ \le f(\bar{r}^\dagger) := \left(w^\circ - \bar{\theta}\bar{r}^\dagger\right)\left[\frac{\lambda\Delta\alpha}{\beta} - \left(w^\circ - \bar{\theta}\bar{r}^\dagger\right)\right]^+, \quad \forall \bar{r}. \tag{B.25}$$

We note that $h(\bar{r}) \equiv 0$ for $\bar{r} \le w^\circ/\bar{\theta}$ and hence (B.25) holds, if $w^\circ/\bar{\theta} \le (w^\circ - \lambda\Delta\alpha/\beta)/\underline{\theta}$. Thus, we just need to show (B.25) holds for the parameter range $w^\circ/\bar{\theta} > (w^\circ - \lambda\Delta\alpha/\beta)/\underline{\theta}$ or equivalently

$$\frac{\alpha_l - \lambda\Delta\alpha}{\alpha_l + \lambda\Delta\alpha} \le \underline{\theta}/\bar{\theta}. \tag{B.26}$$

On the other hand, it is straightforward to see that the quadratic function $(w^\circ - \bar{\theta}\bar{r})\left(\frac{\lambda\Delta\alpha}{\beta} - w^\circ + \underline{\theta}\bar{r}\right)$ is maximized at $\bar{r}^\diamond := \frac{(\bar{\theta}+\underline{\theta})\alpha_l - \Delta\theta\lambda\Delta\alpha}{4\beta\bar{\theta}\underline{\theta}}$ and hence $h(\bar{r}) \le h(\bar{r}^\diamond)$. Hence, direct calculation yields

$$\begin{aligned} f(\bar{r}^\dagger) - h(\bar{r}) \ge & f(\bar{r}^\dagger) - h(\bar{r}^\diamond) = \left(\frac{\lambda\Delta\alpha}{2\beta}\right)^2 - \frac{\left(\alpha\Delta\theta - \sqrt{(\alpha\Delta\theta)^2 + 4\lambda\alpha_l\Delta\alpha\underline{\theta}\Delta\theta}\right)^2}{(4\beta\underline{\theta})^2} - \frac{\left[(\underline{\theta}+\bar{\theta})\lambda\Delta\alpha - \Delta\theta\alpha_l\right]^2}{(4\beta)^2\underline{\theta}\bar{\theta}} \\ = & \frac{2\alpha\Delta\theta}{(4\beta\underline{\theta})^2}\left\{\sqrt{(\alpha\Delta\theta)^2 + 4\lambda\alpha_l\Delta\alpha\underline{\theta}\Delta\theta} - \left(1 + \frac{\underline{\theta}}{2\bar{\theta}}\right)\alpha\Delta\theta\right\}, \end{aligned}$$

which is positive because (B.26) implies that

$$\begin{aligned} & (\alpha\Delta\theta)^2 + 4\lambda\alpha_l\Delta\alpha\underline{\theta}\Delta\theta - \left(1 + \frac{\underline{\theta}}{2\bar{\theta}}\right)^2(\alpha\Delta\theta)^2 = \alpha^2\underline{\theta}\Delta\theta\left\{\frac{4\alpha_l\lambda\Delta\alpha}{\alpha^2} - \frac{(4\bar{\theta}+\underline{\theta})\Delta\theta}{4\bar{\theta}^2}\right\} \\ = & \alpha^2\underline{\theta}\Delta\theta\left\{\frac{(3\bar{\theta}+\underline{\theta})\underline{\theta}}{4\bar{\theta}^2} - \left(\frac{\alpha_l - \lambda\Delta\alpha}{\alpha_l + \lambda\Delta\alpha}\right)^2\right\} \ge \alpha^2(\underline{\theta})^2/\bar{\theta}^2\Delta\theta\left\{\frac{3\bar{\theta}+\underline{\theta}}{4} - \underline{\theta}\right\} = \frac{3\alpha^2(\underline{\theta})^2(\Delta\theta)^2}{4\bar{\theta}^2} > 0. \end{aligned}$$

This concludes the verification of the equilibrium belief.

Finally, we determine the retailer's order quantity as well as unsold inventory. Since $w^\circ - \bar{\theta}\bar{r}^\dagger = \widetilde{u}^\dagger + \frac{\lambda\Delta\alpha}{2\beta} < \lambda\Delta\alpha/\beta$ because $\widetilde{u}^\dagger < \frac{\lambda\Delta\alpha}{2\beta}$, Lemma 1 suggests that all inventory is sold out in the case of high baseline demand realization, and in particular, (3.1) implies that the retailer's stocking quantity $\bar{s}^\dagger = \frac{\lambda^c\beta\bar{\theta}\bar{r}^\dagger + \lambda\alpha_h - \beta w^\circ}{2\lambda}$, which is given by (B.20) and obviously smaller than $s^\circ = \alpha_h/4$. By Lemma 1, again, the unsold inventory in the case of low baseline demand realization is

$$\bar{q}^\dagger = \frac{1}{2}\left[\Delta\alpha - \beta/\lambda\left(w^\circ - \bar{\theta}\bar{r}^\dagger\right)\right] = \frac{1}{2}\left[\Delta\alpha/2 - \beta/\lambda\widetilde{u}^\dagger\right],$$

from which (B.21) follows by (B.24). □

Lemma B.4. *The solution* $(\widetilde{w}^\star, \widetilde{u}^\star)$ *to* (B.5)-(B.7) *satisfies* $\widetilde{w}^\star > -\frac{\alpha}{2\beta}$ *and* $-\frac{\lambda\Delta\alpha}{2\beta} < \widetilde{u}^\star < \frac{\lambda\Delta\alpha}{2\beta}$.

Proof. Suppose $\widetilde{u}^\star = \pm\frac{\lambda\Delta\alpha}{2\beta}$. Then, $(0, \widetilde{u}^\dagger)$ with $\widetilde{u}^\dagger$ given by (B.24) is a feasible solution to (B.5)-(B.7), but

$$\lambda\left(\widetilde{w}^\star\right)^2 + \lambda^c\left(\widetilde{u}^\star\right)^2 \ge \lambda^c\left(\frac{\lambda\Delta\alpha}{2\beta}\right)^2 > \lambda(0)^2 + \lambda^c(\widetilde{u}^\dagger)^2,$$

contradicting the optimality of $(\widetilde{w}^\star, \widetilde{u}^\star)$. Therefore, we must have $-\frac{\lambda\Delta\alpha}{2\beta} < \widetilde{u}^\star < \frac{\lambda\Delta\alpha}{2\beta}$. Subsequently, we must have $\widetilde{w}^\star \ge \widetilde{u}^\star - \frac{\alpha_l}{2\beta} > -\frac{\lambda\Delta\alpha}{2\beta} - \frac{\alpha_l}{2\beta} = -\frac{\alpha}{2\beta}$. □

Lemma B.5. *There exists* $(\widetilde{w},\widetilde{u})$ *such that (i)* $-\frac{\lambda\Delta\alpha}{2\beta}\le\widetilde{u}\le 0$*, (ii)* $\widetilde{u}-\frac{\alpha_l}{2\beta}\le\widetilde{w}\le 0$*, (iii)* (B.6) *holds, and (iv)* $\lambda\widetilde{w}^2+\lambda^c\widetilde{u}^2<\frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\overline{\theta}}$*. Therefore, the optimal solution to* (B.5)-(B.6) *automatically satisfies* (B.7)*.*

Proof of Lemma B.5. For $\widetilde{u}=0$, the left-hand side of (B.6) reduces to the quadratic function $\lambda\widetilde{w}^2-\frac{\lambda^c\Delta\theta\lambda\Delta\alpha}{2\beta\overline{\theta}}\left(\widetilde{w}+\frac{\alpha_l}{2\beta}\right)$ in $\widetilde{w}$, which takes a negative value $-\frac{\lambda^c\alpha_l\Delta\theta\lambda\Delta\alpha}{4\beta^2\overline{\theta}}$ at $\widetilde{w}=0$ and a positive value $\frac{\lambda\alpha_l^2}{4\beta^2}$ at $\widetilde{w}=-\frac{\alpha_l}{2\beta}$. Hence, there exists a root $\widetilde{w}^\flat\in\left(-\frac{\alpha_l}{2\beta},0\right)$ of this the quadratic function such that $\widetilde{w}=\widetilde{w}^\flat$ and $\widetilde{u}=0$ satisfies $-\frac{\lambda\Delta\alpha}{2\beta}\le\widetilde{u}\le 0$, $\widetilde{u}-\frac{\alpha_l}{2\beta}\le\widetilde{w}\le 0$ and (in fact binds) (B.6). Straightforward verification reveals that

$$\lambda(\widetilde{w}^\flat)^2+\lambda^c 0^2=\frac{\lambda^c\Delta\theta\lambda\Delta\alpha}{2\beta\overline{\theta}}\left(\widetilde{w}^\flat+\frac{\alpha_l}{2\beta}\right)<\frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\overline{\theta}}<\frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\underline{\theta}}$$

where the first inequality follows from $\widetilde{w}^\flat<0$ and the second inequality indicates that (B.7) will be satisfied by the optimal solution to (B.5)-(B.6). □

Lemma B.6. *The solution* $(\widetilde{w}^\star,\widetilde{u}^\star)$ *to* (B.5)-(B.7) *is the solution to the following system of equations*

$$\lambda\widetilde{w}^2+\lambda^c\widetilde{u}^2+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\left(\widetilde{w}-\widetilde{u}+\frac{\alpha_l}{2\beta}\right)\left(\widetilde{u}-\frac{\lambda\Delta\alpha}{2\beta}\right)=0, \tag{B.27}$$

$$\lambda^c\widetilde{u}^2-\lambda\widetilde{w}^2+2\lambda\widetilde{w}\widetilde{u}-\frac{\lambda^c\lambda\Delta\alpha}{2\beta}\widetilde{u}-\frac{\lambda\alpha}{2\beta}\widetilde{w}=0, \tag{B.28}$$

such that $\widetilde{w}^\star<0$*,* $\widetilde{u}^\star>0$*,* $0<\lambda\widetilde{w}^\star+\lambda^c\widetilde{u}^\star<\lambda^c\widetilde{u}^\dagger$ *and* $\widetilde{w}^\star-\widetilde{u}^\star<-\widetilde{u}^\dagger$*, where* $\widetilde{u}^\dagger$ *is given by* (B.24)*.*

Proof. By Lemma B.5, solving (B.5)-(B.7) is equivalent to solve the relaxed problem (B.5)-(B.6) by ignoring (B.7). By Lemma B.4, we can also ignore the bound constraint $-\frac{\lambda\Delta\alpha}{2\beta}\le\widetilde{u}\le\frac{\lambda\Delta\alpha}{2\beta}$. Furthermore, we are going to ignore the constraint $\widetilde{w}+\frac{\alpha_l}{2\beta}\ge\widetilde{u}$, which will be verified to hold by the optimal $(\widetilde{w}^\star,\widetilde{u}^\star)$ to the relaxed problem.

The necessary condition for $(\widetilde{w}^\star,\widetilde{u}^\star)$ to be the optimal solution to the relaxed problem is that there exists a Lagrangian multiplier $\xi\ge 0$ associated with (B.6) such that

$$2\lambda\widetilde{w}^\star-\xi\left(2\lambda\widetilde{w}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\widetilde{u}^\star-\frac{\lambda^c\Delta\theta\lambda\Delta\alpha}{2\beta\overline{\theta}}\right)=0, \tag{B.29}$$

$$2\lambda^c\widetilde{u}^\star-\xi\left(\frac{2\lambda^c\underline{\theta}}{\overline{\theta}}\widetilde{u}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\widetilde{w}^\star+\frac{\lambda^c\Delta\theta\alpha}{2\beta\overline{\theta}}\right)=0. \tag{B.30}$$

We claim that $\xi>0$. Otherwise, (B.29) and (B.30) immediately imply that $\widetilde{w}^\star=\widetilde{u}^\star=0$, which can be easily verified to violate (B.6). Therefore, (B.6) must be binding, yielding (B.27), which immediately implies that $\widetilde{w}^\star+\frac{\alpha_l}{2\beta}\ge\widetilde{u}^\star$ because $\widetilde{u}^\star<\frac{\lambda\Delta\alpha}{2\beta}$ by Lemma B.4.

Rearranging terms in (B.29), we have

$$2\lambda(1-\xi)\widetilde{w}^\star=\frac{\xi\lambda^c\Delta\theta}{\overline{\theta}}\left(\widetilde{u}^\star-\frac{\lambda\Delta\alpha}{2\beta}\right)<0, \tag{B.31}$$

where the last inequality follows from Lemma B.4; and rearranging terms in (B.30), we have

$$2\left(\overline{\theta}-\underline{\theta}\xi\right)\widetilde{u}^\star=\xi\Delta\theta\left(\widetilde{w}^\star+\frac{\alpha}{2\beta}\right)>0, \tag{B.32}$$

where the last inequality follows again from Lemma B.4. Therefore, we have $\widetilde{w}^\star\neq 0$ and $\widetilde{u}^\star\neq 0$.

By eliminating ξ from (B.29) and (B.30), we obtain

$$\frac{\lambda\widetilde{w}^\star}{\lambda^c\widetilde{u}^\star}=\frac{2\lambda\widetilde{w}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\widetilde{u}^\star-\frac{\lambda^c\Delta\theta\lambda\Delta\alpha}{2\beta\overline{\theta}}}{\frac{2\lambda^c\underline{\theta}}{\overline{\theta}}\widetilde{u}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\widetilde{w}^\star+\frac{\lambda^c\Delta\theta\alpha}{2\beta\overline{\theta}}}=\frac{2\lambda\widetilde{w}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\left(\widetilde{u}^\star-\frac{\lambda\Delta\alpha}{2\beta}\right)}{2\lambda^c\widetilde{u}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\left[\left(\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}\right)+\left(\frac{\lambda\Delta\alpha}{2\beta}-\widetilde{u}^\star\right)\right]}. \tag{B.33}$$

Rearranging terms of (B.33) yields

$$\lambda\widetilde{w}^\star\left[\left(\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}\right)-\left(\widetilde{u}^\star-\frac{\lambda\Delta\alpha}{2\beta}\right)\right]=\lambda^c\widetilde{u}^\star\left(\widetilde{u}^\star-\frac{\lambda\Delta\alpha}{2\beta}\right),$$

immediately implying (B.28) and

$$\frac{\lambda\widetilde{w}^\star}{\lambda^c\widetilde{u}^\star}=-\frac{\frac{\lambda\Delta\alpha}{2\beta}-\widetilde{u}^\star}{\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}+\frac{\lambda\Delta\alpha}{2\beta}-\widetilde{u}^\star}<0, \tag{B.34}$$

where the negativity follows by noting that $\widetilde{u}^\star<\frac{\lambda\Delta\alpha}{2\beta}$ and $\widetilde{w}^\star+\frac{\alpha_l}{2\beta}\geq\widetilde{u}^\star$.

We have the following three possibilities:

1. If $\xi>\overline{\theta}/\underline{\theta}>1$, then (B.31) and (B.32) imply that $\widetilde{w}^\star>0$ and $\widetilde{u}^\star<0$, respectively. However, (B.27) suggests

$$\lambda(\widetilde{w}^\star)^2+\lambda^c(\widetilde{u}^\star)^2=\frac{\lambda^c\Delta\theta}{\overline{\theta}}\underbrace{\left(\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}\right)}_{>\frac{\alpha_l}{2\beta}}\underbrace{\left(\frac{\lambda\Delta\alpha}{2\beta}-\widetilde{u}^\star\right)}_{>\frac{\lambda\Delta\alpha}{2\beta}}>\frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2\overline{\theta}},$$

contradicting Lemma B.5. Hence, this case can be ruled out.

2. If $\overline{\theta}/\underline{\theta}>\xi>1$, then (B.31) and (B.32) imply that $\widetilde{w}^\star>0$ and $\widetilde{u}^\star>0$, respectively. However, this contradicts (B.34), ruling out this case as well.

3. As such, we must have $\xi<1<\frac{\overline{\theta}}{\underline{\theta}}$, which implies that $\widetilde{w}^\star<0$ and $\widetilde{u}^\star>0$ according to (B.31) and (B.32), respectively. Together with (B.34), implies that $\frac{\lambda\widetilde{w}^\star}{\lambda^c\widetilde{u}^\star}\in(-1,0)$ and $\lambda\widetilde{w}^\star+\lambda^c\widetilde{u}^\star>0$. Therefore, by (B.33),

$$-1<\frac{2\lambda\widetilde{w}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\left(\widetilde{u}^\star-\frac{\lambda\Delta\alpha}{2\beta}\right)}{2\lambda^c\widetilde{u}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\left[\left(\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}\right)+\left(\frac{\lambda\Delta\alpha}{2\beta}-\widetilde{u}^\star\right)\right]}<0, \tag{B.35}$$

which, by multiplying the denominator on both sides of (B.35) and rearranging terms, yields

$$2\lambda\widetilde{w}^\star+2\lambda^c\widetilde{u}^\star+\frac{\lambda^c\Delta\theta}{\overline{\theta}}\left(\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}\right)>0. \tag{B.36}$$

Finally, we demonstrate $\widetilde{w}^\star-\widetilde{u}^\star<-\widetilde{u}^\dagger$ and $\lambda\widetilde{w}^\star+\lambda^c\widetilde{u}^\star<\lambda^c\widetilde{u}^\dagger$. Indeed, both $(\widetilde{w}^\star,\widetilde{u}^\star)$ and $(0,\widetilde{u}^\dagger)$ satisfy the quadric equation (B.27), which can, via the change of variable $\widetilde{z}=\widetilde{w}-\widetilde{u}$, be rewritten as

$$\frac{\overline{\theta}}{\lambda^c}\widetilde{w}^2-(\overline{\theta}+\underline{\theta})\widetilde{w}\widetilde{z}+\underline{\theta}\widetilde{z}^2+\frac{\Delta\theta\alpha_l}{2\beta}\widetilde{w}-\frac{\alpha\Delta\theta}{2\beta}\widetilde{z}-\frac{\Delta\theta\alpha_l\lambda\Delta\alpha}{4\beta^2}=0. \tag{B.37}$$

Let $\widetilde{z}^\star:=\widetilde{w}^\star-\widetilde{u}^\star$ and $\widetilde{z}^\dagger:=-\widetilde{u}^\dagger$. Then, $(\widetilde{w}^\star,\widetilde{z}^\star)$ and $(0,\widetilde{z}^\dagger)$ satisfy (B.37).

Since $\widetilde{w}^\star\in\left(-\frac{\alpha}{2\beta},0\right)$ and $\widetilde{u}^\star,\widetilde{u}^\dagger\in\left(0,\frac{\lambda\Delta\alpha}{2\beta}\right)$ as shown above and in the proof of Proposition B.2, we focus on examining the quadratic curve (B.37) in the region $\Omega:=\left\{(\widetilde{w},\widetilde{z}):-\frac{\alpha}{2\beta}\leq\widetilde{w}\leq0\text{ and }0<\widetilde{w}-\widetilde{z}<\frac{\lambda\Delta\alpha}{2\beta}\right\}$.

Total differentiation of (B.37) yields

$$\frac{d\widetilde{z}}{d\widetilde{w}}=\frac{2\frac{\overline{\theta}}{\lambda^c}\widetilde{w}-(\overline{\theta}+\underline{\theta})\widetilde{z}+\frac{\alpha_l\Delta\theta}{2\beta}}{(\overline{\theta}+\underline{\theta})\widetilde{w}-2\underline{\theta}\widetilde{z}+\frac{\alpha\Delta\theta}{2\beta}}, \tag{B.38}$$

where we notice that

$$(\overline{\theta}+\underline{\theta})\widetilde{w}-2\underline{\theta}\widetilde{z}+\frac{\alpha\Delta\theta}{2\beta}=2\underline{\theta}(\widetilde{w}-\widetilde{z})+\Delta\theta\left(\widetilde{w}+\frac{\alpha}{2\beta}\right)>0.$$

Therefore, the quadratic curve in (B.37) is segmented into (at most) two branches in Ω by the straight line $2\frac{\overline{\theta}}{\lambda^c}\widetilde{w}-(\overline{\theta}+\underline{\theta})\widetilde{z}+\frac{\alpha_l\Delta\theta}{2\beta}=0$: in the region where $2\frac{\overline{\theta}}{\lambda^c}\widetilde{w}-(\overline{\theta}+\underline{\theta})\widetilde{z}+\frac{\alpha_l\Delta\theta}{2\beta}>(<)0$, $\widetilde{z}$ is strictly increasing (decreasing) in $\widetilde{w}$. Since

$$2\frac{\overline{\theta}}{\lambda^c}\widetilde{w}^\star-(\overline{\theta}+\underline{\theta})\widetilde{z}^\star+\frac{\alpha_l\Delta\theta}{2\beta}=2\frac{\overline{\theta}}{\lambda^c}\widetilde{w}^\star-(\overline{\theta}+\underline{\theta})\left(\widetilde{w}^\star-\widetilde{u}^\star\right)+\frac{\alpha_l\Delta\theta}{2\beta}$$

$$=\frac{\bar{\theta}}{\lambda^c}\left[2\lambda\widetilde{w}^\star+2\lambda^c\widetilde{u}^\star+\frac{\lambda^c\Delta\theta}{\bar{\theta}}\left(\widetilde{w}^\star-\widetilde{u}^\star+\frac{\alpha_l}{2\beta}\right)\right]>0 \text{ by (B.36)}$$

and

$$2\frac{\bar{\theta}}{\lambda^c}0-(\bar{\theta}+\underline{\theta})\widetilde{z}^\dagger+\frac{\alpha_l\Delta\theta}{2\beta}=(\bar{\theta}+\underline{\theta})\widetilde{u}^\dagger+\frac{\alpha_l\Delta\theta}{2\beta}>0 \text{ because } \widetilde{u}^\dagger>0,$$

$(\widetilde{w}^\star,\widetilde{z}^\star)$ and $(0,\widetilde{z}^\dagger)$ are on the increasing branch of the quadratic curve in (B.37). Therefore, $\widetilde{w}^\star<0$ immediately suggests that

$$\widetilde{w}^\star-\widetilde{u}^\star=\widetilde{z}^\star<\widetilde{z}^\dagger=-\widetilde{u}^\dagger.$$

As $(\widetilde{w}^\star,\widetilde{u}^\star)$ satisfies (B.27), we must have

$$\lambda^c\left[(\widetilde{u}^*)^2+\frac{\Delta\theta}{\bar{\theta}}\left(\frac{\alpha_l}{2\beta}-\widetilde{u}^*\right)\left(\widetilde{u}^*-\frac{\lambda\Delta\alpha}{2\beta}\right)\right]=-\lambda(\widetilde{w}^*)^2-\frac{\lambda^c\Delta\theta}{\bar{\theta}}\widetilde{w}^*\left(\widetilde{u}^*-\frac{\lambda\Delta\alpha}{2\beta}\right)<0,$$

where the inequality follows from $\widetilde{w}^*<0$ and $\widetilde{u}^*<\frac{\lambda\Delta\alpha}{2\beta}$. Namely, $\widetilde{u}^*$ must lie between the negative and positive roots of the quadratic equation on the left-hand side of (B.23). Thus, we have $\widetilde{u}^*<\widetilde{u}^\dagger$, leading to

$$\lambda\widetilde{w}^\star+\lambda^c\widetilde{u}^\star=\lambda(\widetilde{w}^\star-\widetilde{u}^\star)+\widetilde{u}^\star<-\lambda\widetilde{u}^\dagger+\widetilde{u}^\dagger=\lambda^c\widetilde{u}^\dagger,$$

which completes the proof. □

LEMMA B.7. *$\Pi\left(\overline{w},\overline{r}\,\middle|\,\underline{\theta},\bar{\theta}\right)\leq\Pi\left(\overline{w}^\star,\overline{r}^\star\,\middle|\,\bar{\theta},\bar{\theta}\right)$ for all $\overline{w}\geq\bar{\theta}\overline{r}\geq 0$.*

Proof of Lemma B.7. We first make the following two observations:

Observation 1. The return-price-only signaling strategy $(w^\circ,\overline{r}^\dagger)$ is a feasible solution to (4.3), implying that $\Pi\left(\overline{w}^\star,\overline{r}^\star\,\middle|\,\bar{\theta},\bar{\theta}\right)>\Pi\left(w^\circ,\overline{r}^\dagger\,\middle|\,\bar{\theta},\bar{\theta}\right)$.

Observation 2. Furthermore, $\Pi\left(\overline{w}^\star,\overline{r}^\star\,\middle|\,\bar{\theta},\bar{\theta}\right)>\Pi\left(w^\circ,\overline{r}^\dagger\,\middle|\,\bar{\theta},\bar{\theta}\right)=\frac{\lambda^c\beta}{2\lambda}f\left(\overline{r}^\dagger\right)+\frac{1}{2}\left[-\beta(w^\circ)^2+\alpha w^\circ\right]>\frac{\alpha^2}{8\beta}\geq\frac{1}{2}\left[-\beta\overline{w}^2+\alpha\overline{w}\right]$ for all $\overline{w}\geq 0$, where the function $f(\cdot)$ is defined in the proof of Proposition B.2 showing that $f\left(\overline{r}^\dagger\right)>0$.

We now demonstrate the lemma. For any $\overline{w}-\underline{\theta}\overline{r}\geq\lambda\Delta\alpha/\beta$, (B.1) suggests that $\Pi\left(\overline{w},\overline{r}\,\middle|\,\underline{\theta},\bar{\theta}\right)=\frac{1}{2}\left[-\beta\overline{w}^2+\alpha\overline{w}\right]$ and hence the lemma follows from Observation 2 above. For any $0\leq\overline{w}-\underline{\theta}\overline{r}\leq\lambda\Delta\alpha/\beta$, (B.1) suggests that

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\underline{\theta},\bar{\theta}\right)=\frac{\lambda^c\beta}{2\lambda}(\overline{w}-\bar{\theta}\overline{r})\left(\frac{\lambda\Delta\alpha}{\beta}-\overline{w}+\underline{\theta}\overline{r}\right)+\frac{1}{2}\left[-\beta\overline{w}^2+\alpha\overline{w}\right],\tag{B.39}$$

whose first term, as a quadratic function of $\overline{r}\in\left[\frac{1}{\underline{\theta}}\left(\overline{w}-\lambda\Delta\alpha/\beta\right)^+,\frac{1}{\underline{\theta}}\overline{w}\right]$, achieves its unconstrained maximum at $\overline{r}=\frac{1}{\underline{\theta}}\left(\frac{\bar{\theta}+\underline{\theta}}{2\bar{\theta}}\overline{w}-\frac{\lambda\Delta\alpha}{2\beta}\right)$. We thus examine the following two cases:

1. If $\overline{w}\geq\frac{\bar{\theta}\lambda\Delta\alpha}{\beta\Delta\theta}$, then we have $0\leq\frac{1}{\underline{\theta}}\left(\frac{\bar{\theta}+\underline{\theta}}{2\bar{\theta}}\overline{w}-\frac{\lambda\Delta\alpha}{2\beta}\right)\leq\frac{1}{\underline{\theta}}\left(\overline{w}-\lambda\Delta\alpha/\beta\right)$ and hence the first term in (B.39) is decreasing in $\overline{r}\in\left[\frac{1}{\underline{\theta}}\left(\overline{w}-\lambda\Delta\alpha/\beta\right),\frac{1}{\underline{\theta}}\overline{w}\right]$, suggesting

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\underline{\theta},\bar{\theta}\right)\leq\Pi\left(\overline{w},\frac{1}{\underline{\theta}}\left(\overline{w}-\lambda\Delta\alpha/\beta\right)\,\middle|\,\underline{\theta},\bar{\theta}\right)=\frac{1}{2}\left[-\beta\overline{w}^2+\alpha\overline{w}\right]<\Pi\left(\overline{w}^\star,\overline{r}^\star\,\middle|\,\bar{\theta},\bar{\theta}\right),$$

where the last inequality follows again from Observation 2 above.

2. If $\overline{w} \le \frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}$, then $\frac{1}{\underline{\theta}}\left(\overline{w}-\lambda\Delta\alpha/\beta\right) \le \frac{1}{\underline{\theta}}\left(\frac{\bar{\theta}+\underline{\theta}}{2\bar{\theta}}\overline{w}-\frac{\lambda\Delta\alpha}{2\beta}\right) \le 0$ and hence the first term in (B.39) is a quadratic function in r and can straightforwardly be shown to be decreasing in $\bar{r} \in \left[0, \overline{w}/\bar{\theta}\right]$, suggesting that

$$\Pi\left(\overline{w},\bar{r} \,\middle|\, \underline{\theta},\bar{\theta}\right) \le \Pi\left(\overline{w},0 \,\middle|\, \underline{\theta},\bar{\theta}\right) = \frac{\lambda^c\beta}{2\lambda}\overline{w}\left(\frac{\lambda\Delta\alpha}{\beta}-\overline{w}\right)+\frac{1}{2}\left[-\beta\overline{w}^2+\alpha\overline{w}\right] = -\frac{\beta}{2\lambda}\overline{w}^2+\frac{\alpha_h}{2}\overline{w}, \tag{B.40}$$

which is maximized at $\widehat{w} = \frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}$ if $\alpha_h/\alpha_l \le 2\bar{\theta}/\Delta\theta$ and at $\widehat{w} = \lambda\alpha_h/(2\beta)$ otherwise. In both cases, we claim that $(\widehat{w},0)$ satisfies the first constraint in (4.3), suggesting that

$$\Pi\left(\overline{w}^\star,\bar{r}^\star \,\middle|\, \bar{\theta},\bar{\theta}\right) \ge \Pi\left(\widehat{w},0 \,\middle|\, \bar{\theta},\bar{\theta}\right) \ge \Pi\left(\widehat{w},0 \,\middle|\, \underline{\theta},\bar{\theta}\right) \ge \Pi\left(\overline{w},\bar{r} \,\middle|\, \underline{\theta},\bar{\theta}\right),$$

where the first inequality follows from the optimality of $(\overline{w}^\star,\bar{r}^\star)$ in (4.3) (with the knowledge that the second constraint in (4.3) is nonbinding according to Lemma B.5), the second inequality follows from the monotonicity of $\Pi\left(\widehat{w},0 \,\middle|\, \widehat{\theta},\bar{\theta}\right)$ in $\widehat{\theta}$, and the last inequality is because $(\widehat{w},0)$ maximizes $\Pi\left(\overline{w},\bar{r} \,\middle|\, \underline{\theta},\bar{\theta}\right)$.
Now we verify the claim that $(\widehat{w},0)$ satisfies the first constraint in (4.3), which is, according to Lemma B.3, equivalent to showing that the transformed quantities $\widetilde{w} = \widehat{w} - w^\circ$ and $\widetilde{u} = \bar{\theta}\bar{r}^\circ + \widetilde{w} = \frac{\alpha_l}{2\beta} + \widetilde{w}$ satisfy (B.6), which is immediate by noting that $\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta} = 0$.

3. If $\frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta} \le \overline{w} \le \frac{\bar{\theta}\lambda\Delta\alpha}{\beta\Delta\theta}$, then we have $\frac{1}{\underline{\theta}}\left(\frac{\bar{\theta}+\underline{\theta}}{2\bar{\theta}}\overline{w}-\frac{\lambda\Delta\alpha}{2\beta}\right) \in \left[\frac{1}{\underline{\theta}}\left(\overline{w}-\lambda\Delta\alpha/\beta\right)^+, \frac{1}{\underline{\theta}}\overline{w}\right]$ and hence

$$\Pi\left(\overline{w},\bar{r} \,\middle|\, \underline{\theta},\bar{\theta}\right) \le \Pi\left(\overline{w},\frac{1}{\underline{\theta}}\left(\frac{\bar{\theta}+\underline{\theta}}{2\bar{\theta}}\overline{w}-\frac{\lambda\Delta\alpha}{2\beta}\right) \,\middle|\, \underline{\theta},\bar{\theta}\right) = \frac{\lambda^c\beta}{8\lambda\bar{\theta}\underline{\theta}}\left(\frac{\bar{\theta}\lambda\Delta\alpha}{\beta}-\Delta\theta\overline{w}\right)^2+\frac{1}{2}\left(-\beta\overline{w}^2+\alpha\overline{w}\right). \tag{B.41}$$

- When $\lambda^c(\Delta\theta)^2 \ge 4\lambda\bar{\theta}\underline{\theta}$, the quadratic function of $\overline{w}$ on the right-hand side of (B.41) is convex and hence reaches the maximum at either $\overline{w} = \frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}$ or $\overline{w} = \frac{\bar{\theta}}{\Delta\theta}\frac{\lambda\Delta\alpha}{\beta}$, which correspond to the first two cases, respectively.
- When $\lambda^c(\Delta\theta)^2 < 4\lambda\bar{\theta}\underline{\theta}$ or equivalently $(\Delta\theta)^2 < \lambda\left(\bar{\theta}+\underline{\theta}\right)^2$, the quadratic function of $\overline{w}$ on the right-hand side of (B.41) is concave and achieves its unconstrained maximum at

$$\overline{w} = \frac{\lambda\bar{\theta}\left(2\underline{\theta}\alpha-\lambda^c\Delta\theta\Delta\alpha\right)}{\beta\left[\lambda(\bar{\theta}+\underline{\theta})^2-(\Delta\theta)^2\right]}, \tag{B.42}$$

which can be verified to be within $\left(\frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}, \frac{\bar{\theta}}{\Delta\theta}\frac{\lambda\Delta\alpha}{\beta}\right)$ if and only if $\frac{\Delta\theta}{\bar{\theta}+\underline{\theta}} < \frac{\alpha_l}{\Delta\alpha} < \frac{\lambda(\bar{\theta}+\underline{\theta})}{\Delta\theta}$. Therefore, we consider the following three scenarios:

—When $\frac{\alpha_l}{\Delta\alpha} \le \frac{\Delta\theta}{\bar{\theta}+\underline{\theta}}$, the quadratic function of $\overline{w}$ on the right-hand side of (B.41) is decreasing in $\overline{w} \in \left[\frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}, \frac{\bar{\theta}}{\Delta\theta}\frac{\lambda\Delta\alpha}{\beta}\right]$ and hence achieves its maximum at $\overline{w} = \frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}$, with the corresponding $\bar{r} = 0$, for which the lemma has been shown in Case 1.

—When $\frac{\alpha_l}{\Delta\alpha} \ge \frac{\bar{\theta}}{\Delta\theta}\frac{\lambda\Delta\alpha}{\beta}$, the quadratic function of $\overline{w}$ on the right-hand side of (B.41) is increasing in $\overline{w} \in \left[\frac{\bar{\theta}}{\bar{\theta}+\underline{\theta}}\frac{\lambda\Delta\alpha}{\beta}, \frac{\bar{\theta}}{\Delta\theta}\frac{\lambda\Delta\alpha}{\beta}\right]$ and hence achieves its maximum at $\overline{w} = \frac{\bar{\theta}}{\Delta\theta}\frac{\lambda\Delta\alpha}{\beta}$, making the first term on the right-hand side of (B.41) zero. Hence, Observation 2 above immediately implies the lemma.

—When $\frac{\Delta\theta}{\bar{\theta}+\underline{\theta}} < \frac{\alpha_l}{\Delta\alpha} < \frac{\lambda(\bar{\theta}+\underline{\theta})}{\Delta\theta}$, the quadratic function of $\overline{w}$ on the right-hand side of (B.41) is maximized at (B.42), yielding the maximum value (after some algebra)

$$\underbrace{\frac{\lambda^c\lambda(\Delta\alpha)^2+\alpha^2}{8\beta}}_{\pi^\circ}+\frac{\lambda^c}{8\beta}\left\{\frac{\left[(\bar{\theta}+\underline{\theta})\lambda\Delta\alpha-\alpha_l\Delta\theta\right]^2}{\lambda(\bar{\theta}+\underline{\theta})^2-(\Delta\theta)^2}-\lambda(\Delta\alpha)^2\right\}. \tag{B.43}$$

Therefore, the lemma is equivalent to showing $\Pi\left(\overline{w}^\star, \bar{r}^\star \,\middle|\, \bar{\theta}, \bar{\theta}\right)$ dominates (B.43). By Lemma B.3 and B.5, it suffices to show that there exists $(\widetilde{w}, \widetilde{u})$ feasible to (B.4) and (B.6) such that

$$\lambda \widetilde{w}^2 + \lambda^c \widetilde{u}^2 = \frac{\lambda^c \lambda}{4\beta^2} \left\{ \frac{\left[(\bar{\theta}+\underline{\theta})\lambda\Delta\alpha - \alpha_l \Delta\theta\right]^2}{\lambda(\bar{\theta}+\underline{\theta})^2 - (\Delta\theta)^2} - \lambda(\Delta\alpha)^2 \right\}, \tag{B.44}$$

which can be verified to be greater than zero for $\frac{\Delta\theta}{\bar{\theta}+\underline{\theta}} < \frac{\alpha_l}{\Delta\alpha} < \frac{\lambda(\bar{\theta}+\underline{\theta})}{\Delta\theta}$. Making the following change of variable

$$\widetilde{w} = \frac{1}{\sqrt{\lambda}} \left(x\sqrt{\frac{1+\sqrt{\lambda}}{2}} - y\sqrt{\frac{1-\sqrt{\lambda}}{2}} \right), \quad \widetilde{u} = \frac{1}{\sqrt{\lambda^c}} \left(x\sqrt{\frac{1-\sqrt{\lambda}}{2}} + y\sqrt{\frac{1+\sqrt{\lambda}}{2}} \right), \tag{B.45}$$

we then can straightforwardly verify that

$$\lambda \widetilde{w}^2 + \lambda^c \widetilde{u}^2 = x^2 + y^2, \tag{B.46}$$

and (B.6) is equivalent to

$$\left(\bar{\theta}+\underline{\theta}+\frac{\Delta\theta}{\sqrt{\lambda}}\right)x^2 + \frac{\lambda^c\Delta\theta}{\beta}\frac{\alpha_l - \sqrt{\lambda}\Delta\alpha}{\sqrt{2(1+\sqrt{\lambda})}}x + \left(\bar{\theta}+\underline{\theta}-\frac{\Delta\theta}{\sqrt{\lambda}}\right)y^2 + \frac{\lambda^c\Delta\theta}{\beta}\frac{\alpha_l + \sqrt{\lambda}\Delta\alpha}{\sqrt{2(1-\sqrt{\lambda})}}y - \frac{\lambda^c\Delta\theta\alpha_l\lambda\Delta\alpha}{2\beta^2} \geq 0. \tag{B.47}$$

Obviously, the $(\widetilde{w}, \widetilde{u})$ defined through (B.45) by letting $x = 0$ and $y = -\sqrt{\frac{\lambda^c\lambda}{4\beta^2}\left\{\frac{\left[(\bar{\theta}+\underline{\theta})\lambda\Delta\alpha - \alpha_l\Delta\theta\right]^2}{\lambda(\bar{\theta}+\underline{\theta})^2-(\Delta\theta)^2} - \lambda(\Delta\alpha)^2\right\}}$ satisfies (B.44) by virtue of (B.46). It is also straightforward to verify that such $(\widetilde{w}, \widetilde{u})$ satisfies (B.4).

We now verify that it also satisfies (B.47), which implies that the corresponding $(\widetilde{w}, \widetilde{u})$ must satisfy (B.6). To that end, plugging it to (B.47) renders it to

$$\frac{2(1+\sqrt{\lambda})}{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) - \Delta\theta}\left\{2\lambda\frac{\bar{\theta}+\underline{\theta}}{\Delta\theta}\alpha_l\Delta\alpha - \alpha_l^2 - \lambda(\Delta\alpha)^2\right\} \geq \frac{\left(\alpha_l + \sqrt{\lambda}\Delta\alpha\right)^2}{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) + \Delta\theta},$$

which is equivalent to

$$\frac{2(1+\sqrt{\lambda})}{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) - \Delta\theta}\left\{2\lambda\frac{\bar{\theta}+\underline{\theta}}{\Delta\theta}z - z^2 - \lambda\right\} - \frac{\left(z+\sqrt{\lambda}\right)^2}{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) + \Delta\theta} \geq 0, \tag{B.48}$$

with $z := \alpha_l/\Delta\alpha \in \left(\frac{\Delta\theta}{\bar{\theta}+\underline{\theta}}, \frac{\lambda(\bar{\theta}+\underline{\theta})}{\Delta\theta}\right)$. As the right-hand side of (B.48) is a concave quadratic function in z, to demonstrate that (B.48) holds for all $z \in \left(\frac{\Delta\theta}{\bar{\theta}+\underline{\theta}}, \frac{\lambda(\bar{\theta}+\underline{\theta})}{\Delta\theta}\right)$, we just need to show it holds at the ends of the interval. Indeed, when $z = \frac{\Delta\theta}{\bar{\theta}+\underline{\theta}}$, (B.48) reduces to

$$\frac{2(1+\sqrt{\lambda})}{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) - \Delta\theta}\left\{\frac{\lambda(\bar{\theta}+\underline{\theta})^2}{(\Delta\theta)^2} - 1\right\} - \frac{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) + \Delta\theta}{(\Delta\theta)^2} \geq 0 \quad \Leftrightarrow \quad 2(1+\sqrt{\lambda}) \geq 1,$$

which obviously holds. When $z = \frac{\lambda(\bar{\theta}+\underline{\theta})}{\Delta\theta}$, (B.48) reduces to

$$\frac{2(1+\sqrt{\lambda})}{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) - \Delta\theta}\left\{1 - \frac{(\Delta\theta)^2}{\lambda(\bar{\theta}+\underline{\theta})^2}\right\} - \frac{\sqrt{\lambda}(\bar{\theta}+\underline{\theta}) + \Delta\theta}{\lambda(\bar{\theta}+\underline{\theta})^2} \geq 0 \quad \Leftrightarrow \quad 2(1+\sqrt{\lambda}) \geq 1,$$

which also holds.

Finally, we need to show that $(\widetilde{w}, \widetilde{u})$ identified above also satisfies $-\frac{\lambda\Delta\alpha}{2\beta} \leq \widetilde{u} \leq \frac{\lambda\Delta\alpha}{2\beta}$ and $\widetilde{w} - \widetilde{u} \geq -\frac{\alpha_l}{2\beta}$. Under the change of variable in (B.45) with $x = 0$, this is equivalent to show $y^2 \leq \frac{1-\sqrt{\lambda}}{2\beta^2}(\lambda\Delta\alpha)^2$ and $\left(\sqrt{\frac{1-\sqrt{\lambda}}{2\lambda}} + \sqrt{\frac{1+\sqrt{\lambda}}{2\lambda^c}}\right)y \leq \frac{\alpha_l}{2\beta}$, which are straightforward to hold by $y = -\sqrt{\frac{\lambda^c\lambda}{4\beta^2}\left\{\frac{\left[(\bar{\theta}+\underline{\theta})\lambda\Delta\alpha - \alpha_l\Delta\theta\right]^2}{\lambda(\bar{\theta}+\underline{\theta})^2-(\Delta\theta)^2} - \lambda(\Delta\alpha)^2\right\}}$.

Therefore, $(\widetilde{w}, \widetilde{u})$ defined by (B.45) with $x = 0$ and $y = -\sqrt{\frac{\lambda^c\lambda}{4\beta^2}\left\{\frac{\left[(\bar{\theta}+\underline{\theta})\lambda\Delta\alpha - \alpha_l\Delta\theta\right]^2}{\lambda(\bar{\theta}+\underline{\theta})^2-(\Delta\theta)^2} - \lambda(\Delta\alpha)^2\right\}}$ is a feasible solution to (B.5)-(B.6) that satisfies (B.44), completing the proof. $\square$

Proof of Proposition 1. By Lemma B.3, the solution $(\overline{w}^\star, \overline{r}^\star)$ to (4.3) is given by $\overline{w}^\star = w^\circ + \widetilde{w}^\star$ and $\overline{r}^\star = \overline{r}^\circ + (\widetilde{w}^\star - \widetilde{u}^\star)/\overline{\theta}$, where $(\widetilde{w}^\star, \widetilde{u}^\star)$ is the solution to (B.5)-(B.7). Because $\widetilde{w}^\star < 0$ and $\widetilde{u}^\star > 0$ according to Lemma B.6, we thus have $\overline{w}^\star = w^\circ + \widetilde{w}^\star < w^\circ$ and $\overline{r}^\star = \overline{r}^\circ + (\widetilde{w}^\star - \widetilde{u}^\star)/\overline{\theta} < \overline{r}^\circ$.

By (3.1), the retailer's order quantity under $(\overline{w}^\star, \overline{r}^\star)$ is given by

$$s^\star = s^R\left(\overline{w}^\star, \overline{r}^\star, \overline{\theta}\right) = \frac{\lambda^c \beta \overline{\theta} \overline{r}^\star + \lambda \alpha_h - \beta \overline{w}^\star}{2\lambda} = \underbrace{\frac{\lambda^c \beta \overline{\theta} \overline{r}^\circ + \lambda \alpha_h - \beta \overline{w}^\circ}{2\lambda}}_{\alpha_h/4 = s^\circ} - \frac{\beta}{2\lambda}\left(\lambda \widetilde{w}^\star + \lambda^c \widetilde{u}^\star\right) < s^\circ,$$

where the second equality follows from Lemma B.3 and the last inequality follows from the fact that $\lambda \widetilde{w}^\star + \lambda^c \widetilde{u}^\star > 0$ in Lemma B.6.

Since $\widetilde{u}^\star < \frac{\lambda \Delta \alpha}{2\beta}$, we have $\overline{w}^\star - \overline{\theta} \overline{r}^\star = \widetilde{u}^\star + \frac{\lambda \Delta \alpha}{2\beta} < \frac{\lambda \Delta \alpha}{\beta} = \frac{\alpha_h}{4} = s^\circ$, which, according to Lemma 1, suggests that all inventory is sold out in the case of high baseline demand. Again by Lemma 1 and the fact that $\widetilde{u}^\star = \overline{w}^\star - \overline{\theta} \overline{r}^\star - \frac{\lambda \Delta \alpha}{2\beta}$, we have the unsold inventory in the case of low baseline demand realization to be

$$\bar{q}^\star = \frac{1}{2}\left[\Delta\alpha - \beta/\lambda\left(\overline{w}^\star - \overline{\theta}\overline{r}^\star\right)\right] = \frac{1}{2}\left[\Delta\alpha/2 - \beta/\lambda \widetilde{u}^\star\right] < \frac{\Delta\alpha}{4} = q^\circ,$$

where we use the fact that $\widetilde{u}^\star > 0$ (Lemma B.6) to obtain the inequality.

We now verify that $(\overline{w}^\star, \overline{r}^\star)$ can be sustained as a separating equilibrium by the retailer's posterior belief that the manufacturer is less risky upon contract $(\overline{w}^\star, \overline{r}^\star)$ being offered and is otherwise riskier. To that end, we need to show that neither the less risky nor the riskier manufacturer has incentive to deviate to the off-equilibrium strategies under such a posterior belief.

- The riskier manufacturer's profit of deviating to $(\overline{w}^\star, \overline{r}^\star)$ and hence being mistaken as a less risky type is, by definition, dominated by her equilibrium profit as $(\overline{w}^\star, \overline{r}^\star)$ satisfies the first constraint in (4.3): $\Pi\left(\overline{w}^\star, \overline{r}^\star \,\middle|\, \bar{\theta}, \underline{\theta}\right) \le \pi^\circ$. Among all $(\underline{w}, \underline{r}) \neq (\overline{w}^\star, \overline{r}^\star)$, under which the manufacturer is believed to be of the riskier type, the symmetric-information $(w^\circ, \underline{r}^\circ)$ maximizes her profit $\Pi(\underline{w}, \underline{r} \,|\, \underline{\theta}, \underline{\theta})$ to π°. Therefore, the riskier manufacturer indeed has no incentive to deviate from her symmetric-information contract terms $(w^\circ, \underline{r}^\circ)$.
- For the less risky manufacturer, we need to show that she has no incentive to deviate to any $(\overline{w}, \bar{r}) \neq (\overline{w}^\star, \overline{r}^\star)$. If the deviation takes place, the manufacturer will be mistaken as a riskier type, earning a profit of $\Pi\left(\overline{w}, \overline{r} \,\middle|\, \underline{\theta}, \bar{\theta}\right)$, which is dominated by her equilibrium profit $\Pi\left(\overline{w}^\star, \overline{r}^\star \,\middle|\, \bar{\theta}, \bar{\theta}\right)$ by Lemma B.7. Thus, the less risky manufacturer has no incentive to deviate from $(\overline{w}^\star, \overline{r}^\star)$. This concludes the verification of the equilibrium belief. □

Proof of Proposition 2. First, $\overline{w}^\star < w^\circ < \overline{w}^\ddagger$ follows from Proposition 1 and (B.8) Proposition B.1.

By Proposition B.2, $\overline{r}^\dagger < \overline{r}^\circ < \underline{r}^\circ$ follows from (B.19). To show $\overline{r}^\star < \overline{r}^\dagger$, we recall that by Lemma B.3, the solution $(\overline{w}^\star, \overline{r}^\star)$ to (4.3) is given by $\overline{w}^\star = w^\circ + \widetilde{w}^\star$ and $\overline{r}^\star = \overline{r}^\circ + (\widetilde{w}^\star - \widetilde{u}^\star)/\overline{\theta}$, where $(\widetilde{w}^\star, \widetilde{u}^\star)$ is the solution to (B.5)-(B.7). Because $\widetilde{w}^\star < 0$ and $\widetilde{w}^\star - \widetilde{u}^\star < -\widetilde{u}^\dagger$ according to Lemma B.6, we thus have $\overline{w}^\star = w^\circ + \widetilde{w}^\star < w^\circ$ and $\overline{r}^\star = \overline{r}^\circ + (\widetilde{w}^\star - \widetilde{u}^\star)/\overline{\theta} < \overline{r}^\circ - \widetilde{u}^\dagger/\overline{\theta} = \overline{r}^\dagger$.

The profit rank $\overline{\pi}^\dagger < \overline{\pi}^\star < \pi^\circ$ simply follows from the fact that (4.2) is a relaxed problem of (4.3), which is in turn a relaxed problem of (4.9). To show that $\overline{\pi}^\dagger > \overline{\pi}^\ddagger$, we recognize from (B.5) that it is equivalent to show

$$\pi^\circ - \frac{\beta}{2\lambda}\lambda^c\left(\widetilde{u}^\dagger\right)^2 = \overline{\pi}^\dagger > \overline{\pi}^\ddagger = \pi^\circ - \frac{\beta}{2\lambda}\left(\widetilde{w}^\ddagger\right)^2,$$

or equivalently,

$$\sqrt{\lambda^c}\widetilde{u}^\dagger < \widetilde{w}^\ddagger. \tag{B.49}$$

By (B.12) and (B.24), (B.49) is equivalent to

$$\lambda^c\alpha_l\Delta\theta + \sqrt{(\lambda^c\alpha_l\Delta\theta)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}\Delta\theta} < \sqrt{\lambda^c}\alpha\Delta\theta + \sqrt{\lambda^c(\alpha\Delta\theta)^2 + 4\lambda\lambda^c\Delta\alpha\alpha_l\underline{\theta}\Delta\theta}$$
$$\lambda^c\alpha_l + \sqrt{(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\bar{\theta}/\Delta\theta} < \sqrt{\lambda^c}\alpha + \sqrt{\lambda^c\alpha^2 + 4\lambda\lambda^c\Delta\alpha\alpha_l\underline{\theta}/\Delta\theta},$$

which always holds if the following function in $x \in [0,\infty)$ is positive:

$$\Upsilon(x) := \sqrt{\lambda^c}\alpha - \lambda^c\alpha_l + \sqrt{\lambda^c\alpha^2 + 4\lambda\lambda^c\Delta\alpha\alpha_l x} - \sqrt{(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha(1+x)} > 0. \tag{B.50}$$

We note that

$$\Upsilon'(x) = \frac{2\lambda\lambda^c\Delta\alpha\alpha_l\left[(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha - \lambda^c\alpha^2\right]}{\sqrt{\lambda^c\alpha^2 + 4\lambda\lambda^c\Delta\alpha\alpha_l x}\sqrt{(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha(1+x)}\left[\sqrt{\lambda^c\alpha^2 + 4\lambda\lambda^c\Delta\alpha\alpha_l x} + \sqrt{(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha(1+x)}\right]},$$

whose sign is given by that of $(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha - \lambda^c\alpha^2$. Namely, $\Upsilon(x)$ is a monotonic function in $x \in [0,\infty)$. Therefore, to show (B.50), it suffices to show

$$\Upsilon(0) = \sqrt{\lambda^c}\alpha - \lambda^c\alpha_l - \sqrt{(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha} > 0, \quad \text{and} \tag{B.51}$$
$$\lim_{x\to\infty}\Upsilon(x) > 0. \tag{B.52}$$

Direct calculation reveals that (B.51) hold because

$$\left(\sqrt{\lambda^c}\alpha - \lambda^c\alpha_l\right)^2 - \left((\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha\right) = 4\lambda^c\left[\alpha^2 - \sqrt{\lambda^c}\alpha\alpha_l - \lambda\Delta\alpha\alpha_l\right]$$
$$= 4\lambda^c\left(1 - \sqrt{\lambda^c}\right)\left[\alpha_l^2 + \lambda\Delta\alpha\alpha_l + \left(1 + \sqrt{\lambda^c}\right)\lambda(\Delta\alpha)^2\right] > 0.$$

To see (B.52), we note that

$$\Upsilon(x) = \sqrt{\lambda^c}\alpha - \lambda^c\alpha_l + \frac{\lambda^c\left[\alpha^2 - \lambda^c\alpha_l^2 - 4\lambda\alpha_l\Delta\alpha\right]}{\sqrt{\lambda^c\alpha^2 + 4\lambda\lambda^c\Delta\alpha\alpha_l x} + \sqrt{(\lambda^c\alpha_l)^2 + 4\lambda\lambda^c\alpha_l\Delta\alpha(1+x)}} \to \sqrt{\lambda^c}\alpha - \lambda^c\alpha_l > 0 \quad \text{as } x \to \infty.$$

This completes the proof of (B.49) and hence $\bar{\pi}^\dagger > \bar{\pi}^\ddagger$ holds.

To see that $\bar{s}^\dagger > \bar{s}^\ddagger$, we notice that it is equivalent to

$$s^\circ - \frac{\beta}{2\lambda}\lambda^c\widetilde{u}^\dagger = \bar{s}^\dagger > \bar{s}^\ddagger = s^\circ - \frac{\beta}{2\lambda}\widetilde{w}^\ddagger \quad \Leftrightarrow \quad \lambda^c\widetilde{u}^\dagger < \widetilde{w}^\ddagger,$$

which holds and follows immediately from (B.49).

To see that $s^\star > \bar{s}^\dagger$, we note that

$$\bar{s}^\dagger = \frac{\lambda^c\beta\bar{\theta}\bar{r}^\dagger + \lambda\alpha_h - \beta w^\circ}{2\lambda} = \frac{\lambda^c\beta\bar{\theta}\bar{r}^\circ + \lambda\alpha_h - \beta w^\circ}{2\lambda} - \frac{\beta}{2\lambda}\lambda^c\widetilde{u}^\dagger$$

and hence the result follows from the fact that $\lambda\widetilde{w}^\star + \lambda^c\widetilde{u}^\star < \lambda^c\widetilde{u}^\dagger$ in Lemma B.6. □

Appendix C: Proofs in Section 5

Lemma C.1. *In the demand potential signaling game, the unique equilibrium that survives the intuitive criterion is the most efficient separating equilibrium. In this equilibrium, the low-demand manufacturer offers her symmetric information contract* $(\underline{w}^\circ, r^\circ)$.

Proof of Lemma C.1. We first note that $\Pi\left(w,r \,\middle|\, \widehat{\lambda},\lambda\right)$ is increasing in $\widehat{\lambda}$, as we compute

$$\Pi_{\widehat{\lambda}}\left(w,r \,\middle|\, \widehat{\lambda},\lambda\right) = \begin{cases} \frac{1}{2}\widehat{\lambda}^{-2}\beta(w-r)\left(w-\lambda^c r\right), & \text{if} \quad \beta(w-r)\le\widehat{\lambda}\Delta\alpha, \\ \frac{1}{2}\Delta\alpha w, & \text{if} \quad \beta(w-r)\ge\widehat{\lambda}\Delta\alpha. \end{cases}$$

By the weakened condition of Cho and Sobel (1990) in Engers (1987), the selection of the most efficient separating equilibrium hinges on showing that the marginal rate of substitution (MRS) of *one* of signals (i.e., w or r) for the belief $\widehat{\lambda}$ is monotonic in λ. Indeed, direct calculation reveals that the MRS of r for $\widehat{\lambda}$ is given by

$$-\frac{\Pi_r\left(w,r \,\middle|\, \widehat{\lambda},\lambda\right)}{\Pi_{\widehat{\lambda}}\left(w,r \,\middle|\, \widehat{\lambda},\lambda\right)} = \frac{\lambda^c\left(\widehat{\lambda}\Delta\alpha-\beta(w-r)\right)+\widehat{\lambda}\beta w}{\widehat{\lambda}^{-1}\beta(w-r)(w-\lambda^c r)} - \frac{\widehat{\lambda}}{w-r}, \quad \text{for } 0\le\beta(w-r)\le\widehat{\lambda}\Delta\alpha,$$

which is monotonically decreasing in λ. (For $\beta(w-\widehat{\theta}r)\ge\widehat{\lambda}\Delta\alpha$, $\Pi\left(w,r \,\middle|\, \widehat{\theta},\theta\right)$ is independent of r and λ, and hence is irrelevant.)

The equilibrium strategy for the low-demand manufacturer follows from similar argument as in the proof of Lemma B.2 by recognizing that $\underline{\pi}^\circ = \Pi\left(\underline{w}^\circ, r^\circ \mid \underline{\lambda},\underline{\lambda}\right) \le \Pi\left(\underline{w}^\circ, r^\circ \,\middle|\, \widehat{\lambda},\underline{\lambda}\right)$ for any $\widehat{\lambda}\ge\underline{\lambda}$, because of the monotonicity of $\Pi\left(w,r \,\middle|\, \widehat{\lambda},\lambda\right)$ in $\widehat{\lambda}$. □

Lemma C.2. *Any* $(\overline{w},\overline{r})$ *feasible to* (5.4) *must satisfy* $\overline{w}-\overline{r}\le\bar{\lambda}\Delta\alpha/\beta$.

Proof of Lemma C.2. If $\overline{w}-\overline{r}>\bar{\lambda}\Delta\alpha/\beta$ on the contrary, then the constraints of (5.4) imply that

$$\Pi\left(\underline{w}^\circ, r^\circ \,\middle|\, \underline{\lambda},\bar{\lambda}\right) \le \Pi\left(\overline{w},\overline{r} \,\middle|\, \bar{\lambda},\bar{\lambda}\right) = \frac{1}{2}\overline{w}(\alpha_l+\bar{\lambda}\Delta\alpha-\beta\overline{w}) = \Pi\left(\overline{w},\overline{r} \,\middle|\, \bar{\lambda},\underline{\lambda}\right) \le \underline{\pi}^\circ = \Pi\left(\underline{w}^\circ, r^\circ \mid \underline{\lambda},\underline{\lambda}\right),$$

leading to a contradiction because

$$\Pi\left(\underline{w}^\circ, r^\circ \,\middle|\, \underline{\lambda},\bar{\lambda}\right) - \Pi\left(\underline{w}^\circ, r^\circ \mid \underline{\lambda},\underline{\lambda}\right) = \frac{1}{2}\Delta\lambda r^\circ\left[\Delta\alpha-\beta/\underline{\lambda}(\underline{w}^\circ-r^\circ)\right] = \frac{1}{8\beta}\alpha_l\Delta\alpha\Delta\lambda > 0. \quad \square$$

Road map of remaining proofs. Following the solution strategy similar to that in the returns risk case, Lemma C.3 transforms price decisions into retailer's quantity decisions; Propositions C.1 and C.2 establish the two partial signaling benchmarks formulated in (5.7) and (5.8), respectively (including the verification of supporting off-equilibrium beliefs). To establish the most efficient separating equilibrium formulated in (5.4), the proof of Proposition 3 consists of identifying its direction of distortion and verifying that the pessimistic off-equilibrium belief supports it.

Lemma C.3 **(Change of Variable)**. *For any* $(\overline{w},\overline{r})$ *feasible to* (5.4), *let* $\widetilde{w}:=\overline{w}-\overline{w}^\circ=\overline{w}-\frac{\alpha_l+\bar{\lambda}\Delta\alpha}{2\beta}$ *and* $\widetilde{u}:=\overline{w}-\overline{r}-(\overline{w}^\circ-r^\circ)=\overline{w}-\overline{r}-\frac{\bar{\lambda}\Delta\alpha}{2\beta}$. *Then,* $\overline{w}=\frac{\alpha_l+\bar{\lambda}\Delta\alpha}{2\beta}+\widetilde{w}$, $\overline{r}=\widetilde{w}-\widetilde{u}+\frac{\alpha_l}{2\beta}$, *and*

$$\widetilde{w}+\frac{\alpha_l}{2\beta}\ge\widetilde{u}\ge-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \quad \textit{and} \quad \widetilde{u}\le\frac{\bar{\lambda}\Delta\alpha}{2\beta}. \tag{C.1}$$

Furthermore,

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\bar{\lambda}\right)=\overline{\pi}^{\circ}-\frac{\beta}{2\bar{\lambda}}\left(\bar{\lambda}\widetilde{w}^{2}+\bar{\lambda}^{c}\widetilde{u}^{2}\right) \tag{C.2}$$

$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\underline{\lambda}\right)\leq\underline{\pi}^{\circ}$ *is equivalent to*

$$\bar{\lambda}\widetilde{w}^{2}+\bar{\lambda}^{c}\widetilde{u}^{2}+\Delta\lambda\left(\widetilde{w}-\widetilde{u}+\frac{\alpha_{l}}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}\right)\geq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^{2}+2\alpha_{l}\Delta\alpha\right]}{4\beta^{2}}, \tag{C.3}$$

and $\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\bar{\lambda}\right)\geq\Pi\left(\underline{w}^{\circ},r^{\circ}\,\middle|\,\underline{\lambda},\bar{\lambda}\right)$ *is equivalent to*

$$\bar{\lambda}\widetilde{w}^{2}+\bar{\lambda}^{c}\widetilde{u}^{2}\leq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^{2}+\alpha_{l}\Delta\alpha\right]}{4\beta^{2}}. \tag{C.4}$$

Proof of Lemma C.3. By Lemma C.2, we can restrict to $\overline{w}-\overline{r}\leq\bar{\lambda}\Delta\alpha/\beta$. Thus, direct substitution of $\widetilde{w}$ and $\widetilde{u}$ in (5.1) immediately yields (C.2) and

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\underline{\lambda}\right)=\overline{\pi}^{\circ}-\frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\widetilde{w}^{2}+\bar{\lambda}^{c}\widetilde{u}^{2}+\Delta\lambda\left(\widetilde{w}-\widetilde{u}+\frac{\alpha_{l}}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}\right)\right].$$

Using the symmetric-information expressions in Lemma 3, we thus immediately obtain (C.3) and (C.4). □

Proposition C.1. *The solution to (5.7) is given by*

$$\overline{w}^{\sharp}=\frac{\bar{\alpha}}{2\beta}-\frac{1}{4\beta}\left\{\sqrt{\left(\alpha_{l}\Delta\lambda\right)^{2}+4\bar{\lambda}\Delta\lambda\alpha_{h}\Delta\alpha}-\alpha_{l}\Delta\lambda\right\}<\underline{w}^{\circ}<\overline{w}^{\circ}. \tag{C.5}$$

Contract $(\overline{w}^{\sharp},r^{\circ})$ *can be sustained as a separating equilibrium of the demand potential signaling game if and only if* $\Delta\lambda/\underline{\lambda}\left[1+\bar{\lambda}/(4\underline{\lambda})\right]\leq4\alpha_{h}\Delta\alpha/\alpha_{l}^{2}$. *In this equilibrium, the retailer's order quantity and unsold inventory in case of low baseline demand are given by*

$$\bar{s}^{\sharp}=\frac{\alpha_{h}}{4}+\frac{1}{8\bar{\lambda}}\left\{\sqrt{\left(\alpha_{l}\Delta\lambda\right)^{2}+4\bar{\lambda}\Delta\lambda\alpha_{h}\Delta\alpha}-\alpha_{l}\Delta\lambda\right\}>s^{\circ},\ \textit{and} \tag{C.6}$$

$$\bar{q}^{\sharp}=\frac{\Delta\alpha}{4}+\frac{1}{8\bar{\lambda}}\left\{\sqrt{\left(\alpha_{l}\Delta\lambda\right)^{2}+4\bar{\lambda}\Delta\lambda\alpha_{h}\Delta\alpha}-\alpha_{l}\Delta\lambda\right\}>q^{\circ},\ \textit{respectively;} \tag{C.7}$$

no unsold inventory results from high baseline demand realization.

Proof. We first solve the relaxed problem of (5.7) by ignoring the second constraint and will then show that the solution to the relaxed problem automatically satisfies the ignored constraint. Using the change of variable specified in Lemma C.3, the solution to the relaxed problem $\overline{w}^{\sharp}=\overline{w}^{\circ}+\widetilde{w}^{\sharp}=\frac{\alpha_{l}+\bar{\lambda}\Delta\alpha}{2\beta}+\widetilde{w}^{\sharp}$, where $\widetilde{w}^{\sharp}$ is the solution to the following problem

$$\min_{\frac{\bar{\lambda}\Delta\alpha}{2\beta}\geq\widetilde{w}\geq-\frac{\bar{\lambda}\Delta\alpha}{2\beta}}\quad\widetilde{w}^{2} \tag{C.8}$$

$$\text{subject to}\quad\widetilde{w}^{2}+\frac{\alpha_{l}\Delta\lambda}{2\beta}\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{w}\right)\geq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^{2}+2\alpha_{l}\Delta\alpha\right]}{4\beta^{2}}. \tag{C.9}$$

Straightforward algebra reduces (C.9) to

$$\widetilde{w}^{2}-\frac{\alpha_{l}\Delta\lambda}{2\beta}\widetilde{w}-\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^{2}+\alpha_{l}\Delta\alpha\right]}{4\beta^{2}}\geq0, \tag{C.10}$$

whose left-hand side is a quadratic function of two roots. As this quadratic function is minimized at $\widetilde{w}=\frac{\alpha_{l}\Delta\lambda}{4\beta}>0$, it immediately follows that the smaller (and negative) root

$$\widetilde{w}^{\sharp}=\frac{\alpha_{l}\Delta\lambda}{4\beta}-\sqrt{\left(\frac{\alpha_{l}\Delta\lambda}{4\beta}\right)^{2}+\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^{2}+\alpha_{l}\Delta\alpha\right]}{4\beta^{2}}}\in\left(-\frac{\bar{\lambda}\Delta\alpha}{2\beta},0\right) \tag{C.11}$$

minimizes (C.8). In particular, the verification of $\widetilde{w}^\sharp > -\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ is straightforward. Subsequently, we obtain (C.5) by substituting the expression of $\widetilde{w}^\sharp$ into $\overline{w}^\sharp = \overline{w}^\circ + \widetilde{w}^\sharp$.

To see that $\overline{w}^\sharp < \underline{w}^\circ$, we note that

$$\begin{aligned}\underline{w}^\circ - \overline{w}^\sharp =& \frac{1}{4\beta}\sqrt{(\alpha_l\Delta\lambda)^2 + 4\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]} - \frac{1}{4\beta}\left(2\Delta\lambda\Delta\alpha + \alpha_l\Delta\lambda\right)\\ =& -\frac{\alpha_h\underline{\lambda}\Delta\lambda\Delta\alpha}{\beta\left(\sqrt{(\alpha_l\Delta\lambda)^2 + 4\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]} + 2\Delta\lambda\Delta\alpha + \alpha_l\Delta\lambda\right)} > 0.\end{aligned}$$

We now verify that the ignored constraint is satisfied. By Lemma C.3, it is equivalent to show that

$$\left(\widetilde{w}^\sharp\right)^2 \le \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2}, \tag{C.12}$$

which is equivalent to (by using the fact that $\widetilde{w}^\sharp$ is the root of the right-hand side of (C.10))

$$\left(\widetilde{w}^\sharp\right)^2 = \frac{\alpha_l\Delta\lambda}{2\beta}\widetilde{w}^\sharp + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2} \le \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2} \quad \Leftrightarrow \quad \widetilde{w}^\sharp \le 0,$$

and hence obviously holds.

For contract $(\overline{w}^\sharp, r^\circ)$ to be sustained by some equilibrium belief, we need to show that neither high- nor low-demand manufacturer has an incentive to deviate to any off-equilibrium strategy under that belief.

- For the low-demand manufacturer, we have shown that her profit of deviating to $(\overline{w}^\sharp, r^\circ)$ and hence being mistaken as of high demand potential is dominated by her equilibrium profit, i.e., $\Pi\left(\overline{w}^\sharp, r^\circ \,\middle|\, \bar{\lambda}, \underline{\lambda}\right) \le \underline{\pi}^\circ$. As any other contract $(\underline{w}, \underline{r})$ induces a belief that she is of low demand potential, the symmetric-information $(\underline{w}^\circ, r^\circ)$ maximizes her profit $\Pi(\underline{w}, \underline{r} \,|\, \underline{\lambda}, \underline{\lambda})$ to $\underline{\pi}^\circ$. Therefore, the low-demand manufacturer indeed has no incentive to deviate away from her symmetric-information contract $(\underline{w}^\circ, r^\circ)$.
- For the high-demand manufacturer whose return price is restricted to r°, it suffices to show that she has no incentive to deviate her wholesale price to any $\overline{w} \neq \overline{w}^\sharp$ and thus to be mistaken as of low demand potential $\underline{\lambda}$, i.e., $\Pi\left(\overline{w}, r^\circ \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \le \Pi\left(\overline{w}^\sharp, r^\circ \,\middle|\, \bar{\lambda}, \bar{\lambda}\right)$ for all $\overline{w}$. Indeed, if this condition fails, no other deviation belief $\widehat{\lambda}$ can support $\overline{w}^\sharp$, because $\Pi\left(\overline{w}, r^\circ \,\middle|\, \widehat{\lambda}, \bar{\lambda}\right)$ is non-decreasing in $\widehat{\lambda}$ as pointed out in the proof of Lemma C.1 and hence $\Pi\left(\overline{w}, r^\circ \,\middle|\, \widehat{\lambda}, \bar{\lambda}\right) \ge \Pi\left(\overline{w}, r^\circ \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) > \Pi\left(\overline{w}^\sharp, r^\circ \,\middle|\, \bar{\lambda}, \bar{\lambda}\right)$ for all $\widehat{\lambda} \ge \underline{\lambda}$.

—For any $\overline{w} \le r^\circ + \underline{\lambda}\Delta\alpha/\beta = \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$, (5.1) implies that

$$\begin{aligned}\Pi\left(\overline{w}, r^\circ \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) =& \frac{\beta}{2}\left\{\overline{w}\left(\frac{\alpha_l + \underline{\lambda}\Delta\alpha}{\beta} - \overline{w}\right) + \underline{\lambda}^{-1}\left(\underline{\lambda}^c\overline{w} - \bar{\lambda}^c r^\circ\right)\left(\frac{\underline{\lambda}\Delta\alpha}{\beta} + r^\circ - \overline{w}\right)\right\}\\ =& \frac{\beta}{2}\left\{-\underline{\lambda}^{-1}\overline{w}^2 + \left[\frac{\Delta\alpha}{\beta} + \frac{\alpha_l}{\beta}\left(1 + \frac{\underline{\lambda}^c + \bar{\lambda}^c}{2\underline{\lambda}}\right)\right]\overline{w} - \frac{\bar{\lambda}^c}{\underline{\lambda}}\frac{\alpha_l}{2\beta}\left(\frac{\underline{\lambda}\Delta\alpha}{\beta} + \frac{\alpha_l}{2\beta}\right)\right\},\end{aligned}$$

which reaches its (unconstrained) maximum (calculated below) at $\overline{w} = \frac{\underline{\lambda}}{2}\left[\frac{\Delta\alpha}{\beta} + \frac{\alpha_l}{\beta}\left(1 + \frac{\underline{\lambda}^c + \bar{\lambda}^c}{2\underline{\lambda}}\right)\right] \le \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$:

$$\begin{aligned}&\frac{\beta}{2}\left\{\frac{\underline{\lambda}}{4}\left[\frac{\Delta\alpha}{\beta} + \frac{\alpha_l}{\beta}\left(1 + \frac{\underline{\lambda}^c + \bar{\lambda}^c}{2\underline{\lambda}}\right)\right]^2 - \frac{\bar{\lambda}^c}{\underline{\lambda}}\frac{\alpha_l}{2\beta}\left(\frac{\underline{\lambda}\Delta\alpha}{\beta} + \frac{\alpha_l}{2\beta}\right)\right\}\\ =& \frac{1}{8\beta}\left\{\underline{\lambda}(\Delta\alpha)^2 + (\underline{\lambda} + \bar{\lambda})\alpha_l\Delta\alpha + \left[1 + \frac{(\Delta\lambda)^2}{4\underline{\lambda}}\right]\alpha_l^2\right\}.\end{aligned} \tag{C.13}$$

On the other hand, (C.2) suggests that

$$\Pi\left(\overline{w}^\sharp, r^\circ \,\middle|\, \bar{\lambda}, \bar{\lambda}\right) = \overline{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left(\widetilde{w}^\sharp\right)^2$$

$$= \frac{\bar{\lambda}\left((\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right) + \alpha_l^2}{8\beta} - \frac{\beta}{2\bar{\lambda}}\left[\frac{\alpha_l\Delta\lambda}{2\beta}\widetilde{w}^\sharp + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2}\right]$$

$$= \frac{1}{8\beta}\left\{\underline{\lambda}(\Delta\alpha)^2 + (\underline{\lambda} + \bar{\lambda})\alpha_l\Delta\alpha + \alpha_l^2\right\} - \frac{\alpha_l\Delta\lambda}{4\bar{\lambda}}\widetilde{w}^\sharp, \tag{C.14}$$

where the second equality follows from (5.3) and the fact that $\widetilde{w}^\sharp$ is the root of the right-hand side of (C.10).

Therefore, to show that $\Pi\left(\overline{w}, r^\circ \mid \underline{\lambda}, \bar{\lambda}\right) \le \Pi\left(\overline{w}^\sharp, r^\circ \mid \bar{\lambda}, \bar{\lambda}\right)$ for all $\overline{w} \le \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$, we need, by (C.13) and (C.14),

$$\frac{1}{8\beta}\left\{\underline{\lambda}(\Delta\alpha)^2 + (\underline{\lambda} + \bar{\lambda})\alpha_l\Delta\alpha + \left[1 + \frac{(\Delta\lambda)^2}{4\underline{\lambda}}\right]\alpha_l^2\right\} \le \frac{1}{8\beta}\left\{\underline{\lambda}(\Delta\alpha)^2 + (\underline{\lambda} + \bar{\lambda})\alpha_l\Delta\alpha + \alpha_l^2\right\} - \frac{\alpha_l\Delta\lambda}{4\bar{\lambda}}\widetilde{w}^\sharp,$$

or equivalently,

$$\widetilde{w}^\sharp = \frac{\alpha_l\Delta\lambda}{4\beta} - \sqrt{\left(\frac{\alpha_l\Delta\lambda}{4\beta}\right)^2 + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2}} \le -\frac{\bar{\lambda}\alpha_l\Delta\lambda}{8\beta\underline{\lambda}},$$

which holds if and only if $\Delta\lambda/\underline{\lambda}\left[1 + \bar{\lambda}/(4\underline{\lambda})\right] \le 4\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]/\alpha_l^2$, the assumption in the proposition.

—For $\overline{w} \ge r^\circ + \underline{\lambda}\Delta\alpha/\beta = \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$, (5.1) implies that

$$\Pi\left(\overline{w}, r^\circ \mid \underline{\lambda}, \bar{\lambda}\right) = \frac{\beta}{2}\overline{w}\left(\frac{\alpha_l + \underline{\lambda}\Delta\alpha}{\beta} - \overline{w}\right),$$

whose unconstrained maximum is achieved at $\overline{w} = \frac{\alpha_l + \underline{\lambda}\Delta\alpha}{2\beta} < \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$. Therefore, the maximum of $\Pi\left(\overline{w}, r^\circ \mid \underline{\lambda}, \bar{\lambda}\right)$ over $\overline{w} \ge \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$ is achieved at $\overline{w} = \alpha_l/(2\beta) + \underline{\lambda}\Delta\alpha/\beta$, leading us back to the previous case.

Finally, we determine the retailer's order quantity as well as unsold inventory. Since $\overline{w}^\sharp - r^\circ = \overline{w}^\circ + \widetilde{w}^\sharp - \frac{\alpha_l}{2\beta} = \widetilde{w}^\sharp + \frac{\bar{\lambda}\Delta\alpha}{2\beta} \le \frac{\bar{\lambda}\Delta\alpha}{\beta}$ because $\widetilde{w}^\sharp < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$, all inventory is sold out in the case of high baseline demand realization and, in particular, (3.1) implies that the retailer's order quantity is

$$\bar{s}^\sharp = s^R\left(\overline{w}^\sharp, r^\circ, 1, \bar{\lambda}\right) = \frac{\bar{\lambda}^c\beta r^\circ + \bar{\lambda}\alpha_h - \beta\overline{w}^\sharp}{2\bar{\lambda}} > \frac{\bar{\lambda}^c\beta r^\circ + \bar{\lambda}\alpha_h - \beta\overline{w}^\circ}{2\bar{\lambda}} = s^\circ,$$

which yields (C.6) by substituting the expression of $\overline{w}^\sharp$ given in (C.5). By Lemma 1, again, the unsold inventory in the case of low baseline demand realization is

$$\bar{q}^\sharp = \frac{1}{2}\left[\Delta\alpha - \beta/\bar{\lambda}\left(\overline{w}^\sharp - r^\circ\right)\right] = \frac{1}{2}\left[\Delta\alpha/2 - \beta/\bar{\lambda}\widetilde{w}^\sharp\right],$$

from which (C.7) follows by (C.11). □

Proposition C.2. *The solution to* (5.8) *is given by*

$$\overline{r}^\flat = \frac{\alpha_l}{2\beta} + \frac{1}{4\beta\underline{\lambda}^c}\left\{\sqrt{(\Delta\lambda\overline{\alpha})^2 + 4\underline{\lambda}^c\bar{\lambda}\Delta\lambda\alpha_h\Delta\alpha} - \Delta\lambda\overline{\alpha}\right\} \in \left(r^\circ, \overline{w}^\circ\right). \tag{C.15}$$

Contract $\left(\overline{w}^\circ, \overline{r}^\flat\right)$ *can always be sustained as a separating equilibrium of the demand potential signaling game, in which the retailer's order quantity and unsold inventory in case of low baseline demand are given by*

$$\bar{s}^\flat = \frac{\alpha_h}{4} + \frac{\bar{\lambda}^c}{8\bar{\lambda}\underline{\lambda}^c}\left\{\sqrt{(\Delta\lambda\overline{\alpha})^2 + 4\underline{\lambda}^c\bar{\lambda}\Delta\lambda\alpha_h\Delta\alpha} - \Delta\lambda\overline{\alpha}\right\} > s^\circ, \text{ and} \tag{C.16}$$

$$\bar{q}^\flat = \frac{\Delta\alpha}{4} + \frac{1}{8\bar{\lambda}\underline{\lambda}^c}\left\{\sqrt{(\Delta\lambda\overline{\alpha})^2 + 4\underline{\lambda}^c\bar{\lambda}\Delta\lambda\alpha_h\Delta\alpha} - \Delta\lambda\overline{\alpha}\right\} > q^\circ, \text{ respectively;} \tag{C.17}$$

no unsold inventory results from high baseline demand realization.

Proof. We first solve the relaxed problem of (5.8) by ignoring the second constraint and will then show that the solution to the relaxed problem automatically satisfies the ignored constraint. Using the change of variable specified in Lemma C.3, the solution to the relaxed problem $\bar{r}^\flat = \frac{\alpha_l}{2\beta} - \widetilde{u}^\flat$, where $\widetilde{u}^\flat$ is the solution to the following problem

$$\min_{\widetilde{u}\in\left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta},\,\frac{\bar{\lambda}\Delta\alpha}{2\beta}\wedge\frac{\alpha_l}{2\beta}\right]} \quad \widetilde{u}^2 \tag{C.18}$$

$$\text{subject to} \quad \bar{\lambda}^c\widetilde{u}^2 + \Delta\lambda\left(\frac{\alpha_l}{2\beta}-\widetilde{u}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}\right) \geq \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}. \tag{C.19}$$

Straightforward algebra reduces (C.19) to

$$\underline{\lambda}^c\widetilde{u}^2 - \frac{\Delta\lambda\overline{\alpha}}{2\beta}\widetilde{u} - \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{4\beta^2} \geq 0, \tag{C.20}$$

whose left-hand side is a quadratic function of two roots. As this quadratic function is minimized at $\widetilde{u} = \frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c} > 0$, it immediately follows that the smaller (and negative) root

$$\widetilde{u}^\flat = \frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c} - \sqrt{\left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c}\right)^2 + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{4\beta^2\underline{\lambda}^c}} < 0 \tag{C.21}$$

minimizes (C.18). Subsequently, we obtain (C.15) by substituting the expression of $\widetilde{u}^\flat$ into $\bar{r}^\flat = \frac{\alpha_l}{2\beta} - \widetilde{u}^\flat$. In particular, we note that $\bar{r}^\flat = r^\circ - \widetilde{u}^\flat > r^\circ$ and that $\bar{r}^\flat < \overline{w}^\circ$ is equivalent to $\widetilde{u}^\flat > -\frac{\bar{\lambda}\Delta\alpha}{2\beta}$, which indeed holds because

$$\widetilde{u}^\flat > -\frac{\bar{\lambda}\Delta\alpha}{2\beta} \quad\Leftrightarrow\quad \left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c}+\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)^2 > \left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c}\right)^2 + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{4\beta^2\underline{\lambda}^c} \quad\Leftrightarrow\quad \underline{\lambda}^c\bar{\lambda} > \bar{\lambda}^c\Delta\lambda.$$

We now verify that the ignored constraint is satisfied. Since $\widetilde{u}^\flat$ is a root of the right-hand side of (C.20), we have

$$\bar{\lambda}^c\left(\widetilde{u}^\flat\right)^2 < \underline{\lambda}^c\left(\widetilde{u}^\flat\right)^2 = \frac{\Delta\lambda\overline{\alpha}}{2\beta}\widetilde{u}^\flat + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{4\beta^2} < \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{4\beta^2}, \tag{C.22}$$

which shows that the ignored constraint holds by Lemma C.3.

We claim that contract $\left(\overline{w}^\circ, \bar{r}^\flat\right)$ can be sustained as a separating equilibrium by the retailer's posterior belief that the manufacturer is of high demand potential upon such a contract being offered and is otherwise of low demand potential. To that end, we need to show that neither high- nor low-demand manufacturer has incentive to deviate to any off-equilibrium strategy under such a posterior belief.

- For the low-demand manufacturer, we have shown that her profit of deviating to $\left(\overline{w}^\circ, \bar{r}^\flat\right)$ and hence being mistaken as of high demand potential is dominated by her equilibrium profit, i.e., $\Pi\left(\overline{w}^\circ, \bar{r}^\flat \,\middle|\, \bar{\lambda}, \underline{\lambda}\right) \leq \underline{\pi}^\circ$. As any other contract $(\underline{w}, \underline{r})$ induces a belief that she is of low demand potential, the symmetric-information $(\underline{w}^\circ, r^\circ)$ maximizes her profit $\Pi\left(\underline{w}, \underline{r} \,|\, \underline{\lambda}, \underline{\lambda}\right)$ to $\underline{\pi}^\circ$. Therefore, the low-demand manufacturer indeed has no incentive to deviate away from her symmetric-information contract $(\underline{w}^\circ, r^\circ)$.
- For the high-demand manufacturer whose wholesale price is restricted to $\overline{w}^\circ$, it suffices to show that she has no incentive to deviate her return price to any $\bar{r} \neq \bar{r}^\flat$ and thus to be mistaken as of low demand potential, i.e., $\Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \leq \Pi\left(\overline{w}^\circ, \bar{r}^\flat \,\middle|\, \bar{\lambda}, \bar{\lambda}\right)$ for all $\bar{r}$.

—For any $\bar{r}$ such that $\overline{w}^\circ - \bar{r} > \underline{\lambda}\Delta\alpha/\beta$, we have, by (C.2),

$$\begin{aligned}\Pi\left(\overline{w}^\circ, \bar{r}^\flat \,\middle|\, \bar{\lambda}, \bar{\lambda}\right) - \Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) =& \overline{\pi}^\circ - \frac{\beta\bar{\lambda}^c}{2\bar{\lambda}}\left(\widetilde{u}^\flat\right)^2 - \frac{\beta}{2}\left[\left(\frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right)^2 - \frac{\Delta\lambda\Delta\alpha\left(\alpha_l + \bar{\lambda}\Delta\alpha\right)}{2\beta^2}\right] \\ =& \frac{\beta\bar{\lambda}^c}{2\bar{\lambda}}\left[\frac{\bar{\lambda}^2(\Delta\alpha)^2}{4\beta^2} + \frac{\bar{\lambda}\Delta\lambda\left[\bar{\lambda}(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{2\beta^2\bar{\lambda}^c} - \left(\widetilde{u}^\flat\right)^2\right] \\ >& \frac{\beta\bar{\lambda}^c}{2\bar{\lambda}}\left[\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2\underline{\lambda}^c} - \left(\widetilde{u}^\flat\right)^2\right] \geq 0,\end{aligned}$$

where the last inequality follows from (C.22).

—For any $\bar{r}$ such that $0 \leq \overline{w}^\circ - \bar{r} \leq \underline{\lambda}\Delta\alpha/\beta$, direct calculation from (5.1) yields

$$\begin{aligned}\Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) =& \frac{\overline{\alpha}^2}{8\beta} - \frac{\Delta\lambda\Delta\alpha\overline{\alpha}}{4\beta} + \frac{\beta\bar{\lambda}^c}{2\underline{\lambda}}\left(\underline{\lambda}^c/\bar{\lambda}^c\overline{w}^\circ - \bar{r}\right)\left(\underline{\lambda}\Delta\alpha/\beta - \overline{w}^\circ + \bar{r}\right) \\ =& \overline{\pi}^\circ - \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2}{8\beta} - \frac{\Delta\lambda\Delta\alpha\overline{\alpha}}{4\beta} + \frac{\beta\bar{\lambda}^c}{2\underline{\lambda}}\left(\underline{\lambda}^c/\bar{\lambda}^c\overline{w}^\circ - \bar{r}\right)\left(\underline{\lambda}\Delta\alpha/\beta - \overline{w}^\circ + \bar{r}\right).\end{aligned}$$

Here, the quadratic equation $\left(\underline{\lambda}^c/\bar{\lambda}^c\overline{w}^\circ - \bar{r}\right)\left(\underline{\lambda}\Delta\alpha/\beta - \overline{w}^\circ + \bar{r}\right)$ in $\bar{r}$ achieves its unconstrained maximum $\left(\frac{\Delta\lambda}{2\bar{\lambda}^c}\overline{w}^\circ + \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right)^2$ at $\bar{r} = \frac{\underline{\lambda}^c + \bar{\lambda}^c}{2\bar{\lambda}^c}\overline{w}^\circ - \frac{\underline{\lambda}\Delta\alpha}{2\beta}$, which surely satisfies $\overline{w}^\circ - \bar{r} \leq \underline{\lambda}\Delta\alpha/\beta$ and is smaller than $\overline{w}^\circ$ if and only if $\Delta\alpha/\bar{\alpha} \geq \Delta\lambda/(2\bar{\lambda}^c\underline{\lambda})$. Thus, we consider the following two cases:

∗ When $\Delta\alpha/\bar{\alpha} \leq \Delta\lambda/(2\bar{\lambda}^c\underline{\lambda})$, the quadratic equation $\left(\underline{\lambda}^c/\bar{\lambda}^c\overline{w}^\circ - \bar{r}\right)\left(\underline{\lambda}\Delta\alpha/\beta - \overline{w}^\circ + \bar{r}\right)$ reaches it maximum $\frac{\Delta\lambda}{\bar{\lambda}^c}\frac{\bar{\alpha}}{2\beta}\frac{\underline{\lambda}\Delta\alpha}{\beta}$ at $\bar{r} = \overline{w}^\circ$, leading to

$$\Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \leq \overline{\pi}^\circ - \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2}{8\beta} - \frac{\Delta\lambda\Delta\alpha\overline{\alpha}}{4\beta} + \frac{\beta\bar{\lambda}^c}{2\underline{\lambda}}\frac{\Delta\lambda}{\bar{\lambda}^c}\frac{\bar{\alpha}}{2\beta}\frac{\underline{\lambda}\Delta\alpha}{\beta} = \overline{\pi}^\circ - \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2}{8\beta}.$$

Therefore, $\Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \leq \Pi\left(\overline{w}^\circ, \bar{r}^\flat \,\middle|\, \bar{\lambda}, \bar{\lambda}\right) = \overline{\pi}^\circ - \frac{\beta\bar{\lambda}^c}{2\bar{\lambda}}\left(\widetilde{u}^\flat\right)^2$ holds if

$$\left(\widetilde{u}^\flat\right)^2 \leq \left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)^2 \quad \Leftrightarrow \quad \widetilde{u}^\flat \geq -\frac{\bar{\lambda}\Delta\alpha}{2\beta},$$

which is, by (C.21), equivalent to

$$\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c} + \frac{\bar{\lambda}\Delta\alpha}{2\beta} \geq \sqrt{\left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c}\right)^2 + \frac{\bar{\lambda}\Delta\lambda\Delta\alpha\left(\bar{\lambda}^c\Delta\alpha + \bar{\alpha}\right)}{4\beta^2\underline{\lambda}^c}} \quad \Leftrightarrow \quad \underline{\lambda}^c\bar{\lambda} \geq \underline{\lambda}^c\Delta\lambda.$$

The last inequality above obviously holds.

∗ When $\Delta\alpha/\bar{\alpha} \geq \Delta\lambda/(2\bar{\lambda}^c\underline{\lambda})$, we have

$$\Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \leq \overline{\pi}^\circ - \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2}{8\beta} - \frac{\Delta\lambda\Delta\alpha\overline{\alpha}}{4\beta} + \frac{\beta\bar{\lambda}^c}{2\underline{\lambda}}\left(\frac{\Delta\lambda}{2\bar{\lambda}^c}\frac{\overline{\alpha}}{2\beta} + \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right)^2.$$

Therefore, $\Pi\left(\overline{w}^\circ, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \leq \Pi\left(\overline{w}^\circ, \bar{r}^\flat \,\middle|\, \bar{\lambda}, \bar{\lambda}\right) = \overline{\pi}^\circ - \frac{\beta\bar{\lambda}^c}{2\bar{\lambda}}\left(\widetilde{u}^\flat\right)^2$ holds if

$$\left(\widetilde{u}^\flat\right)^2 \leq \frac{\bar{\lambda}^2(\Delta\alpha)^2}{4\beta^2} + \frac{\bar{\lambda}\Delta\lambda\Delta\alpha\overline{\alpha}}{2\beta^2\bar{\lambda}^c} - \frac{\bar{\lambda}}{\underline{\lambda}}\left(\frac{\Delta\lambda}{2\bar{\lambda}^c}\frac{\overline{\alpha}}{2\beta} + \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right)^2 = \frac{\bar{\lambda}\Delta\lambda(\Delta\alpha)^2}{4\beta^2} + \frac{\bar{\lambda}\Delta\lambda\Delta\alpha\overline{\alpha}}{4\beta^2\bar{\lambda}^c} - \frac{\bar{\lambda}}{\underline{\lambda}}\left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\bar{\lambda}^c}\right)^2.$$

By noticing that $\widetilde{u}^\flat$ binds (C.20), the above inequality can be rewritten as

$$\frac{\Delta\lambda\overline{\alpha}}{2\beta\underline{\lambda}^c}\widetilde{u}^\flat + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2\underline{\lambda}^c} \leq \frac{\bar{\lambda}\Delta\lambda(\Delta\alpha)^2}{4\beta^2} + \frac{\bar{\lambda}\Delta\lambda\Delta\alpha\overline{\alpha}}{4\beta^2\bar{\lambda}^c} - \frac{\bar{\lambda}}{\underline{\lambda}}\left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\bar{\lambda}^c}\right)^2,$$

which is, by (C.21), equivalent to

$$\frac{\Delta\lambda\overline{\alpha}}{2\beta\underline{\lambda}^c}\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c} - \frac{\Delta\lambda\overline{\alpha}}{2\beta\underline{\lambda}^c}\sqrt{\left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\underline{\lambda}^c}\right)^2 + \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2\underline{\lambda}^c}} \leq \frac{\bar{\lambda}(\Delta\lambda)^2(\Delta\alpha)^2}{4\beta^2\underline{\lambda}^c} + \frac{\bar{\lambda}(\Delta\lambda)^2\Delta\alpha\overline{\alpha}}{4\beta^2\bar{\lambda}^c\underline{\lambda}^c} - \frac{\bar{\lambda}}{\underline{\lambda}}\left(\frac{\Delta\lambda\overline{\alpha}}{4\beta\bar{\lambda}^c}\right)^2.$$

Letting $x := 2\Delta\alpha/\bar{\alpha} \geq \Delta\lambda/(\bar{\lambda}^c\underline{\lambda})$, we can reduce the above inequality to

$$2 + \frac{\bar{\lambda}}{\underline{\lambda}}\left(\frac{\underline{\lambda}^c}{\bar{\lambda}^c}\right)^2 - \underline{\lambda}^c\bar{\lambda}\left(x^2 + 2/\bar{\lambda}^c x\right) - 2\sqrt{1 + \frac{\bar{\lambda}^c}{\Delta\lambda}\underline{\lambda}^c\bar{\lambda}\left(x^2 + 2/\bar{\lambda}^c x\right)} \leq 0.$$

The left-hand side of the above inequality is a decreasing function in x, and hence we just need to show it holds for $x := 2\Delta\alpha/\bar{\alpha} = \Delta\lambda/(\bar{\lambda}^c\underline{\lambda})$, which returns to the previous case.

Finally, we determine the retailer's order quantity as well as unsold inventory. Since $\overline{w}^\circ - \bar{r}^\flat = \widetilde{u}^\flat + \frac{\bar{\lambda}\Delta\alpha}{2\beta} < \bar{\lambda}\Delta\alpha/\beta$ because $\widetilde{u}^\flat < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$, Lemma 1 suggests that all inventory is sold out in the case of high baseline demand realization, and in particular, (3.1) implies that the retailer's order quantity is

$$\bar{s}^\flat = s^R\left(\overline{w}^\circ, \bar{r}^\flat, 1, \bar{\lambda}\right) = \frac{\bar{\lambda}^c\beta\bar{r}^\flat + \bar{\lambda}\alpha_h - \beta\overline{w}^\circ}{2\bar{\lambda}} > \frac{\bar{\lambda}^c\beta r^\circ + \bar{\lambda}\alpha_h - \beta\overline{w}^\circ}{2\bar{\lambda}} = s^\circ,$$

which yields (C.16) by substituting the expression of $\bar{r}^\flat$ given in (C.15). Again, by Lemma 1, the unsold inventory in the case of low baseline demand realization is

$$\bar{q}^\flat = \frac{1}{2}\left[\Delta\alpha - \beta/\bar{\lambda}\left(\overline{w}^\circ - \bar{r}^\flat\right)\right] = \frac{1}{2}\left[\Delta\alpha/2 - \beta/\bar{\lambda}\widetilde{u}^\flat\right],$$

from which (C.17) follows by (C.21). □

Proof of Proposition 3. We first claim that *we can ignore the second constraint in* (5.4). Indeed, as the contract $\left(\overline{w}^\circ, \bar{r}^\flat\right)$ identified in Proposition C.2 satisfies the first constraint in (5.4), the optimal objective value from the relaxed problem must dominate $\Pi\left(\overline{w}^\circ, \bar{r}^\flat \mid \bar{\lambda}, \bar{\lambda}\right) \geq \Pi\left(\underline{w}^\circ, r^\circ \mid \underline{\lambda}, \bar{\lambda}\right)$, i.e., the ignored constraint must be satisfied by the optimal solution to the relaxed problem.

Therefore, $(\overline{w}^{\star\star}, \bar{r}^{\star\star})$ can be identified by ignoring the second constraint in (5.4). Using the change of variable specified in Lemma C.3, we have $\overline{w}^{\star\star} = \overline{w}^\circ + \widetilde{w}^{\star\star} = \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta} + \widetilde{w}^{\star\star}$ and $\bar{r}^{\star\star} = \frac{\alpha_l}{2\beta} + \widetilde{w}^{\star\star} - \widetilde{u}^{\star\star}$, where $(\widetilde{w}^{\star\star}, \widetilde{u}^{\star\star})$ is the solution to the following problem

$$\min_{\widetilde{w} + \frac{\alpha_l}{2\beta} \geq \widetilde{u} \geq -\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \widetilde{u} \leq \frac{\bar{\lambda}\Delta\alpha}{2\beta}} \quad \bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2, \quad \text{subject to (C.3).} \tag{C.23}$$

We claim that $-\frac{\bar{\lambda}\Delta\alpha}{2\beta} < \widetilde{u}^{\star\star} < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and hence the bound constraint on $\widetilde{u}$ can be ignored. Suppose $\widetilde{u}^{\star\star} = \pm\frac{\bar{\lambda}\Delta\alpha}{2\beta}$. Then consider a feasible solution $(0, \widetilde{u}^\flat)$ to the relaxed problem identified in the proof of Proposition C.2. We then have

$$\bar{\lambda}\left(\widetilde{w}^{\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star}\right)^2 \geq \bar{\lambda}^c\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)^2 > \bar{\lambda}\left(0\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^\flat\right)^2,$$

where the strict inequality follows from the fact that $-\frac{\bar{\lambda}\Delta\alpha}{2\beta} < \widetilde{u}^\flat < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$. This immediately contradicts the optimality of $(\widetilde{w}^{\star\star}, \widetilde{u}^{\star\star})$, proving that $-\frac{\bar{\lambda}\Delta\alpha}{2\beta} < \widetilde{u}^{\star\star} < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$.

We now solve (C.23) by ignoring the constraint $\widetilde{w} + \frac{\alpha_l}{2\beta} \geq \widetilde{u}$, which will be verified to be satisfied by the optimal solution. The necessary condition for the optimality of $(\widetilde{w}^{\star\star}, \widetilde{u}^{\star\star})$ is that there exists a Lagarangian multiplier $\xi \geq 0$ associated with (C.3) such that

$$2\bar{\lambda}\widetilde{w}^{\star\star} - \xi\left[2\bar{\lambda}\widetilde{w}^{\star\star} + \Delta\lambda\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star}\right)\right] = 0, \tag{C.24}$$

$$2\bar{\lambda}^c\widetilde{u}^{\star\star} - \xi\left[2\bar{\lambda}^c\widetilde{u}^{\star\star} + \Delta\lambda\left(2\widetilde{u}^{\star\star} - \widetilde{w}^{\star\star} - \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right)\right] = 0. \tag{C.25}$$

We first note that $\xi > 0$. Otherwise, (C.24) and (C.25) suggest that $\widetilde{w}^{\star\star} = \widetilde{u}^{\star\star} = 0$, violating (C.3). Hence, (C.3) must be binding, from which we have

$$\begin{aligned}\Delta\lambda\left(\widetilde{w}^{\star\star} - \widetilde{u}^{\star\star} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star}\right) =& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \left[\bar{\lambda}\left(\widetilde{w}^{\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star}\right)^2\right]\\ \geq& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \left[\bar{\lambda}0^2 + \bar{\lambda}^c\left(\widetilde{u}^{\flat}\right)^2\right]\\ =& \Delta\lambda\left(\frac{\alpha_l}{2\beta} - \widetilde{u}^{\flat}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\flat}\right) > 0\end{aligned}$$

where the inequality follows from the optimality of $(\widetilde{w}^{\star\star}, \widetilde{u}^{\star\star})$, the last equality follows from the fact that $(0, \widetilde{u}^{\flat})$ also binds (C.3), and the last inequality follows from the fact that $\widetilde{u}^{\flat} < 0$. Subsequently, the ignored constraint $\widetilde{w}^{\star\star} + \frac{\alpha_l}{2\beta} > \widetilde{u}^{\star\star}$ is satisfied.

On the one hand, rearranging terms of (C.24) yields

$$2(1-\xi)\bar{\lambda}\widetilde{w}^{\star\star} = \xi\Delta\lambda\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star}\right) > 0, \tag{C.26}$$

where the inequality follows from the fact that $\widetilde{u}^{\star\star} < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$; and rearranging terms of (C.25) yields

$$2(\bar{\lambda}^c - \xi\underline{\lambda}^c)\widetilde{u}^{\star\star} = -\xi\Delta\lambda\left(\widetilde{w}^{\star\star} + \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right) < 0, \tag{C.27}$$

where the last inequality follows from the fact that $\widetilde{w}^{\star\star} + \frac{\alpha_l}{2\beta} > \widetilde{u}^{\star\star} > -\frac{\bar{\lambda}\Delta\alpha}{2\beta}$. Therefore, we must have $\widetilde{w}^{\star\star} \neq 0$ and $\widetilde{u}^{\star\star} \neq 0$.

On the other hand, eliminating ξ from (C.24) and (C.25) yields

$$\frac{\bar{\lambda}\widetilde{w}^{\star\star}}{\bar{\lambda}^c\widetilde{u}^{\star\star}} = \frac{2\bar{\lambda}\widetilde{w}^{\star\star} + \Delta\lambda\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star}\right)}{2\bar{\lambda}^c\widetilde{u}^{\star\star} + \Delta\lambda\left(2\widetilde{u}^{\star\star} - \widetilde{w}^{\star\star} - \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right)}, \tag{C.28}$$

or equivalently,

$$\frac{\bar{\lambda}\widetilde{w}^{\star\star}}{\bar{\lambda}^c\widetilde{u}^{\star\star}} = -\frac{\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star}}{\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star} + \widetilde{w}^{\star\star} - \widetilde{u}^{\star\star} + \frac{\alpha_l}{2\beta}} \in (-1, 0), \tag{C.29}$$

where the bound follows from the fact that $\widetilde{u}^{\star\star} < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and $\widetilde{w}^{\star\star} + \frac{\alpha_l}{2\beta} > \widetilde{u}^{\star\star}$.

Suggested by (C.26) and (C.27), we consider the following three possibilities:

1. If $\xi > 1 > \bar{\lambda}^c/\underline{\lambda}^c$, then (C.26) and (C.27) suggest that $\widetilde{w}^{\star\star} < 0$ and $\widetilde{u}^{\star\star} > 0$, respectively. However, this would imply, by the binding constraint (C.3), that

$$\begin{aligned}\bar{\lambda}\left(\widetilde{w}^{\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star}\right)^2 =& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \Delta\lambda\left(\widetilde{w}^{\star\star} - \widetilde{u}^{\star\star} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star}\right)\\ >& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \frac{\bar{\lambda}\Delta\lambda\alpha_l\Delta\alpha}{4\beta^2} = \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2},\end{aligned}$$

contradicting the optimality of $(\widetilde{w}^{\star\star}, \widetilde{u}^{\star\star})$, because another feasible solution $(0, \widetilde{u}^{\flat})$ identified in the proof of Proposition C.2 yields an even lower value:

$$\bar{\lambda}0^2 + \bar{\lambda}^c\left(\widetilde{u}^{\flat}\right)^2 \leq \frac{\bar{\lambda}^c}{\underline{\lambda}^c}\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2} < \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right]}{4\beta^2},$$

where the first inequality follows from (C.22). As such, this case can be ruled out.

2. If $1 > \xi > \bar{\lambda}^c/\underline{\lambda}^c$, then (C.26) and (C.27) suggest that $\widetilde{w}^{\star\star} > 0$ and $\widetilde{u}^{\star\star} > 0$, respectively. This, however, contradicts (C.29) and hence can be ruled out.

3. As such, we must have $1 > \bar{\lambda}^c/\underline{\lambda}^c > \xi$, which implies that $\widetilde{w}^{\star\star} > 0$ and $\widetilde{u}^{\star\star} < 0$ according to (C.26) and (C.27), respectively. Therefore, it follows immediately that $\overline{w}^{\star\star} > \overline{w}^{\circ}$ and $\bar{r}^{\star\star} = \alpha_l/(2\beta) + \widetilde{w}^{\star\star} - \widetilde{u}^{\star\star} > \alpha_l/(2\beta) = r^{\circ}$. It also follows from (C.29) that

$$\bar{\lambda}\widetilde{w}^{\star\star} + \bar{\lambda}^c\widetilde{u}^{\star\star} < 0 \tag{C.30}$$

and, together with (C.28), that

$$\left(\bar{\lambda} + \underline{\lambda}\right)\widetilde{w}^{\star\star} + \left(\bar{\lambda}^c + \underline{\lambda}^c\right)\widetilde{u}^{\star\star} - \frac{\alpha_l \Delta\lambda}{2\beta} < 0. \tag{C.31}$$

By Lemma 1 and the fact that $\widetilde{u}^{\star\star} = \overline{w}^{\star\star} - \bar{r}^{\star\star} - \frac{\bar{\lambda}\Delta\alpha}{2\beta}$, we have the unsold inventory in the case of low baseline demand realization to be

$$\bar{q}^{\star\star} = \frac{1}{2}\left[\Delta\alpha - \beta/\lambda\left(\overline{w}^{\star\star} - \bar{r}^{\star\star}\right)\right] = \frac{1}{2}\left[\Delta\alpha/2 - \beta/\lambda\widetilde{u}^{\star\star}\right] > \frac{\Delta\alpha}{4} = q^{\circ},$$

where the inequality follows from the fact that $\widetilde{u}^{\star\star} < 0$.

Finally, we verify that the equilibrium can be sustained by the retailer's posterior equilibrium belief that the manufacturer is of high demand potential upon contract $(\overline{w}^{\star\star}, \bar{r}^{\star\star})$ being offered and is otherwise of low demand potential. To that end, we need to show that neither high- nor low-demand manufacturer has incentive to deviate to the off-equilibrium strategies under the specified belief.

- The low-demand-potential manufacturer's profit of deviating to $(\overline{w}^{\star\star}, \bar{r}^{\star\star})$ and hence being mistaken as of high demand potential is, by definition, dominated by her equilibrium profit according to the constraints of (5.4): $\Pi\left(\overline{w}^{\star\star}, \bar{r}^{\star\star} \,\middle|\, \bar{\lambda}, \underline{\lambda}\right) \le \underline{\pi}^{\circ}$. Among all $(\underline{w}, \underline{r}) \neq (\overline{w}^{\star\star}, \bar{r}^{\star\star})$, under which the manufacturer is believed to be of low demand potential, the symmetric-information $(\underline{w}^{\circ}, r^{\circ})$ maximizes her profit $\Pi\left(\underline{w}, \underline{r} \,|\, \underline{\lambda}, \underline{\lambda}\right)$ to $\underline{\pi}^{\circ}$. Therefore, the low-demand manufacturer indeed has no incentive to deviate from her symmetric-information contract terms $(\underline{w}^{\circ}, r^{\circ})$.
- For high-demand manufacturer, we need to show that she has no incentive to deviate to any $(\overline{w}, \bar{r}) \neq (\overline{w}^{\star\star}, \bar{r}^{\star\star})$ and hence to be mistaken as of low demand potential, namely

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \le \Pi\left(\overline{w}^{\star\star}, \bar{r}^{\star\star} \,\middle|\, \bar{\lambda}, \bar{\lambda}\right). \tag{C.32}$$

For the rest of the proof, we are to establish (C.32) and hence concludes the verification of the equilibrium belief.

1. For $\overline{w} - \bar{r} \ge \underline{\lambda}\Delta\alpha/\beta$, (5.1) yields

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) = \frac{1}{2}\overline{w}\left(\alpha_l + \underline{\lambda}\Delta\alpha - \beta\overline{w}\right) \le \frac{\left(\alpha_l + \underline{\lambda}\Delta\alpha\right)^2}{8\beta}.$$

On the other hand, the optimality of $(\overline{w}^{\star\star}, \bar{r}^{\star\star})$ suggests

$$\Pi\left(\overline{w}^{\star\star}, \bar{r}^{\star\star} \,\middle|\, \bar{\lambda}, \bar{\lambda}\right) > \Pi\left(\overline{w}^{\circ}, \bar{r}^{\flat} \,\middle|\, \bar{\lambda}, \bar{\lambda}\right) > \frac{1}{2}\overline{w}^{\circ}\left(\alpha_l + \bar{\lambda}\Delta\alpha - \beta\overline{w}^{\circ}\right) = \frac{\left(\alpha_l + \bar{\lambda}\Delta\alpha\right)^2}{8\beta} \tag{C.33}$$

from which and the previous inequality (C.32) then follows.

2. For $0 \le \overline{w} - \overline{r} \le \underline{\lambda}\Delta\alpha/\beta < \bar{\lambda}\Delta\alpha/\beta$, (5.1) implies

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) = \frac{1}{2}\overline{w}\left(\alpha_l + \underline{\lambda}\Delta\alpha - \beta\overline{w}\right) + \frac{\beta}{2\underline{\lambda}}\left(\underline{\lambda}^c\overline{w} - \bar{\lambda}^c\overline{r}\right)\left(\overline{r} - \overline{w} + \frac{\underline{\lambda}\Delta\alpha}{\beta}\right), \tag{C.34}$$

in which the second term, as a quadratic function of $\overline{r}$, achieves its unconstrained maximum at $\overline{r} = \frac{\bar{\lambda}^c + \underline{\lambda}^c}{2\bar{\lambda}^c}\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta} > \overline{w} - \underline{\lambda}\Delta\alpha/\beta$. Thus, we consider the following two cases.

(a) If $\frac{\bar{\lambda}^c + \underline{\lambda}^c}{2\bar{\lambda}^c}\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta} \ge \overline{w}$, or equivalently, $\overline{w} \ge \frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\Delta\lambda}$, the quadratic function of $\overline{r}$ in the second term of (C.34) is increasing in $\overline{r} \in [\overline{w} - \underline{\lambda}\Delta\alpha/\beta, \overline{w}]$ and thus

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \le \Pi\left(\overline{w}, \overline{w} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) = \frac{1}{2}\overline{w}\left(\alpha_l + \underline{\lambda}\Delta\alpha - \beta\overline{w}\right) + \frac{\Delta\lambda\Delta\alpha}{2}\overline{w} = \frac{1}{2}\overline{w}\left(\alpha_l + \bar{\lambda}\Delta\alpha - \beta\overline{w}\right) \le \frac{\left(\alpha_l + \bar{\lambda}\Delta\alpha\right)^2}{8\beta}.$$

Thus, (C.32) again follows from (C.33).

(b) If $\overline{w} - \underline{\lambda}\Delta\alpha/\beta < \frac{\bar{\lambda}^c + \underline{\lambda}^c}{2\bar{\lambda}^c}\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta} \le \overline{w}$, or equivalently, $0 \le \overline{w} \le \frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\Delta\lambda}$, we then have

$$\begin{aligned}\Pi\left(\overline{w}, \overline{r} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right) \le& \Pi\left(\overline{w}, \frac{\bar{\lambda}^c + \underline{\lambda}^c}{2\bar{\lambda}^c}\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta} \,\middle|\, \underline{\lambda}, \bar{\lambda}\right)\\ =& \frac{\beta}{2\underline{\lambda}\bar{\lambda}^c}\left\{\frac{(\underline{\lambda} + \bar{\lambda})^2 - 4\underline{\lambda}}{4}\overline{w}^2 + \frac{\underline{\lambda}\bar{\lambda}^c}{\beta}\left(\alpha_l + \frac{\underline{\lambda} + \bar{\lambda}}{2}\Delta\alpha\right)\overline{w} + \left(\frac{\underline{\lambda}\bar{\lambda}^c\Delta\alpha}{2\beta}\right)^2\right\}.\end{aligned} \tag{C.35}$$

Therefore, if $(\underline{\lambda} + \bar{\lambda})^2 \ge 4\underline{\lambda}$, the quadratic function (C.35) is convex and achieves its maximum at $\overline{w} = \frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\Delta\lambda}$, which falls back to the previous case and hence is proved.

If $(\underline{\lambda} + \bar{\lambda})^2 < 4\underline{\lambda}$, the quadratic function (C.35) is concave and achieves its unconstrained maximum at

$$\overline{w} = \frac{2\underline{\lambda}\bar{\lambda}^c}{\beta\left[4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2\right]}\left(\alpha_l + \frac{\underline{\lambda} + \bar{\lambda}}{2}\Delta\alpha\right) > 0, \tag{C.36}$$

which will be smaller than $\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\Delta\lambda}$ if and only if

$$\frac{\alpha_l}{\Delta\alpha} + \frac{\underline{\lambda} + \bar{\lambda}}{2} < \frac{4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2}{2\Delta\lambda} \quad\Leftrightarrow\quad 0 \le \frac{\alpha_l}{\Delta\alpha} < \frac{2}{\Delta\lambda}\left(\underline{\lambda} - \frac{\bar{\lambda}(\underline{\lambda} + \bar{\lambda})}{2}\right). \tag{C.37}$$

Therefore, if (C.37) does not hold, the quadratic function (C.35) achieves its maximum at $\overline{w} = \frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\Delta\lambda}$, which again falls back to the previous case and hence is proved.

In the remaining proof, we will work under (C.37), which implies the maximum of the quadratic function (C.35) to be

$$\frac{\underline{\lambda}\bar{\lambda}^c}{2\beta}\left\{\frac{1}{4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2}\left(\alpha_l + \frac{\underline{\lambda} + \bar{\lambda}}{2}\Delta\alpha\right)^2 + \frac{(\Delta\alpha)^2}{4}\right\}.$$

By (C.2), to obtain (C.32), it suffices to show

$$\begin{aligned}\overline{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\left(\tilde{w}^{\star\star}\right)^2 + \bar{\lambda}^c\left(\tilde{u}^{\star\star}\right)^2\right] \ge& \frac{\underline{\lambda}\bar{\lambda}^c}{2\beta}\left\{\frac{1}{4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2}\left(\alpha_l + \frac{\underline{\lambda} + \bar{\lambda}}{2}\Delta\alpha\right)^2 + \frac{(\Delta\alpha)^2}{4}\right\}\\ \Leftrightarrow\quad \bar{\lambda}\left(\tilde{w}^{\star\star}\right)^2 + \bar{\lambda}^c\left(\tilde{u}^{\star\star}\right)^2 \le& \frac{\bar{\lambda}}{4\beta^2}\left\{\bar{\lambda}^c\bar{\lambda}\left(\Delta\alpha\right)^2 - \frac{\left[\left(2\underline{\lambda} - \bar{\lambda}(\bar{\lambda} + \underline{\lambda})\right)\Delta\alpha - \Delta\lambda\alpha_l\right]^2}{4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2}\right\},\end{aligned} \tag{C.38}$$

where the right-hand side of (C.38) is positive and, in fact, greater than

$$\frac{\bar{\lambda}}{4\beta^2}\left(\Delta\alpha\right)^2\left\{\bar{\lambda}^c\bar{\lambda} - \frac{\left[2\underline{\lambda} - \bar{\lambda}(\bar{\lambda} + \underline{\lambda})\right]^2}{4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2}\right\} = \frac{\bar{\lambda}\left(\Delta\alpha\right)^2}{4\beta^2\left[4\underline{\lambda} - (\underline{\lambda} + \bar{\lambda})^2\right]}\left[\frac{\underline{\lambda}\bar{\lambda}^c}{2} + \frac{1}{2}\left(\underline{\lambda} - \frac{\bar{\lambda}(\underline{\lambda} + \bar{\lambda})}{2}\right)\right] > 0,$$

where the inequalities follow from (C.37).

By (C.23), to show (C.38), we just need to show that there exists $(\widetilde{w},\widetilde{u})$ satisfying (C.1) and (C.3) such that

$$\bar{\lambda}\widetilde{w}^2+\bar{\lambda}^c\widetilde{u}^2=\frac{\bar{\lambda}}{4\beta^2}\left\{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2-\frac{(B\Delta\alpha-\Delta\lambda\alpha_l)^2}{A}\right\}, \tag{C.39}$$

where we adopt the abbreviation $A:=4\underline{\lambda}-(\underline{\lambda}+\bar{\lambda})^2$ and $B:=2\underline{\lambda}-\bar{\lambda}(\bar{\lambda}+\underline{\lambda})$ for notational convenience.

To that end, we make the following change of variable

$$\widetilde{w}=\sqrt{\frac{1}{\bar{\lambda}}}\left(x\sqrt{\frac{1+\bar{\lambda}^{\frac{1}{2}}}{2}}-y\sqrt{\frac{1-\bar{\lambda}^{\frac{1}{2}}}{2}}\right),\quad \widetilde{u}=\sqrt{\frac{1}{\bar{\lambda}^c}}\left(x\sqrt{\frac{1-\bar{\lambda}^{\frac{1}{2}}}{2}}+y\sqrt{\frac{1+\bar{\lambda}^{\frac{1}{2}}}{2}}\right), \tag{C.40}$$

we then can straightforwardly verify that

$$\bar{\lambda}\widetilde{w}^2+\bar{\lambda}^c\widetilde{u}^2=x^2+y^2, \tag{C.41}$$

and (C.3) can then be written as

$$\left(\bar{\lambda}^c+\underline{\lambda}^c-\frac{\Delta\lambda}{\bar{\lambda}^{\frac{1}{2}}}\right)x^2-\frac{\bar{\lambda}^c\Delta\lambda}{\beta}\frac{\alpha_l-\bar{\lambda}^{\frac{1}{2}}\Delta\alpha}{\sqrt{2(1+\bar{\lambda}^{\frac{1}{2}})}}x+\left(\bar{\lambda}^c+\underline{\lambda}^c+\frac{\Delta\lambda}{\bar{\lambda}^{\frac{1}{2}}}\right)y^2-\frac{\bar{\lambda}^c\Delta\lambda}{\beta}\frac{\alpha_l+\bar{\lambda}^{\frac{1}{2}}\Delta\alpha}{\sqrt{2(1-\bar{\lambda}^{\frac{1}{2}})}}y\geq\frac{\bar{\lambda}^c\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{2\beta^2}. \tag{C.42}$$

Obviously, the $(\widetilde{w},\widetilde{u})$ defined through (C.40) by letting $x=0$ and $y=-\frac{\bar{\lambda}^{\frac{1}{2}}}{2\beta}\sqrt{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2-\frac{(B\Delta\alpha-\Delta\lambda\alpha_l)^2}{A}}$ satisfies (C.39) by virtue of (C.41). It is also straightforward to verify that such $(\widetilde{w},\widetilde{u})$ satisfies (C.1).

We now verify that it also satisfies (C.42), which implies that the corresponding $(\widetilde{w},\widetilde{u})$ must satisfy (C.3). To that end, plugging it to (C.42) renders it to

$$\underbrace{\frac{1}{AB}\left\{\left(\bar{\lambda}^{\frac{1}{2}}\Delta\lambda-B\right)(B-\Delta\lambda z)^2+A\bar{\lambda}^c\bar{\lambda}^{\frac{1}{2}}\left(B\bar{\lambda}^{\frac{1}{2}}-\Delta\lambda z^2\right)\right\}}_{\Psi_1(z)}+\underbrace{\sqrt{2\bar{\lambda}^c\left(1+\bar{\lambda}^{\frac{1}{2}}\right)}(z+\bar{\lambda}^{\frac{1}{2}})\sqrt{\bar{\lambda}^c\bar{\lambda}-\frac{(B-\Delta\lambda z)^2}{A}}}_{\Psi_2(z)}\geq 0, \tag{C.43}$$

where $z:=\alpha_l/\Delta\alpha<B/\Delta\lambda$ according to (C.37).

When $B\leq\bar{\lambda}^{\frac{1}{2}}\Delta\lambda$ (i.e., $4\underline{\lambda}^2\leq\bar{\lambda}(\bar{\lambda}+\underline{\lambda})^2$), we immediately have the first term on the left-hand side of (C.43) $\Psi_1(z)\geq 0$, and hence (C.43) holds.

When $B>\bar{\lambda}^{\frac{1}{2}}\Delta\lambda$ (i.e., $4\underline{\lambda}^2>\bar{\lambda}(\bar{\lambda}+\underline{\lambda})^2$), we recognize that

$$\Psi_2(z)\geq\Psi_3(z):=\sqrt{2\bar{\lambda}^c\left(1+\bar{\lambda}^{\frac{1}{2}}\right)}\left(z+\bar{\lambda}^{\frac{1}{2}}\right)\left((\bar{\lambda}^c\bar{\lambda})^{\frac{1}{2}}-\frac{B-\Delta\lambda z}{\sqrt{A}}\right),$$

which suggest that it suffices to show

$$\Psi_1(z)+\Psi_3(z)\geq 0,\ \text{for } z\in[0,B/\Delta\lambda]. \tag{C.44}$$

Direct calculation reveals that

$$\Psi_1(z)+\Psi_3(z)=\underbrace{\bar{\lambda}^c\bar{\lambda}-\frac{B\left(B-\bar{\lambda}^{\frac{1}{2}}\Delta\lambda\right)}{A}+\sqrt{2\bar{\lambda}\bar{\lambda}^c\left(1+\bar{\lambda}^{\frac{1}{2}}\right)}\left((\bar{\lambda}^c\bar{\lambda})^{\frac{1}{2}}-\frac{B}{\sqrt{A}}\right)}_{\text{constant}}$$
$$+\underbrace{\left[\frac{2\Delta\lambda\left(B-\bar{\lambda}^{\frac{1}{2}}\Delta\lambda\right)}{A}+\sqrt{2\bar{\lambda}^c\left(1+\bar{\lambda}^{\frac{1}{2}}\right)}\left((\bar{\lambda}^c\bar{\lambda})^{\frac{1}{2}}-\frac{B-\bar{\lambda}^{\frac{1}{2}}\Delta\lambda}{\sqrt{A}}\right)\right]z}_{>0}$$

$$+\Delta\lambda\left(\sqrt{\frac{2\bar{\lambda}^c\left(1+\bar{\lambda}^{\frac{1}{2}}\right)}{A}}-\frac{\bar{\lambda}^c\bar{\lambda}^{\frac{1}{2}}}{B}-\frac{\Delta\lambda\left(B-\bar{\lambda}^{\frac{1}{2}}\Delta\lambda\right)}{AB}\right)z^2.$$

If the coefficient of z^2 is nonnegative, then $\Psi_1(z)+\Psi_3(z)$ is increasing in z and hence (C.44) follows from

$$\Psi_1(0)+\Psi_3(0)>\Psi_1(0)=\frac{\Delta\lambda}{A}\left[\left(1+\bar{\lambda}^{\frac{1}{2}}\right)B+2\underline{\lambda}\bar{\lambda}^c\right]>0. \tag{C.45}$$

If the coefficient of z^2 is negative, then $\Psi_1(z)+\Psi_3(z)$ is concave in z, and hence (C.44) follows from (C.45) and

$$\Psi_1\left(B/\Delta\lambda\right)+\Psi_3\left(B/\Delta\lambda\right)=\bar{\lambda}^c\bar{\lambda}^{\frac{1}{2}}\left[\sqrt{2\left(1+\bar{\lambda}^{\frac{1}{2}}\right)}\left(B/\Delta\lambda+2\bar{\lambda}^{\frac{1}{2}}\right)-\left(B/\Delta\lambda-\bar{\lambda}^{\frac{1}{2}}\right)\right]>0.$$

This completes the proof. □

Proof of Proposition 4. First, $\overline{w}^{\star\star}>\overline{w}^{\circ}>\underline{w}^{\circ}>\overline{w}^{\sharp}$ follows from Proposition 3 and (C.5) in Proposition C.1.

By Proposition C.2, $\bar{r}^{\flat}>r^{\circ}$ follows from (C.15). To show that $\bar{r}^{\star\star}=\alpha_l/(2\beta)+\widetilde{w}^{\star\star}-\widetilde{u}^{\star\star}>\bar{r}^{\flat}=\alpha_l/(2\beta)-\widetilde{u}^{\flat}$, we now demonstrate that

$$\widetilde{w}^{\star\star}-\widetilde{u}^{\star\star}>-\widetilde{u}^{\flat}, \tag{C.46}$$

where $\widetilde{u}^{\flat}$ is identified in the proof of Proposition C.2. Since both $\left(0,\widetilde{u}^{\flat}\right)$ and $(\widetilde{w}^{\star\star},\widetilde{u}^{\star\star})$ bind the constraint (C.3), $\left(0,\widetilde{z}^{\flat}=-\widetilde{u}^{\flat}\right)$ and $(\widetilde{w}^{\star\star},\widetilde{z}^{\star\star}=\widetilde{w}^{\star\star}-\widetilde{u}^{\star\star})$ lie on the same quadratic curve in the $(\widetilde{w},\widetilde{z})$-space given by

$$\bar{\lambda}\widetilde{w}^2+\bar{\lambda}^c\left(\widetilde{w}-\widetilde{z}\right)^2+\Delta\lambda\left(\widetilde{z}+\frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}+\widetilde{z}-\widetilde{w}\right)=\text{constant},$$

or equivalently,

$$\widetilde{w}^2+\underline{\lambda}^c\widetilde{z}^2-(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{w}\widetilde{z}-\frac{\alpha_l\Delta\lambda}{2\beta}\widetilde{w}+\frac{(\alpha_l+\bar{\lambda}\Delta\alpha)\Delta\lambda}{2\beta}\widetilde{z}=\text{constant}. \tag{C.47}$$

Since $\widetilde{w}^{\star\star}>0$ and $\widetilde{u}^{\star\star},\widetilde{u}^{\flat}\in\left(-\bar{\lambda}\Delta\alpha/(2\beta),0\right)$, we just need to focus on the region $\Omega:=\left\{(\widetilde{w},\widetilde{z}):\widetilde{w}\geq 0\text{ and }\widetilde{z}\in\left[\widetilde{w},\widetilde{w}+\bar{\lambda}\Delta\alpha/(2\beta)\right]\right\}$.

Total differentiation of (C.47) yields

$$\frac{d\widetilde{z}}{d\widetilde{w}}=\frac{2\widetilde{w}-(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{z}-\frac{\alpha_l\Delta\lambda}{2\beta}}{(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{w}-2\underline{\lambda}^c\widetilde{z}-\frac{(\alpha_l+\bar{\lambda}\Delta\alpha)\Delta\lambda}{2\beta}}, \tag{C.48}$$

where we note that the denominator $(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{w}-2\underline{\lambda}^c\widetilde{z}-\frac{(\alpha_l+\bar{\lambda}\Delta\alpha)\Delta\lambda}{2\beta}=2\underline{\lambda}^c(\widetilde{w}-\widetilde{z})-\Delta\lambda\left(\widetilde{w}+\frac{\alpha_l+\bar{\lambda}\Delta\alpha}{2\beta}\right)<0$. Therefore, the region Ω is divided by the straight line $2\widetilde{w}-(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{z}-\frac{\alpha_l\Delta\lambda}{2\beta}=0$ into two segments: in the segment where $2\widetilde{w}-(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{z}-\frac{\alpha_l\Delta\lambda}{2\beta}<(>)0$, $\widetilde{z}$ is strictly increasing (decreasing) in $\widetilde{w}$.

Since

$$2\widetilde{w}^{\star\star}-(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{z}^{\star\star}-\frac{\alpha_l\Delta\lambda}{2\beta}=\left(\bar{\lambda}+\underline{\lambda}\right)\widetilde{w}^{\star\star}+\left(\bar{\lambda}^c+\underline{\lambda}^c\right)\widetilde{u}^{\star\star}-\frac{\alpha_l\Delta\lambda}{2\beta}<0\quad\text{by (C.31)},$$

and

$$2*0-(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{z}^{\flat}-\frac{\alpha_l\Delta\lambda}{2\beta}=(\bar{\lambda}^c+\underline{\lambda}^c)\widetilde{u}^{\flat}-\frac{\alpha_l\Delta\lambda}{2\beta}<0,$$

as shown in the proof of Proposition C.2, both $\left(0,\widetilde{z}^{\flat}\right)$ and $(\widetilde{w}^{\star\star},\widetilde{z}^{\star\star})$ lie on the increasing branch of the quadratic curve in (C.47). Therefore, $\widetilde{w}^{\star\star}>0$ suggests $\widetilde{w}^{\star\star}-\widetilde{u}^{\star\star}=\widetilde{z}^{\star\star}>\widetilde{z}^{\flat}=-\widetilde{u}^{\flat}$, namely (C.46).

The profit rank $\overline{\pi}^\flat < \overline{\pi}^{\star\star} < \overline{\pi}^\circ$ simply follows from the fact that (4.2) is a relaxed problem of (5.4), which is in turn a relaxed problem of (5.8). To show that $\overline{\pi}^\flat > \overline{\pi}^\sharp$, we recognize from (C.2) that it is equivalent to show

$$\overline{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\bar{\lambda}^c\left(\widetilde{u}^\flat\right)^2 = \overline{\pi}^\flat > \overline{\pi}^\sharp = \overline{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left(\widetilde{w}^\sharp\right)^2,$$

or equivalently, because both $\widetilde{w}^\sharp < 0$ and $\widetilde{u}^\flat < 0$,

$$\widetilde{w}^\sharp < \sqrt{\bar{\lambda}^c}\widetilde{u}^\flat. \tag{C.49}$$

By (C.11) and (C.21), (C.49) is equivalent to

$$\sqrt{(\alpha_l\Delta\lambda)^2 + 4\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]} - \alpha_l\Delta\lambda > \sqrt{\bar{\lambda}^c}\left(\sqrt{\left(\frac{\Delta\lambda\overline{\alpha}}{\underline{\lambda}^c}\right)^2 + \frac{4\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]}{\underline{\lambda}^c}} - \frac{\Delta\lambda\overline{\alpha}}{\underline{\lambda}^c}\right)$$

$$\Leftrightarrow \quad \sqrt{(\Delta\lambda\overline{\alpha})^2 + 4\underline{\lambda}^c\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]} + \Delta\lambda\overline{\alpha} > \sqrt{\bar{\lambda}^c(\alpha_l\Delta\lambda)^2 + 4\bar{\lambda}^c\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+\alpha_l\Delta\alpha\right]} + \sqrt{\bar{\lambda}^c}\alpha_l\Delta\lambda,$$

which obviously holds because $\bar{\alpha} > \alpha_l > \sqrt{\bar{\lambda}^c}\alpha_l$ and $\underline{\lambda}^c > \bar{\lambda}^c$. Therefore, we have shown (C.49) and hence $\overline{\pi}^\flat > \overline{\pi}^\sharp$.

To see that $\bar{s}^\sharp > \bar{s}^\flat$, we notice that it is equivalent to

$$s^\circ - \frac{\beta}{2\bar{\lambda}}\widetilde{w}^\sharp = \bar{s}^\sharp > \bar{s}^\flat = s^\circ - \frac{\beta}{2\bar{\lambda}}\bar{\lambda}^c\widetilde{u}^\flat \quad \Leftrightarrow \quad \widetilde{w}^\sharp < \bar{\lambda}^c\widetilde{u}^\flat,$$

which holds and follows immediately from (C.49).

To show $s^\circ < \bar{s}^{\star\star} < \bar{s}^\flat$, we first claim that $\widetilde{u}^{\star\star} > \widetilde{u}^\flat$. Since both $\left(0,\widetilde{u}^\flat\right)$ and $\left(\widetilde{w}^{\star\star},\widetilde{u}^{\star\star}\right)$ bind the constraint (C.3), we have

$$\begin{aligned}
\bar{\lambda}^c\left(\widetilde{u}^\flat\right)^2 + \Delta\lambda\left(\frac{\alpha_l}{2\beta}-\widetilde{u}^\flat\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^\flat\right) =& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}\\
>& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2} - \left[\bar{\lambda}\left(\widetilde{w}^{\star\star}\right)^2 + \Delta\lambda\widetilde{w}^{\star\star}\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star}\right)\right]\\
=& \bar{\lambda}^c\left(\widetilde{u}^{\star\star}\right)^2 + \Delta\lambda\left(\frac{\alpha_l}{2\beta}-\widetilde{u}^{\star\star}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star}\right),
\end{aligned}$$

where the inequality follows from the fact that $\widetilde{w}^{\star\star} > 0 > \widetilde{u}^{\star\star}$. That is, the quadratic function

$$\bar{\lambda}^c\widetilde{u}^2 + \Delta\lambda\left(\frac{\alpha_l}{2\beta}-\widetilde{u}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}\right) - \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2},$$

has $\widetilde{u}^\flat < 0$ as its smaller root while takes negative value at $\widetilde{u}^{\star\star} < 0$, immediately suggesting that $\widetilde{u}^{\star\star} > \widetilde{u}^\flat$. We thus have

$$\bar{\lambda}\widetilde{w}^{\star\star} + \bar{\lambda}^c\widetilde{u}^{\star\star} = \bar{\lambda}\left(\widetilde{w}^{\star\star}-\widetilde{u}^{\star\star}\right) + \widetilde{u}^{\star\star} > -\bar{\lambda}\widetilde{u}^\flat + \widetilde{u}^\flat = \bar{\lambda}^c\widetilde{u}^\flat, \tag{C.50}$$

where we also used (C.46) to obtain the inequality.

To evaluate the retailer's stocking quantity as well as unsold inventory, we first note that $\widetilde{u}^{\star\star} < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and hence $\overline{w}^{\star\star} - \overline{r}^{\star\star} = \widetilde{u}^{\star\star} + \frac{\bar{\lambda}\Delta\alpha}{2\beta} < \frac{\bar{\lambda}\Delta\alpha}{\beta}$, which, according to Lemma 1, suggests that all inventory is sold out in the case of high baseline demand. In particular, (3.1) suggests that the retailer orders

$$\begin{aligned}
\bar{s}^\flat =& s^R\left(\overline{w}^\circ,\overline{r}^\flat,1,\bar{\lambda}\right) = \frac{\bar{\lambda}^c\beta\overline{r}^\flat + \bar{\lambda}\alpha_h - \beta\overline{w}^\circ}{2\bar{\lambda}} = \frac{\bar{\lambda}^c\beta r^\circ + \bar{\lambda}\alpha_h - \beta\overline{w}^\circ}{2\bar{\lambda}} - \frac{\beta}{2\bar{\lambda}}\bar{\lambda}^c\widetilde{u}^\flat,\\
\bar{s}^{\star\star} =& s^R\left(\overline{w}^{\star\star},\overline{r}^{\star\star},1,\bar{\lambda}\right) = \frac{\bar{\lambda}^c\beta\overline{r}^{\star\star} + \bar{\lambda}\alpha_h - \beta\overline{w}^{\star\star}}{2\bar{\lambda}} = \underbrace{\frac{\bar{\lambda}^c\beta r^\circ + \bar{\lambda}\alpha_h - \beta\overline{w}^\circ}{2\bar{\lambda}}}_{s^\circ} - \frac{\beta}{2\bar{\lambda}}\left(\bar{\lambda}\widetilde{w}^{\star\star} + \bar{\lambda}^c\widetilde{u}^{\star\star}\right).
\end{aligned}$$

Therefore, $\bar{s}^{\star\star} > s^\circ$ follows from (C.30) and $\bar{s}^{\star\star} < \bar{s}^\flat$ from (C.50). □

Appendix D: Proofs in Section 6

The retailer's prior belief is that the manufacturer is of a low-risk high-demand type $(\bar{\theta}, \bar{\lambda})$ with probability $\upsilon \in (0,1)$ and is of a high-risk low-demand type $(\underline{\theta}, \underline{\lambda})$ with probability $\upsilon^c \in (0,1)$. Under a separating equilibrium, the retailer forms belief $\widehat{\upsilon} = \widehat{\upsilon}(w,r) := \mathbb{P}\left[(\theta,\lambda) = (\bar{\theta},\bar{\lambda}) \,\middle|\, (w,r)\right] \in \{0,1\}$ upon being offered contract (w,r). Accordingly, we denote

$$\widehat{\theta} = \widehat{\theta}(w,r) =: \mathbb{E}\left[\theta \mid (w,r)\right] = \widehat{\upsilon}\bar{\theta} + \widehat{\upsilon}^c\underline{\theta}, \quad \text{and} \quad \widehat{\lambda} = \widehat{\lambda}(w,r) =: \mathbb{E}\left[\lambda \mid (w,r)\right] = \widehat{\upsilon}\bar{\lambda} + \widehat{\upsilon}^c\underline{\lambda}.$$

Consequently, the retailer's ordering strategy is characterized by Lemma 1 and the manufacturer's reduced-form profit function is given by

$$\Pi\left(w,r \mid \widehat{\upsilon},(\theta,\lambda)\right) = \frac{1}{2}w\left(\alpha_l + \widehat{\lambda}\Delta\alpha - \beta w\right) + \frac{1}{2}\left(\widehat{\lambda}^c w - \lambda^c\theta r\right)\left[\Delta\alpha - \beta/\widehat{\lambda}(w - \widehat{\theta}r)\right]^+. \tag{D.1}$$

Similar to Lemmas 2 and 3, we can specialize (D.3) $\widehat{\upsilon} \in \{0,1\}$ (and hence $\widehat{\lambda} = \lambda$ and $\widehat{\theta}_l = \widehat{\theta}_h = \theta$) to obtain the symmetric-information contract

$$w^\circ(\theta,\lambda) = \frac{\alpha_l + \lambda\Delta\alpha}{2\beta} \quad \text{and} \quad r^\circ(\theta,\lambda) = \frac{\alpha_l}{2\beta\theta}, \tag{D.2}$$

yielding (6.1) and (6.2).

Consequently, the retailer's ordering strategy is characterized by Lemma 1 and the manufacturer's reduced-form profit function is given by

$$\Pi\left(w,r \mid \widehat{\upsilon},(\theta,\lambda)\right) = \frac{1}{2}w\left(\alpha_l + \widehat{\lambda}\Delta\alpha - \beta w\right) + \frac{1}{2}\left(\widehat{\lambda}^c w - \lambda^c\theta r\right)\left[\Delta\alpha - \beta/\widehat{\lambda}(w - \widehat{\theta}r)\right]^+. \tag{D.3}$$

As will be verified later (by identifying the supporting equilibrium belief), manufacturer of type $(\underline{\theta}, \underline{\lambda})$ offers the symmetric-information contract $(\underline{w}^\circ, \underline{r}^\circ)$ and hence earns $\underline{\pi}^\circ$, establishing the first statement of Proposition 5. Thus, manufacturer of type $(\bar{\theta}, \bar{\lambda})$ needs to distinguish herself from type $(\underline{\theta}, \underline{\lambda})$, and, in the most efficient separating equilibrium, offers the buyback contract according to

$$\begin{aligned} &\max_{\overline{w} \ge \bar{\theta}\bar{r} \ge 0} \quad \Pi\left(\overline{w}, \bar{r} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) \\ &\text{subject to} \quad \Pi\left(\overline{w}, \bar{r} \mid 1, (\underline{\theta}, \underline{\lambda})\right) \le \underline{\pi}^\circ \quad \text{and} \quad \Pi\left(\overline{w}, \bar{r} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) \ge \Pi\left(\underline{w}^\circ, \underline{r}^\circ \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right), \end{aligned} \tag{D.4}$$

where the two IC constraints are the non-mimicry condition for type $(\underline{\theta}, \underline{\lambda})$ and $(\bar{\theta}, \bar{\lambda})$, respectively.

Similar to Lemmas B.3 and C.3, we obtain the following (proof omitted)

Lemma D.1 **(Change of Variable)**. *Let* $\delta := \left(\underline{\theta}\underline{\lambda}^c - \bar{\theta}\bar{\lambda}^c\right)/\bar{\theta}$, $\widetilde{w} := \overline{w} - \frac{\bar{\alpha}}{2\beta}$ *and* $\widetilde{u} := \overline{w} - \bar{\theta}\bar{r} - \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ *(that is,* $\overline{w} = \overline{w}^\circ + \widetilde{w}$ *and* $\bar{r} = \bar{r}^\circ + (\widetilde{w} - \widetilde{u})/\bar{\theta}$*). Then,*

$$\overline{w} \ge \bar{\theta}\bar{r} \ge 0 \quad \Leftrightarrow \quad \widetilde{w} \ge \widetilde{u} - \frac{\alpha_l}{2\beta} \text{ and } -\frac{\bar{\lambda}\Delta\alpha}{2\beta} \le \widetilde{u} \le \frac{\bar{\lambda}\Delta\alpha}{2\beta}, \tag{D.5}$$

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) = \bar{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left(\bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2\right), \tag{D.6}$$

$$\Pi\left(\overline{w}, \bar{r} \mid 1, (\underline{\theta}, \underline{\lambda})\right) \le \underline{\pi}^\circ \quad \Leftrightarrow \quad \bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2 + \delta\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right) \ge \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}, \tag{D.7}$$

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) \ge \Pi\left(\underline{w}^\circ, \underline{r}^\circ \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) \quad \Leftrightarrow \quad \bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2 \le \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\frac{\bar{\theta}}{\underline{\theta}}\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2}. \tag{D.8}$$

As before, Lemma D.1 implies that the solution to (D.4) is given by $\overline{w}^{\star\star\star} = \overline{w}^{\circ} + \widetilde{w}^{\star\star\star}$ and $\bar{r}^{\star\star\star} = \bar{r}^{\circ} + (\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star})/\bar{\theta}$, where $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ is the solution to

$$\min_{\widetilde{w} \geq \widetilde{u} - \frac{\alpha_l}{2\beta}, -\frac{\bar{\lambda}\Delta\alpha}{2\beta} \leq \widetilde{u} \leq \frac{\bar{\lambda}\Delta\alpha}{2\beta}} \bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2, \quad \text{subject to (D.7)}, \tag{D.9}$$

provided that $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ satisfies (D.8), i.e., the optimal objective value of (D.9) is bounded below by the left-hand side of (D.8). We note that if $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ does not satisfy (D.8), then (D.4) is infeasible and hence the most efficient separating equilibrium does not exist.

Straightforward verification yields the following

LEMMA D.2. *Parameter $\delta := \left(\underline{\theta}\underline{\lambda}^c - \bar{\theta}\bar{\lambda}^c\right)/\bar{\theta}$ satisfies the following properties: i) $\delta + \bar{\lambda}^c = \underline{\lambda}^c\underline{\theta}/\bar{\theta} > 0$, ii) $\delta = \Delta\lambda - \underline{\lambda}^c\Delta\theta/\bar{\theta}$, iii) $\delta\bar{\theta}/\underline{\theta} = \Delta\lambda - \bar{\lambda}^c\Delta\theta/\underline{\theta}$, iv) $\delta \geq 0 \Leftrightarrow \Delta\theta/\bar{\theta} \leq \Delta\lambda/\underline{\lambda}^c$, v) $\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta^2}{4\underline{\lambda}\bar{\lambda}^c} \geq 1 \Leftrightarrow \Delta\theta/\bar{\theta} \leq 1 - \left[\bar{\lambda}^c/\left(1-\sqrt{\underline{\lambda}}\right)^2\right] (< \Delta\lambda/\underline{\lambda}^c)$ or $\Delta\theta/\bar{\theta} \geq 1 - \left[\bar{\lambda}^c/\left(1+\sqrt{\underline{\lambda}}\right)^2\right] (> \Delta\lambda/\underline{\lambda}^c)$.*

D.1. Case of $\delta < 0$ (i.e., $\underline{\theta}\underline{\lambda}^c < \bar{\theta}\bar{\lambda}^c$)

LEMMA D.3. *For $\delta < 0$, there exists a feasible solution $(\widetilde{w}, \widetilde{u})$ to (D.9) such that*

$$\bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2 \leq \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2}, \tag{D.10}$$

and hence the optimal solution to (D.9), $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$, must satisfy (D.8).

Proof. Since the objective of (D.9) is to minimize the left-hand side of (D.8), we can ignore constraint (D.8) once the feasibility of (D.9) is established. Below, we identify such a feasible solution satisfying (D.10), stronger than (D.8).

- If $\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] - \delta\alpha_l\Delta\alpha \leq \bar{\lambda}(\Delta\alpha)^2$, then

$$\widetilde{w} = \widetilde{u} = \frac{1}{2\beta}\sqrt{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] - \delta\bar{\lambda}\alpha_l\Delta\alpha} \in \left(0, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$$

binds (D.10) and also satisfies (D.7):

$$\begin{aligned}
&\bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2 + \delta\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right) \\
&= \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} + \delta\left[\underbrace{\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)}_{=\frac{\alpha_l}{2\beta}}\underbrace{\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right)}_{\leq\frac{\bar{\lambda}\Delta\alpha}{2\beta}} - \frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2}\right] \geq \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}.
\end{aligned}$$

- Otherwise, $\widetilde{u} = \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and

$$\widetilde{w} = \frac{1}{2\beta}\sqrt{\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] - \delta\alpha_l\Delta\alpha - \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2} > \widetilde{u} = \frac{\bar{\lambda}\Delta\alpha}{2\beta}$$

bind (D.10) and also satisfy (D.7):

$$\begin{aligned}
&\bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2 + \delta\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right) \\
&= \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} + \delta\left[\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\underbrace{\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right)}_{=0} - \frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2}\right] \geq \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}. \quad \square
\end{aligned}$$

LEMMA D.4. *For $\delta<0$, (D.7) must be binding at the optimal solution to (D.9).*

Proof. Suppose $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ satisfy (D.7) with strict inequality. Since $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}=0$ does not satisfy (D.7), we thus consider the following scenarios:

- If $\widetilde{u}^{\star\star\star}>0$, there must exist $\epsilon>0$ such that $\widetilde{u}^{\star\star\star}-\epsilon>0$ and, by continuity of the left-hand side of (D.7) in $\widetilde{u}$,

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}-\epsilon\right)^2+\delta\left(\widetilde{w}^{\star\star\star}-\widetilde{u}^{\star\star\star}+\epsilon+\frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star\star}+\epsilon\right)\geq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}.$$

As $\widetilde{u}^{\star\star\star}-\epsilon<\widetilde{u}^{\star\star\star}\leq\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and $\widetilde{w}^{\star\star\star}\geq\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}>\widetilde{u}^{\star\star\star}-\epsilon-\frac{\alpha_l}{2\beta}$, thus $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star}-\epsilon)$ is a feasible solution to (D.9) but contradicts the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ because $\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}-\epsilon\right)^2<\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2$.

- If $\widetilde{u}^{\star\star\star}<0$ and $\widetilde{w}^{\star\star\star}>\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$, then there exists $\epsilon>0$ such that $\widetilde{u}^{\star\star\star}+\epsilon<0$, $\widetilde{w}^{\star\star\star}\geq\widetilde{u}^{\star\star\star}+\epsilon-\frac{\alpha_l}{2\beta}$ and, by continuity of the left-hand side of (D.7) in $\widetilde{u}$,

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}+\epsilon\right)^2+\delta\left(\widetilde{w}^{\star\star\star}-\widetilde{u}^{\star\star\star}-\epsilon+\frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star\star}-\epsilon\right)\geq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}.$$

That is, $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star}+\epsilon)$ is a feasible solution to (D.9) but contradicts the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ because $\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}+\epsilon\right)^2<\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2$.

- If $\widetilde{u}^{\star\star\star}<0$ and $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}<0$, then there exists $\epsilon>0$ such that $\widetilde{u}^{\star\star\star}+\epsilon<0$, $\widetilde{w}^{\star\star\star}+\epsilon<0$ and, by continuity of the left-hand side of (D.7) in $(\widetilde{w},\widetilde{u})$,

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}+\epsilon\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}+\epsilon\right)^2+\delta\left(\widetilde{w}^{\star\star\star}-\widetilde{u}^{\star\star\star}+\frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star\star}-\epsilon\right)\geq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}.$$

That is, $(\widetilde{w}^{\star\star\star}+\epsilon,\widetilde{u}^{\star\star\star}+\epsilon)$ is a feasible solution to (D.9) but contradicts the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ because $\bar{\lambda}\left(\widetilde{w}^{\star\star\star}+\epsilon\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}+\epsilon\right)^2<\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2$.

- If $\widetilde{w}^{\star\star\star}<0$, using arguments similar to the case of $\widetilde{u}^{\star\star\star}>0$, one can show that there exists $\epsilon>0$ such that $(\widetilde{w}^{\star\star\star}+\epsilon,\widetilde{u}^{\star\star\star})$ is a feasible solution to (D.9) but contradicts the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$.

- If $\widetilde{w}^{\star\star\star}>0$ and $\widetilde{w}^{\star\star\star}>\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$, using arguments similar to the case of $\widetilde{u}^{\star\star\star}<0$ and $\widetilde{w}^{\star\star\star}>\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$, one can show that there exists $\epsilon>0$ such that $(\widetilde{w}^{\star\star\star}-\epsilon,\widetilde{u}^{\star\star\star})$ is a feasible solution to (D.9) but contradicts the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$.

- If $\widetilde{w}^{\star\star\star}>0$ and $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$, which implies that $\widetilde{u}^{\star\star\star}>0$, then using arguments similar to the case of $\widetilde{u}^{\star\star\star}<0$ and $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$, one can show that there exists $\epsilon>0$ such that $(\widetilde{w}^{\star\star\star}-\epsilon,\widetilde{u}^{\star\star\star}-\epsilon)$ is a feasible solution to (D.9) but contradicts the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$. ☐

LEMMA D.5. *For $\delta<0$, the optimal solution to (D.9) must satisfy $\widetilde{u}^{\star\star\star}=\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ or $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}<0$ if and only if*

$$\Delta\lambda\geq\frac{\bar{\lambda}^c\min\left\{\alpha_l^2,\bar{\lambda}\left(\Delta\alpha\right)^2\right\}}{\Delta\alpha(\alpha_l+\alpha_h)}. \tag{D.11}$$

Proof. We first claim that $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ satisfies $\widetilde{u}^{\star\star\star}=\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ or $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$ if and only if

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2=\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}. \tag{D.12}$$

The necessity of (D.12) follows from the fact that $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ must bind (D.7) by Lemma D.4 and that $\left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right) = 0$. To see the sufficiency of (D.12), we note, by (D.7), that

$$\begin{aligned}\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2 \geq & \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right) \\ \geq & \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2},\end{aligned}$$

which immediately implies that $\widetilde{u}^{\star\star\star} = \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ or $\widetilde{w}^{\star\star\star} = \widetilde{u}^{\star\star\star} - \frac{\alpha_l}{2\beta}$.

It is straightforward to verify that $\widetilde{w}^{\star\star\star}$ exists such that $\widetilde{u}^{\star\star\star} = \frac{\bar{\lambda}\Delta\alpha}{2\beta}$, $\widetilde{w}^{\star\star\star} \geq \widetilde{u}^{\star\star\star} - \frac{\alpha_l}{2\beta} = \frac{\bar{\lambda}\Delta\alpha - \alpha_l}{2\beta}$, and (D.12) hold, if and only if

$$\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + [(\bar{\lambda}\Delta\alpha - \alpha_l)^+]^2 \leq \Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] = \Delta\lambda\Delta\alpha(\alpha_l + \alpha_h). \tag{D.13}$$

There exists $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ such that $\widetilde{u}^{\star\star\star} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$, $\widetilde{w}^{\star\star\star} = \widetilde{u}^{\star\star\star} - \frac{\alpha_l}{2\beta}$ and (D.12) hold, if and only if there exists $\widetilde{u} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$ such that $g(\widetilde{u}) = \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$, where

$$g(\widetilde{u}) := \bar{\lambda}\left(\widetilde{u} - \frac{\alpha_l}{2\beta}\right)^2 + \bar{\lambda}^c\widetilde{u}^2 = \left(\widetilde{u} - \frac{\bar{\lambda}\alpha_l}{2\beta}\right)^2 + \bar{\lambda}\bar{\lambda}^c\frac{\alpha_l^2}{4\beta^2}. \tag{D.14}$$

It is straightforward to verify that $g\left(-\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right) > \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$.

- If $\Delta\alpha \leq \alpha_l$, then $g(\widetilde{u})$ is monotonically decreasing in $\widetilde{u} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$. Thus, there exists $\widetilde{u} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$ such that $g(\widetilde{u}) = \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$ if and only if $g\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right) \leq \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$, which is equivalent to

$$\bar{\lambda}(\Delta\alpha)^2 - 2\bar{\lambda}\alpha_l\Delta\alpha + \alpha_l^2 \leq \Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]. \tag{D.15}$$

Since $\bar{\lambda}(\Delta\alpha)^2 - 2\bar{\lambda}\alpha_l\Delta\alpha + \alpha_l^2 \geq \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 = \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + [(\bar{\lambda}\Delta\alpha - \alpha_l)^+]^2$, (D.15) also implies (D.13), which is equivalent to (D.11).

- If $\Delta\alpha > \alpha_l$, then $g(\widetilde{u})$ reaches its minimum at $\frac{\bar{\lambda}\alpha_l}{2\beta} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\alpha_l}{2\beta}\right] \subset \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$. Thus, there exists $\widetilde{u} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$ such that $g(\widetilde{u}) = \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$ if and only if $g\left(\frac{\bar{\lambda}\alpha_l}{2\beta}\right) \leq \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$, which is equivalent to

$$\bar{\lambda}^c\alpha_l^2 \leq \Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] = \Delta\lambda\Delta\alpha(\alpha_l + \alpha_h). \tag{D.16}$$

It is straightforward to verify that

—if $\Delta\alpha > \alpha_l \geq \sqrt{\bar{\lambda}}\Delta\alpha$, then $\bar{\lambda}^c\alpha_l^2 \geq \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 = \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + [(\bar{\lambda}\Delta\alpha - \alpha_l)^+]^2$ and hence (D.16) also implies (D.13), which is equivalent to (D.11);

—if $\sqrt{\bar{\lambda}}\Delta\alpha > \alpha_l \geq \bar{\lambda}\Delta\alpha$, then $\bar{\lambda}^c\alpha_l^2 \leq \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 = \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + [(\bar{\lambda}\Delta\alpha - \alpha_l)^+]^2$ and hence (D.13) implies (D.16), which is equivalent to (D.11);

—if $\alpha_l < \bar{\lambda}\Delta\alpha$, then $\bar{\lambda}^c\alpha_l^2 \leq \bar{\lambda}(\Delta\alpha)^2 - 2\bar{\lambda}\alpha_l\Delta\alpha + \alpha_l^2 = \bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + [(\bar{\lambda}\Delta\alpha - \alpha_l)^+]^2$ and hence (D.13) again implies (D.16), which is equivalent to (D.11). □

LEMMA D.6. *For $\delta < 0$, if (D.11) does not hold, the optimal solution to (D.9) must satisfy $0 < \widetilde{u}^{\star\star\star} < \frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and $\widetilde{u}^{\star\star\star} - \frac{\alpha_l}{2\beta} < \widetilde{w}^{\star\star\star} < 0$.*

Proof. If (D.11) does not hold, Lemma D.5 implies that $\widetilde{u}^{\star\star\star} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)$ and $\widetilde{w}^{\star\star\star} > \widetilde{u}^{\star\star\star} - \frac{\alpha_l}{2\beta}$. Hence, the necessary condition for the optimality of $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ is for there to exist a Lagrangian multiplier $\xi \geq 0$ associated with (D.7) such that

$$2\bar{\lambda}\widetilde{w}^{\star\star\star} - \xi\left[2\bar{\lambda}\widetilde{w}^{\star\star\star} + \delta\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right)\right] = 0, \tag{D.17}$$

$$2\bar{\lambda}^c\widetilde{u}^{\star\star\star} - \xi\left[2\bar{\lambda}^c\widetilde{u}^{\star\star\star} + \delta\left(2\widetilde{u}^{\star\star\star} - \widetilde{w}^{\star\star\star} - \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right)\right] \geq 0, \text{ with “=” if } \widetilde{u}^{\star\star\star} > -\frac{\bar{\lambda}\Delta\alpha}{2\beta}. \tag{D.18}$$

We first note that $\xi > 0$ and hence (D.7) must be binding. Otherwise, (D.17) and (D.18) suggest that $\widetilde{w}^{\star\star\star} = 0$ and $\widetilde{u}^{\star\star\star} \geq 0$ with “=” if $\widetilde{u}^{\star\star\star} > -\frac{\bar{\lambda}\Delta\alpha}{2\beta}$. Thus, we must have $\widetilde{w}^{\star\star\star} = \widetilde{u}^{\star\star\star} = 0$, which then violates (D.7).

Using the properties in Lemma D.2 and rearranging terms of (D.17) and (D.18) yields

$$2\bar{\lambda}(1-\xi)\widetilde{w}^{\star\star\star} = \xi\delta\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right) < 0, \tag{D.19}$$

$$2(\bar{\theta}\bar{\lambda}^c - \xi\underline{\theta\lambda}^c)/\bar{\theta}\widetilde{u}^{\star\star\star} \geq -\xi\delta\left(\widetilde{w}^{\star\star\star} + \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right) > 0, \tag{D.20}$$

where the strict inequality follows from the fact that $\widetilde{w}^{\star\star\star} + \frac{\alpha_l}{2\beta} > \widetilde{u}^{\star\star\star} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)$. Therefore, we must have $\widetilde{w}^{\star\star\star} \neq 0$ and $\widetilde{u}^{\star\star\star} \neq 0$. Together with the fact that $\delta < 0$ or equivalently $\bar{\theta}\bar{\lambda}^c > \underline{\theta\lambda}^c$, we consider the following three possibilities:

1. If $\xi > \bar{\theta}\bar{\lambda}^c/(\underline{\theta\lambda}^c) > 1$, then (D.19) and (D.20) suggest that $\widetilde{w}^{\star\star\star} > 0$ and $\widetilde{u}^{\star\star\star} < 0$, respectively. However, as (D.7) is binding, this would imply that

$$\begin{aligned}\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2 =& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right)\\ >& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2},\end{aligned}$$

contradicting the optimality of $(\widetilde{w}^{\star\star}, \widetilde{u}^{\star\star})$ by Lemma D.3. Hence, this case can be ruled out.

2. If $\bar{\theta}\bar{\lambda}^c/(\underline{\theta\lambda}^c) > \xi > 1$, (D.19) and (D.20) suggest that $\widetilde{w}^{\star\star\star} > 0$ and $\widetilde{u}^{\star\star\star} > 0$, respectively. However, this leads to contradiction, because eliminating ξ and δ from (D.17) and (D.18) would yield a contradiction

$$\bar{\lambda}^c \underbrace{\widetilde{u}^{\star\star\star}}_{>0} \underbrace{\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right)}_{>0} \leq \bar{\lambda} \underbrace{\widetilde{w}^{\star\star\star}}_{>0} \underbrace{\left(2\widetilde{u}^{\star\star\star} - \widetilde{w}^{\star\star\star} - \frac{\alpha_l + \bar{\lambda}\Delta\alpha}{2\beta}\right)}_{<0}.$$

Hence, this case again can be ruled out.

3. As such, we must have $\bar{\theta}\bar{\lambda}^c/(\underline{\theta\lambda}^c) > 1 > \xi$, which implies that $\widetilde{w}^{\star\star\star} < 0$ and $\widetilde{u}^{\star\star\star} > 0$ according to (D.19) and (D.20), respectively, establishing the lemma. □

Lemma D.7. *For $\delta < 0$, if (D.11) does not hold, then*

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2 < \frac{\bar{\lambda}^c\bar{\lambda}^2\left(\Delta\alpha\right)^2}{4\beta^2}. \tag{D.21}$$

Proof. If (D.11) does not hold, it is straight forward to verify that the quadratic function in $\widetilde{u}$

$$\bar{\lambda}^c\widetilde{u}^2 + \delta\left(\frac{\alpha_l}{2\beta} - \widetilde{u}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right) = \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$$

has a root $\widetilde{u}^\flat \in \left(0, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)$. That is, $\left(0, \widetilde{u}^\flat\right)$ binds (D.7). Thus, the optimality of $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ must imply that

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2 \le \bar{\lambda}0^2 + \bar{\lambda}^c\left(\widetilde{u}^\flat\right)^2 < \frac{\bar{\lambda}^c\bar{\lambda}^2\left(\Delta\alpha\right)^2}{4\beta^2},$$

establishing (D.21). □

Proof of Proposition 5.1 and Corollary 1.1. For $\delta < 0$, Lemma D.3 implies that the optimal solution $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ to (D.9) must satisfy (D.8). Thus, the most efficient equilibrium, if exists, must be given by $\overline{w}^{\star\star\star} = \overline{w}^\circ + \widetilde{w}^{\star\star\star}$ and $\bar{r}^{\star\star\star} = \bar{r}^\circ + \left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star}\right)/\bar{\theta}$. By Lemma D.5 and D.6, if $\widetilde{u}^{\star\star\star} = \frac{\bar{\lambda}\Delta\alpha}{2\beta}$, then manufacturer $\left(\bar{\theta}, \bar{\lambda}\right)$ induces the retailer's unsold inventory to be

$$\frac{1}{2}\left[\Delta\alpha - \beta/\bar{\lambda}\left(\overline{w}^{\star\star\star} - \bar{\theta}\bar{r}^{\star\star\star}\right)\right]^+ = \frac{1}{2}\left[\Delta\alpha - \beta/\bar{\lambda}\left(\widetilde{u}^{\star\star\star} + \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)\right]^+ = 0;$$

otherwise, $\widetilde{w}^{\star\star\star} < 0$ and $-\frac{\alpha_l}{2\beta} \le \widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star} < 0$, which implies that $\overline{w}^{\star\star\star} = \overline{w}^\circ + \widetilde{w}^{\star\star\star} < \overline{w}^\circ$ and $\bar{r}^{\star\star\star} = \bar{r}^\circ + \left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star}\right)/\bar{\theta} \in [0, \bar{r}^\circ)$. We now show that if

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) \le \Pi\left(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) = \bar{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\right], \tag{D.22}$$

then $(\overline{w}^{\star\star\star}, \overline{w}^{\star\star\star})$ can be sustained as a separating equilibrium by the retailer's posterior belief that the manufacturer is of type $\left(\bar{\theta}, \bar{\lambda}\right)$ upon contract $(\overline{w}^{\star\star\star}, \overline{w}^{\star\star\star})$ being offered and is otherwise of type $(\underline{\theta}, \underline{\lambda})$.

- The manufacturer $(\underline{\theta}, \underline{\lambda})$'s profit of deviating to $(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star})$ and hence being mistaken as of type $\left(\bar{\theta}, \bar{\lambda}\right)$ is, by definition, dominated by her equilibrium profit according to the constraints of (D.4): $\Pi\left(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star} \mid 1, (\underline{\theta}, \underline{\lambda})\right) \le \underline{\pi}^\circ$. Among all $(\underline{w}, \underline{r}) \ne (\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star})$, under which the manufacturer is believed to be of type $(\underline{\theta}, \underline{\lambda})$, the symmetric-information contract $(\underline{w}^\circ, \underline{r}^\circ)$ maximizes her profit: $\Pi\left(\underline{w}, \underline{r} \mid 0, (\underline{\theta}, \underline{\lambda})\right) < \Pi\left(\underline{w}^\circ, \underline{r}^\circ \mid 0, (\underline{\theta}, \underline{\lambda})\right)$. Therefore, the manufacturer $(\underline{\theta}, \underline{\lambda})$ indeed has no incentive to deviate from her symmetric-information contract terms $(\underline{w}^\circ, \underline{r}^\circ)$.
- For manufacturer $\left(\bar{\theta}, \bar{\lambda}\right)$, we need to show that she has no incentive to deviate to any $(\overline{w}, \bar{r}) \ne (\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star})$ and hence to be mistaken as of type $(\underline{\theta}, \underline{\lambda})$, namely the condition (D.22).

Since condition $\Delta\theta/\bar{\theta} > \Delta\lambda/\underline{\lambda}^c \ge \bar{\lambda}^c\left[\alpha_l^2 \wedge \bar{\lambda}\left(\Delta\alpha\right)^2\right]/\left[\underline{\lambda}^c\Delta\alpha(\alpha_l + \alpha_h)\right]$ implies $\delta < 0$ and (D.11), Lemma D.5 implies (D.12) and hence

$$\Pi\left(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) = \bar{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\right] = \bar{\pi}^\circ - \frac{\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{8\beta} = \underline{\pi}^\circ.$$

On the other hand, (D.3) implies that (after some simple algebra)

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) = \Pi\left(\overline{w}, \bar{r} \mid 0, (\underline{\theta}, \underline{\lambda})\right) + \frac{1}{2}\delta\bar{\theta}\bar{r}\left[\Delta\alpha - \beta/\underline{\lambda}(\overline{w} - \underline{\theta}\bar{r})\right]^+ \le \Pi\left(\overline{w}, \bar{r} \mid 0, (\underline{\theta}, \underline{\lambda})\right) \le \underline{\pi}^\circ,$$

where we used the fact that $\delta < 0$. Therefore, (D.22) holds.

The rest of the proof is to establish (D.22) when $\Delta\theta/\bar{\theta} \ge 1 - \left[\bar{\lambda}^c/\left(1 + \sqrt{\underline{\lambda}}\right)^2\right]$ but (D.11) does not hold, and hence concludes the verification of the equilibrium belief.

1. For $\overline{w} - \underline{\theta}\bar{r} \ge \underline{\lambda}\Delta\alpha/\beta$, (D.3) yields

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) = \frac{1}{2}\overline{w}\left(\underbrace{\alpha_l + \underline{\lambda}\Delta\alpha}_{\underline{\alpha}} - \beta\overline{w}\right) \le \frac{\underline{\alpha}^2}{8\beta} < \frac{\bar{\alpha}^2}{8\beta}.$$

On the other hand, Lemma D.7 implies that

$$\Pi\left(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) = \bar{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\right] > \frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2 + \bar{\alpha}^2}{8\beta} - \frac{\beta}{2\bar{\lambda}}\frac{\bar{\lambda}^c\bar{\lambda}^2\left(\Delta\alpha\right)^2}{4\beta^2} = \frac{\bar{\alpha}^2}{8\beta},$$

thus establishing (D.22).

2. For $0 \le \overline{w} - \underline{\theta}\bar{r} \le \underline{\lambda}\Delta\alpha/\beta$, (D.3) implies

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) = \frac{1}{2}\overline{w}\left(\alpha_l + \underline{\lambda}\Delta\alpha - \beta\overline{w}\right) + \frac{\beta}{2\underline{\lambda}}\left(\underline{\lambda}^c\overline{w} - \bar{\theta}\bar{\lambda}^c\bar{r}\right)\left(\underline{\theta}\bar{r} - \overline{w} + \frac{\underline{\lambda}\Delta\alpha}{\beta}\right), \tag{D.23}$$

in which the second term, as a quadratic function of $\bar{r} \in \left[1/\underline{\theta}\left(\overline{w} - \underline{\lambda}\Delta\alpha/\beta\right)^+, 1/\underline{\theta}\overline{w}\right]$, achieves its unconstrained maximum at

$$\bar{r} = 1/\underline{\theta}\left[\frac{\bar{\theta}\bar{\lambda}^c + \underline{\theta}\underline{\lambda}^c}{2\bar{\theta}\bar{\lambda}^c}\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right] = 1/\underline{\theta}\left[\left(1 + \frac{\delta}{2\bar{\lambda}^c}\right)\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right] < 1/\underline{\theta}\overline{w}.$$

Thus, we consider the following three cases.

(a) For $\overline{w} \ge -\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta} \ge \underline{\lambda}\Delta\alpha/\beta \ge \left(1 + \frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta}$ (by Lemma D.2), we have $0 \le 1/\underline{\theta}\left[\left(1 + \frac{\delta}{2\bar{\lambda}^c}\right)\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right] \le 1/\underline{\theta}\left(\overline{w} - \underline{\lambda}\Delta\alpha/\beta\right)$ and hence the second term of (D.23) is decreasing in $\bar{r} \in \left[1/\underline{\theta}\left(\overline{w} - \underline{\lambda}\Delta\alpha/\beta\right), 1/\underline{\theta}\overline{w}\right]$, implying

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) \le \Pi\left(\overline{w}, 1/\underline{\theta}\left(\overline{w} - \underline{\lambda}\Delta\alpha/\beta\right) \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) = \frac{1}{2}\overline{w}\left[\alpha_l + \underline{\lambda}\Delta\alpha - \beta\overline{w}\right].$$

We thus fall back to Case 1, establishing (D.22).

(b) For $\overline{w} \le \left(1 + \frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta} \le \underline{\lambda}\Delta\alpha/\beta$, we have $1/\underline{\theta}\left(\overline{w} - \underline{\lambda}\Delta\alpha/\beta\right) \le 1/\underline{\theta}\left[\left(1 + \frac{\delta}{2\bar{\lambda}^c}\right)\overline{w} - \frac{\underline{\lambda}\Delta\alpha}{2\beta}\right] \le 0$ and hence the second term of (D.23) is decreasing in $\bar{r} \in [0, 1/\underline{\theta}\overline{w}]$, implying

$$\Pi\left(\overline{w}, \bar{r} \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) \le \Pi\left(\overline{w}, 0 \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) = -\frac{\beta}{2\underline{\lambda}}\overline{w}^2 + \frac{\alpha_h}{2}\overline{w} \le \frac{\underline{\lambda}\alpha_h^2}{8\beta} = \Pi\left(\frac{\underline{\lambda}\alpha_h}{2\beta}, 0 \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right). \tag{D.24}$$

- If $\underline{\lambda}\alpha_l \le \left(\bar{\lambda} + \Delta\lambda\right)\Delta\alpha$, then $(\overline{w}, \bar{r}) = \left(\frac{\underline{\lambda}\alpha_h}{2\beta}, 0\right)$ is a feasible solution to (D.4), because $\widetilde{w} = \frac{\underline{\lambda}\alpha_h}{2\beta} - \frac{\bar{\alpha}}{2\beta} = -\frac{\underline{\lambda}^c\alpha_l + \Delta\lambda\Delta\alpha}{2\beta}$ and $\widetilde{u} = \frac{\underline{\lambda}\alpha_h}{2\beta} - \frac{\bar{\lambda}\Delta\alpha}{2\beta} = \widetilde{w} + \frac{\alpha_l}{2\beta} = \frac{\underline{\lambda}\alpha_l - \Delta\lambda\Delta\alpha}{2\beta} \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, \frac{\bar{\lambda}\Delta\alpha}{2\beta}\right]$ satisfy (D.7):

$$\begin{aligned}
&\bar{\lambda}\left(-\frac{\underline{\lambda}^c\alpha_l + \Delta\lambda\Delta\alpha}{2\beta}\right)^2 + \bar{\lambda}^c\left(\frac{\underline{\lambda}\alpha_l - \Delta\lambda\Delta\alpha}{2\beta}\right)^2 \\
&= \frac{1}{4\beta^2}\left\{\underbrace{\left[\bar{\lambda}(\underline{\lambda}^c)^2 + \bar{\lambda}^c\underline{\lambda}^2\right]}_{\ge\bar{\lambda}\bar{\lambda}^c}\alpha_l^2 + (\Delta\lambda\Delta\alpha)^2 + 2\left(\bar{\lambda}\underline{\lambda}^c - \bar{\lambda}^c\underline{\lambda}\right)\Delta\lambda\alpha_l\Delta\alpha\right\} \ge \frac{\bar{\lambda}\bar{\lambda}^c}{4\beta^2}\alpha_l^2 > \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2},
\end{aligned}$$

where the last inequality follows from the fact that (D.11) does not hold. Thus, the optimality of $(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star})$ implies

$$\Pi\left(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star} \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) \ge \Pi\left(\frac{\underline{\lambda}\alpha_h}{2\beta}, 0 \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right). \tag{D.25}$$

Therefore, (D.22) follows from (D.24) and (D.25) by noting that

$$\Pi\left(\frac{\underline{\lambda}\alpha_h}{2\beta}, 0 \,\middle|\, 1, (\bar{\theta}, \bar{\lambda})\right) - \Pi\left(\frac{\underline{\lambda}\alpha_h}{2\beta}, 0 \,\middle|\, 0, (\bar{\theta}, \bar{\lambda})\right) = \frac{\beta}{2}\left(\frac{\underline{\lambda}^c}{\underline{\lambda}} - \frac{\bar{\lambda}^c}{\bar{\lambda}}\right)\left(\frac{\underline{\lambda}\alpha_h}{2\beta}\right)^2 > 0.$$

- If $\underline{\lambda}\alpha_l \ge \left(\bar{\lambda} + \Delta\lambda\right)\Delta\alpha$, to establish (D.22) it suffices to show

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2 \le \frac{2\bar{\lambda}}{\beta}\left[\overline{\pi}^\circ - \frac{\underline{\lambda}\alpha_h^2}{8\beta}\right] = \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} + \frac{\bar{\lambda}\underline{\lambda}^c\alpha_l^2}{4\beta^2},$$

which holds by (D.21):

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2 < \frac{\bar{\lambda}^c\bar{\lambda}^2(\Delta\alpha)^2}{4\beta^2} \le \frac{\bar{\lambda}^c\bar{\lambda}^2}{4\beta^2}\left(\frac{\underline{\lambda}\alpha_l}{\bar{\lambda}}\right)^2 \le \frac{\bar{\lambda}\underline{\lambda}^c\alpha_l^2}{4\beta^2}.$$

(c) For $-\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta} \geq \overline{w} \geq \left(1+\frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta}$, we have $1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right] \geq 1/\underline{\theta}\left(\overline{w}-\underline{\lambda}\Delta\alpha/\beta\right)^+$ and hence

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right) \leq & \Pi\left(\overline{w},1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)\\ =&\frac{1}{2}\overline{w}\left(\alpha_l+\underline{\lambda}\Delta\alpha-\beta\overline{w}\right)+\frac{\beta}{2}\frac{\bar{\theta}\bar{\lambda}^c}{\underline{\theta}\underline{\lambda}}\left[\frac{\delta}{2\bar{\lambda}^c}\overline{w}+\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]^2\\ =&\frac{1}{2}\left\{\beta\left[\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta^2}{4\underline{\lambda}\bar{\lambda}^c}-1\right]\overline{w}^2+\left[\alpha_l+\left(\underline{\lambda}+\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta}{2}\right)\Delta\alpha\right]\overline{w}+\frac{\bar{\theta}\bar{\lambda}^c\underline{\lambda}\left(\Delta\alpha\right)^2}{4\beta\underline{\theta}}\right\}. \end{aligned}\tag{D.26}$$

By Lemma D.2, $\Delta\theta/\bar{\theta}\geq 1-\left[\bar{\lambda}^c/\left(1+\sqrt{\underline{\lambda}}\right)^2\right]$ implies that both $\delta<0$ and $\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta^2}{4\underline{\lambda}\bar{\lambda}^c}\geq 1$. Hence, the quadratic function of $\overline{w}$ in (D.35) is convex and hence reaches its maximum at either $\overline{w}=-\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta}$ or $\overline{w}=\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta}$, corresponding to the above two cases respectively, for which (D.22) has been established. □

D.2. Case of $\delta>0$ (i.e., $\underline{\theta}\underline{\lambda}^c>\bar{\theta}\bar{\lambda}^c$)

LEMMA D.8. *For $\delta>0$, the optimal solution to (D.9) that satisfies (D.8) must satisfy $\widetilde{w}^{\star\star\star}>0$ and $\widetilde{u}^{\star\star\star}<0$.*

Proof. First, we claim that $\widetilde{u}^{\star\star\star}<\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and $\widetilde{w}^{\star\star\star}>\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$. Otherwise, $\widetilde{u}^{\star\star\star}=\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ or $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}-\frac{\alpha_l}{2\beta}$, so (D.7) and (D.8) imply

$$\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}\leq\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\leq\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}-\delta\frac{\bar{\theta}}{\underline{\theta}}\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2},$$

leading to a contradiction. Thus, the necessary condition for the optimality of $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ is for there to exist a Lagrangian multiplier $\xi\geq 0$ associated with (D.7) such that

$$2\bar{\lambda}\widetilde{w}^{\star\star\star}-\xi\left[2\bar{\lambda}\widetilde{w}^{\star\star\star}+\delta\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star\star}\right)\right]=0,\tag{D.27}$$

$$2\bar{\lambda}^c\widetilde{u}^{\star\star\star}-\xi\left[2\bar{\lambda}^c\widetilde{u}^{\star\star\star}+\delta\left(2\widetilde{u}^{\star\star\star}-\widetilde{w}^{\star\star\star}-\frac{\alpha_l+\bar{\lambda}\Delta\alpha}{2\beta}\right)\right]\geq 0,\ \text{with “=” if }\widetilde{u}^{\star\star\star}>-\frac{\bar{\lambda}\Delta\alpha}{2\beta}.\tag{D.28}$$

We first note that $\xi>0$ and hence (D.7) must be binding. Otherwise, (D.27) and (D.28) suggest that $\widetilde{w}^{\star\star\star}=0$ and $\widetilde{u}^{\star\star\star}\geq 0$ with "=" if $\widetilde{u}^{\star\star\star}>-\frac{\bar{\lambda}\Delta\alpha}{2\beta}$. Thus, we must have $\widetilde{w}^{\star\star\star}=\widetilde{u}^{\star\star\star}=0$, which then violates (D.7).

Using the properties in Lemma D.2 and rearranging terms of (D.27) and (D.28) yields

$$2\bar{\lambda}(1-\xi)\widetilde{w}^{\star\star\star}=\xi\delta\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}^{\star\star\star}\right)>0,\tag{D.29}$$

$$2(\bar{\theta}\bar{\lambda}^c-\xi\underline{\theta}\underline{\lambda}^c)/\bar{\theta}\widetilde{u}^{\star\star\star}\geq-\xi\delta\left(\widetilde{w}^{\star\star\star}+\frac{\alpha_l+\bar{\lambda}\Delta\alpha}{2\beta}\right)\in(-\infty,0),\ \text{with “=” if }\widetilde{u}^{\star\star\star}>-\frac{\bar{\lambda}\Delta\alpha}{2\beta},\tag{D.30}$$

where the strict inequality follows from the fact that $\widetilde{w}^{\star\star\star}+\frac{\alpha_l}{2\beta}>\widetilde{u}^{\star\star\star}\in\left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta},\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)$.

- If $\widetilde{u}^{\star\star\star}=-\frac{\bar{\lambda}\Delta\alpha}{2\beta}<0$, then the binding (D.7) implies

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\delta\frac{\bar{\lambda}\Delta\alpha}{\beta}\widetilde{w}^{\star\star\star}=A:=\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}-\bar{\lambda}^c\frac{\left(\bar{\lambda}\Delta\alpha\right)^2}{4\beta^2}-\delta\frac{2\bar{\lambda}\bar{\alpha}\Delta\alpha}{4\beta^2}.\tag{D.31}$$

We claim that $A\geq 0$. Otherwise, it is straightforward to verify that the following quadratic equation

$$\bar{\lambda}^c\widetilde{u}^2+\delta\left(\frac{\alpha_l}{2\beta}-\widetilde{u}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta}-\widetilde{u}\right)=\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}$$

has a unique root $\widetilde{u}^{\natural} \in \left(-\frac{\bar\lambda\Delta\alpha}{2\beta}, 0\right)$, and hence $(0, \widetilde{u}^{\natural})$ binds (D.7). However, this immediately contradicts the optimality of $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ because

$$\bar\lambda\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar\lambda^c\left(\widetilde{u}^{\star\star\star}\right)^2 \ge \bar\lambda^c\left(\frac{\bar\lambda\Delta\alpha}{2\beta}\right)^2 > \bar\lambda(0)^2 + \bar\lambda^c\left(\widetilde{u}^{\natural}\right)^2.$$

Therefore, (D.31) immediately implies that $\widetilde{w}^{\star\star\star} = \sqrt{\left(\frac{\delta\Delta\alpha}{2\beta}\right)^2 + A/\bar\lambda} - \frac{\delta\Delta\alpha}{2\beta} \ge 0$. On the other hand, (D.29) implies that $\widetilde{w}^{\star\star\star} \ne 0$. Hence, we must have $\widetilde{w}^{\star\star\star} > 0$, establishing the lemma.

- If $\widetilde{u}^{\star\star\star} > -\frac{\bar\lambda\Delta\alpha}{2\beta}$, then (D.28) and (D.30) are binding. Together with the fact that $\delta > 0$ or equivalently $\bar\theta\bar\lambda^c < \underline{\theta\lambda}^c$, we consider the following three possibilities:

1. If $\xi > 1 > \bar\theta\bar\lambda^c/(\underline{\theta\lambda}^c)$, then (D.29) and binding (D.30) suggest that $\widetilde{w}^{\star\star\star} < 0$ and $\widetilde{u}^{\star\star\star} > 0$, respectively. However, as (D.7) is binding, this would imply that

$$\begin{aligned}\bar\lambda\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar\lambda^c\left(\widetilde{u}^{\star\star\star}\right)^2 =& \frac{\bar\lambda\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar\lambda\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right)\\ >& \frac{\bar\lambda\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\frac{\bar\lambda\alpha_l\Delta\alpha}{4\beta^2} > \frac{\bar\lambda\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\frac{\bar\theta}{\underline\theta}\frac{\bar\lambda\alpha_l\Delta\alpha}{4\beta^2},\end{aligned}$$

violating (D.8). Hence, this case can be ruled out.

2. If $1 > \xi > \bar\theta\bar\lambda^c/(\underline{\theta\lambda}^c)$, (D.29) and binding (D.30) suggest that $\widetilde{w}^{\star\star\star} > 0$ and $\widetilde{u}^{\star\star\star} > 0$, respectively. However, this leads to contradiction, because eliminating ξ and δ from (D.27) and binding (D.28) would yield a contradiction

$$\bar\lambda^c\underbrace{\widetilde{u}^{\star\star\star}}_{>0}\underbrace{\left(\frac{\bar\lambda\Delta\alpha}{2\beta} - \widetilde{u}^{\star\star\star}\right)}_{>0} = \bar\lambda\underbrace{\widetilde{w}^{\star\star\star}}_{>0}\underbrace{\left(2\widetilde{u}^{\star\star\star} - \widetilde{w}^{\star\star\star} - \frac{\alpha_l + \bar\lambda\Delta\alpha}{2\beta}\right)}_{<0}.$$

Hence, this case again can be ruled out.

3. As such, we must have $1 > \bar\theta\bar\lambda^c/(\underline{\theta\lambda}^c) > \xi$, which implies that $\widetilde{w}^{\star\star\star} > 0$ and $\widetilde{u}^{\star\star\star} < 0$ according to (D.19) and binding (D.20), respectively, establishing the lemma. ☐

Lemma D.9. *For $\delta > 0$, the optimal solution to (D.9) must satisfy (D.8) if $\Delta\theta/\underline\theta \le 2\Delta\alpha/\alpha_l$.*

Proof. Since the objective of (D.9) is to minimize the left-hand side of (D.8), we thus just need to identify a feasible solution to (D.9) that satisfies (D.8).

- For $\Delta\theta/\underline\theta \le \Delta\alpha/\alpha_l$, it is straightforward to verify that $\widetilde{u} = -\bar\lambda\frac{\Delta\theta}{\underline\theta}\frac{\alpha_l}{2\beta}$ and

$$\begin{aligned}\widetilde{w} =& \frac{1}{2\beta}\sqrt{\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] - \delta\frac{\bar\theta}{\underline\theta}\alpha_l\Delta\alpha - \bar\lambda\bar\lambda^c\left(\frac{\Delta\theta}{\underline\theta}\alpha_l\right)^2}\\ \text{(by Lemma D.2)} \quad =& \frac{1}{2\beta}\sqrt{\left(\bar\lambda^c\frac{\Delta\theta}{\underline\theta}\alpha_l\right)^2 + \Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right] + \bar\lambda^c\frac{\Delta\theta}{\underline\theta}\alpha_l^2\left(\frac{\Delta\alpha}{\alpha_l} - \frac{\Delta\theta}{\underline\theta}\right)}\end{aligned}$$

satisfy $\widetilde{u} \in \left[-\frac{\bar\lambda\Delta\alpha}{2\beta}, 0\right]$, $\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta} \ge \bar\lambda^c\frac{\Delta\theta}{\underline\theta}\frac{\alpha_l}{2\beta} + \bar\lambda\frac{\Delta\theta}{\underline\theta}\frac{\alpha_l}{2\beta} + \frac{\alpha_l}{2\beta} = \frac{\bar\theta}{\underline\theta}\frac{\alpha_l}{2\beta}$, and bind (D.8). Subsequently, $(\widetilde{w}, \widetilde{u})$ also satisfies (D.7):

$$\begin{aligned}&\bar\lambda\widetilde{w}^2 + \bar\lambda^c\widetilde{u}^2 + \delta\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar\lambda\Delta\alpha}{2\beta} - \widetilde{u}\right)\\ =& \frac{\bar\lambda\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\frac{\bar\theta}{\underline\theta}\frac{\bar\lambda\alpha_l\Delta\alpha}{4\beta^2} + \delta\underbrace{\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)}_{\ge\frac{\bar\theta}{\underline\theta}\frac{\alpha_l}{2\beta}}\underbrace{\left(\frac{\bar\lambda\Delta\alpha}{2\beta} - \widetilde{u}\right)}_{\ge\frac{\bar\lambda\Delta\alpha}{2\beta}} \ge \quad \frac{\bar\lambda\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}.\end{aligned}$$

- For $\Delta\alpha/\alpha_l \le \Delta\theta/\underline{\theta} \le 2\Delta\alpha/\alpha_l$, it is straightforward to verify that $\widetilde{u} = -\frac{\bar{\lambda}\Delta\alpha}{2\beta}$ and

$$
\begin{aligned}
\widetilde{w} &= \frac{1}{2\beta}\sqrt{\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right] - \delta\frac{\bar{\theta}}{\underline{\theta}}\alpha_l\Delta\alpha - \bar{\lambda}\bar{\lambda}^c(\Delta\alpha)^2} \\
\text{(by Lemma D.2)} \quad &= \frac{1}{2\beta}\sqrt{\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right] + \bar{\lambda}^c\frac{\Delta\theta}{\underline{\theta}}\alpha_l\Delta\alpha - \bar{\lambda}\bar{\lambda}^c(\Delta\alpha)^2} \\
&\ge \frac{1}{2\beta}\sqrt{\Delta\lambda\left[(\Delta\alpha)^2 + \alpha_l\Delta\alpha\right] + \left(\bar{\lambda}^c\Delta\alpha\right)^2} \ge \frac{\bar{\lambda}^c\Delta\alpha}{2\beta} > 0 > \widetilde{u} - \frac{\alpha_l}{2\beta},
\end{aligned}
$$

bind (D.8). Subsequently, $(\widetilde{w}, \widetilde{u})$ also satisfies (D.7):

$$
\begin{aligned}
&\bar{\lambda}\widetilde{w}^2 + \bar{\lambda}^c\widetilde{u}^2 + \delta\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right) \\
=& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} - \delta\underbrace{\frac{\bar{\theta}}{\underline{\theta}}}_{\le 1 + \frac{2\Delta\alpha}{\alpha_l}}\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2} + \delta\underbrace{\left(\widetilde{w} - \widetilde{u} + \frac{\alpha_l}{2\beta}\right)}_{\ge \frac{\Delta\alpha + \alpha_l}{2\beta}}\underbrace{\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right)}_{= \frac{2\bar{\lambda}\Delta\alpha}{2\beta}} \\
\ge& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} + \delta\left[\frac{2\bar{\lambda}\Delta\alpha\left[\Delta\alpha + \alpha_l\right]}{4\beta^2} - \left(1 + \frac{2\Delta\alpha}{\alpha_l}\right)\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2}\right] \\
=& \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} + \delta\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2} > \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}. \quad \square
\end{aligned}
$$

LEMMA D.10. *Suppose $\delta > 0$. There exists $\widetilde{u}^\diamond \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, 0\right]$ such that $(0, \widetilde{u}^\diamond)$ binds (D.7) if and only if*

$$
\Delta\theta/\bar{\theta} \le \left(\underline{\lambda}^c\bar{\lambda} - \bar{\lambda}^c\Delta\lambda\right)\Delta\alpha/\left(2\underline{\lambda}^c\bar{\alpha}\right). \tag{D.32}
$$

Furthermore, (D.32) implies that $\Delta\theta/\underline{\theta} \le 2\Delta\alpha/\alpha_l$ and hence that the optimal solution to (D.9) must satisfy (D.8).

Proof of Lemma D.10. Substituting $\widetilde{w} = 0$ into the left-hand side of (D.9) results in a quadratic convex function in $\widetilde{u}$,

$$
\bar{\lambda}^c\widetilde{u}^2 + \delta\left(\frac{\alpha_l}{2\beta} - \widetilde{u}\right)\left(\frac{\bar{\lambda}\Delta\alpha}{2\beta} - \widetilde{u}\right),
$$

which takes value of $\delta\frac{\bar{\lambda}\alpha_l\Delta\alpha}{4\beta^2} < \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2}$ (the right-hand side of (D.9)) at $\widetilde{u} = 0$ and value of $\bar{\lambda}^c\frac{\bar{\lambda}^2(\Delta\alpha)^2}{4\beta^2} + \delta\frac{2\bar{\lambda}\Delta\alpha\left(\alpha_l + \bar{\lambda}\Delta\alpha\right)}{4\beta^2}$ at $\widetilde{u} = -\frac{\bar{\lambda}\Delta\alpha}{2\beta}$. Therefore, there exists $\widetilde{u}^\diamond \in \left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta}, 0\right]$ such that $(0, \widetilde{u}^\diamond)$ binds (D.9) if and only if

$$
\bar{\lambda}^c\frac{\bar{\lambda}^2(\Delta\alpha)^2}{4\beta^2} + \delta\frac{2\bar{\lambda}\Delta\alpha\left(\alpha_l + \bar{\lambda}\Delta\alpha\right)}{4\beta^2} \ge \frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2 + 2\alpha_l\Delta\alpha\right]}{4\beta^2} \text{ (the right-hand side of (D.9))},
$$

which reduces to (D.32) through straightforward verification. Finally, by direct verification, (D.32) implies that $\Delta\theta/\bar{\theta} \le 2\Delta\alpha/(2\Delta\alpha + \alpha_l)$, which is equivalent to $\Delta\theta/\underline{\theta} \le 2\Delta\alpha/\alpha_l$, and hence the last statement in the lemma follows from Lemma D.9. $\square$

Proof of Proposition 5.2 and Corollary 1.2. For $\delta > 0$, Lemma D.8 implies that the most efficient separating equilibrium, if exists, must be given by $\overline{w}^{\star\star\star} = \overline{w}^\circ + \widetilde{w}^{\star\star\star} > \overline{w}^\circ$ and $\bar{r}^{\star\star\star} = \bar{r}^\circ + \left(\widetilde{w}^{\star\star\star} - \widetilde{u}^{\star\star\star}\right)/\bar{\theta} > \bar{r}^\circ$, where $(\widetilde{w}^{\star\star\star}, \widetilde{u}^{\star\star\star})$ is the optimal solution to (D.9) that satisfies (D.8). We now show that if

$$
\Pi\left(\overline{w}, \bar{r} \mid 0, (\bar{\theta}, \bar{\lambda})\right) \le \Pi\left(\overline{w}^{\star\star\star}, \bar{r}^{\star\star\star} \mid 1, (\bar{\theta}, \bar{\lambda})\right) = \bar{\pi}^\circ - \frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2 + \bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\right], \tag{D.33}
$$

then $(\overline{w}^{\star\star\star}, \overline{w}^{\star\star\star})$ can be sustained as a separating equilibrium by the retailer's posterior belief that the manufacturer is of type $(\bar{\theta}, \bar{\lambda})$ upon contract $(\overline{w}^{\star\star\star}, \overline{w}^{\star\star\star})$ being offered and is otherwise of type $(\underline{\theta}, \underline{\lambda})$.

- The manufacturer $(\underline{\theta},\underline{\lambda})$'s profit of deviating to $(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star})$ and hence being mistaken as of type $(\bar{\theta},\bar{\lambda})$ is, by definition, dominated by her equilibrium profit according to the constraints of (D.4): $\Pi\left(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star}\mid 1,(\underline{\theta},\underline{\lambda})\right)\le\underline{\pi}^{\circ}$. Among all $(\underline{w},\underline{r})\neq(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star})$, under which the manufacturer is believed to be of type $(\underline{\theta},\underline{\lambda})$, the symmetric-information contract $(\underline{w}^{\circ},\underline{r}^{\circ})$ maximizes her profit: $\Pi\left(\underline{w},\underline{r}\mid 0,(\underline{\theta},\underline{\lambda})\right)<\Pi\left(\underline{w}^{\circ},\underline{r}^{\circ}\mid 0,(\underline{\theta},\underline{\lambda})\right)$. Therefore, the manufacturer $(\underline{\theta},\underline{\lambda})$ indeed has no incentive to deviate from her symmetric-information contract terms $(\underline{w}^{\circ},\underline{r}^{\circ})$.
- For manufacturer $(\bar{\theta},\bar{\lambda})$, we need to show that she has no incentive to deviate to any $(\overline{w},\bar{r})\neq(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star})$ and hence to be mistaken as of type $(\underline{\theta},\underline{\lambda})$, namely the condition (D.33). The rest of the proof is to establish (D.33) under condition $\Delta\theta/\bar{\theta}\le\min\left\{\left(\underline{\lambda}^{c}\bar{\lambda}-\bar{\lambda}^{c}\Delta\lambda\right)\Delta\alpha/\left(2\underline{\lambda}^{c}\bar{\alpha}\right),1-\left[\bar{\lambda}^{c}/\left(1-\sqrt{\underline{\lambda}}\right)^{2}\right]\right\}$ and hence concludes the verification of the equilibrium belief.

1. For $\overline{w}-\underline{\theta}\overline{r}\ge\underline{\lambda}\Delta\alpha/\beta$, (D.3) yields

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)=\frac{1}{2}\overline{w}\left(\underbrace{\alpha_l+\underline{\lambda}\Delta\alpha}_{\underline{\alpha}}-\beta\overline{w}\right)\le\frac{\underline{\alpha}^2}{8\beta}<\underline{\pi}^{\circ}.$$

Thus, to show (D.33), it suffices to show that

$$\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\le\frac{2\bar{\lambda}}{\beta}\left(\overline{\pi}^{\circ}-\underline{\pi}^{\circ}\right)=\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2},$$

which indeed holds as $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ must satisfy (D.8) (with $\delta>0$).

2. For $0\le\overline{w}-\underline{\theta}\overline{r}\le\underline{\lambda}\Delta\alpha/\beta$, (D.3) implies

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)=\frac{1}{2}\overline{w}\left(\alpha_l+\underline{\lambda}\Delta\alpha-\beta\overline{w}\right)+\frac{\beta}{2\underline{\lambda}}\left(\underline{\lambda}^c\overline{w}-\bar{\theta}\bar{\lambda}^c\overline{r}\right)\left(\underline{\theta}\overline{r}-\overline{w}+\frac{\underline{\lambda}\Delta\alpha}{\beta}\right),\tag{D.34}$$

in which the second term, as a quadratic function of $\bar{r}\in\left[1/\underline{\theta}\left(\overline{w}-\underline{\lambda}\Delta\alpha/\beta\right)^{+},1/\underline{\theta}\overline{w}\right]$, achieves its unconstrained maximum at

$$\bar{r}=1/\underline{\theta}\left[\frac{\bar{\theta}\bar{\lambda}^c+\underline{\theta}\underline{\lambda}^c}{2\bar{\theta}\bar{\lambda}^c}\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]=1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]>1/\underline{\theta}\left(\overline{w}-\underline{\lambda}\Delta\alpha/\beta\right).$$

Thus, we consider the following three cases.

(a) For $\overline{w}\ge\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta}$, we have $1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]\ge1/\underline{\theta}\overline{w}$ and hence the second term of (D.34) is increasing in $\bar{r}\in\left[1/\underline{\theta}\left(\overline{w}-\underline{\lambda}\Delta\alpha/\beta\right)^{+},1/\underline{\theta}\overline{w}\right]$, implying

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)\le\Pi\left(\overline{w},1/\underline{\theta}\overline{w}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)=\frac{1}{2}\overline{w}\left[\alpha_l+\left(\underline{\lambda}+\frac{\bar{\theta}}{\underline{\theta}}\delta\right)\Delta\alpha-\beta\overline{w}\right]\le\frac{1}{8\beta}\left[\alpha_l+\left(\underline{\lambda}+\frac{\bar{\theta}}{\underline{\theta}}\delta\right)\Delta\alpha\right]^2\le\frac{\bar{\alpha}^2}{8\beta}.$$

On the other hand, condition $\Delta\theta/\bar{\theta}\le\min\left\{\left(\underline{\lambda}^{c}\bar{\lambda}-\bar{\lambda}^{c}\Delta\lambda\right)\Delta\alpha/\left(2\underline{\lambda}^{c}\bar{\alpha}\right),1-\left[\bar{\lambda}^{c}/\left(1-\sqrt{\underline{\lambda}}\right)^{2}\right]\right\}$ implies $\delta>0$ (by Lemma D.2) and condition (D.32). Thus, Lemma D.10 implies that, by (D.6),

$$\Pi\left(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star}\,\middle|\,1,(\bar{\theta},\bar{\lambda})\right)=\overline{\pi}^{\circ}-\frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}(\widetilde{w}^{\star\star\star})^2+\bar{\lambda}^c(\widetilde{u}^{\star\star\star})^2\right]\ge\overline{\pi}^{\circ}-\frac{\beta}{2\bar{\lambda}}\bar{\lambda}^c(\widetilde{u}^{\circ})^2\ge\overline{\pi}^{\circ}-\frac{\bar{\lambda}^c\bar{\lambda}(\Delta\alpha)^2}{8\beta}=\frac{\bar{\alpha}^2}{8\beta},$$

immediately implying that (D.33) holds.

(b) For $0\le\overline{w}\le\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta}<\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta}$, we have $1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]\le0$ and hence the second term of (D.34) is decreasing in $\bar{r}\in[0,1/\underline{\theta}\overline{w}]$, implying

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)\le\Pi\left(\overline{w},0\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)==-\frac{\beta}{2\underline{\lambda}}\overline{w}^2+\frac{\alpha_h}{2}\overline{w}\le\frac{\underline{\lambda}\alpha_h^2}{8\beta}\le\frac{\bar{\alpha}^2}{8\beta}\le\Pi\left(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star}\,\middle|\,1,(\bar{\theta},\bar{\lambda})\right),$$

where the last inequality follows from the same argument in part (a). Thus, (D.33) again holds.

(c) For $\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta} \le \overline{w}\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta}$, we have $1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right] \le 1/\underline{\theta}\overline{w}$ and hence

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\mid 0,(\bar{\theta},\bar{\lambda})\right) \le &\Pi\left(\overline{w},1/\underline{\theta}\left[\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)\overline{w}-\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]\,\Big|\, 0,(\bar{\theta},\bar{\lambda})\right)\\ =&\frac{1}{2}\overline{w}\left(\alpha_l+\underline{\lambda}\Delta\alpha-\beta\overline{w}\right)+\frac{\beta}{2}\frac{\bar{\theta}\bar{\lambda}^c}{\underline{\theta}\underline{\lambda}}\left[\frac{\delta}{2\bar{\lambda}^c}\overline{w}+\frac{\underline{\lambda}\Delta\alpha}{2\beta}\right]^2\\ =&\frac{1}{2}\left\{\beta\left[\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta^2}{4\underline{\lambda}\bar{\lambda}^c}-1\right]\overline{w}^2+\left[\alpha_l+\left(\underline{\lambda}+\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta}{2}\right)\Delta\alpha\right]\overline{w}+\frac{\bar{\theta}\bar{\lambda}^c\underline{\lambda}\left(\Delta\alpha\right)^2}{4\beta\underline{\theta}}\right\}.\end{aligned} \tag{D.35}$$

Again by Lemma D.2, condition $\Delta\theta/\bar{\theta} \le \min\left\{\left(\underline{\lambda}^c\bar{\lambda}-\bar{\lambda}^c\Delta\lambda\right)\Delta\alpha/\left(2\underline{\lambda}^c\bar{\alpha}\right), 1-\left[\bar{\lambda}^c/\left(1-\sqrt{\underline{\lambda}}\right)^2\right]\right\}$ implies that $\frac{\bar{\theta}}{\underline{\theta}}\frac{\delta^2}{4\underline{\lambda}\bar{\lambda}^c} \ge 1$. Hence, the quadratic function of $\overline{w}$ in (D.35) is convex and hence reaches its maximum at either $\overline{w}=\frac{\bar{\lambda}^c\underline{\lambda}\Delta\alpha}{\beta\delta}$ or $\overline{w}=\left(1+\frac{\delta}{2\bar{\lambda}^c}\right)^{-1}\frac{\underline{\lambda}\Delta\alpha}{2\beta}$, corresponding to the above two cases respectively, for which (D.33) has been established. □

D.3. Case of $\delta=0$ (i.e., $\underline{\theta}\underline{\lambda}^c=\bar{\theta}\bar{\lambda}^c$)

Lemma D.11. *There exist at least two solutions to*

$$\bar{\lambda}\widetilde{w}^2+\bar{\lambda}^c\widetilde{u}^2=\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}, \tag{D.36}$$

both satisfying (D.5), *one of which satisfies* $\widetilde{w}<0$ *and* $\widetilde{w}-\widetilde{u}<0$ *and the other of which satisfies* $\widetilde{w}>0$ *and* $\widetilde{w}-\widetilde{u}>0$.

Proof. It is straightforward to see that function $g(\widetilde{u}) := \left[\bar{\lambda}\left(1+\frac{\alpha_l}{\bar{\lambda}\Delta\alpha}\right)^2+\bar{\lambda}^c\right]\widetilde{u}^2$ is monotonically decreasing in $\widetilde{u}\in\left[-\frac{\bar{\lambda}\Delta\alpha}{2\beta},0\right]$ with

$$g\left(-\frac{\bar{\lambda}\Delta\alpha}{2\beta}\right)=\frac{\bar{\lambda}^2\left(\Delta\alpha\right)^2+2\bar{\lambda}^2\alpha_l\Delta\alpha+\bar{\lambda}\alpha_l^2}{4\beta^2}>\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2},\quad\text{and}\quad g(0)=0<\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}.$$

Thus, the Intermediate Value Theorem implies that there is a unique $\widetilde{u}\in\left(-\frac{\bar{\lambda}\Delta\alpha}{2\beta},0\right)$ such that $g(\widetilde{u})=\frac{\bar{\lambda}\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{4\beta^2}$. Let $\widetilde{w}=\left(1+\frac{\alpha_l}{\bar{\lambda}\Delta\alpha}\right)\widetilde{u}<\widetilde{u}$. Then, $(\widetilde{w},\widetilde{u})$ specified as such is a solution to (D.36) with $\widetilde{w}<0$ and $\widetilde{w}-\widetilde{u}<0$.

To show the existence of the other solution to (D.36), we simply let $\widetilde{w}=\frac{1}{2\beta}\sqrt{\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}$ and $\widetilde{u}=0$, which automatically satisfy $\widetilde{w}>0$ and $\widetilde{w}-\widetilde{u}>0$. □

Proof of Proposition 5.3 and Corollary 1.3. When $\delta=0$, it is straightforward to see that the optimal solution $(\widetilde{w}^{\star\star\star},\widetilde{u}^{\star\star\star})$ to (D.9) is the solution to (D.36) that satisfies (D.5). By Lemma D.11, at least two of such solutions exist, one with $\widetilde{w}^{\star\star\star}<0$ and $\widetilde{w}^{\star\star\star}-\widetilde{u}^{\star\star\star}<0$ and the other one with $\widetilde{w}^{\star\star\star}>0$ and $\widetilde{w}^{\star\star\star}-\widetilde{u}^{\star\star\star}>0$. Correspondingly, there exist two solutions to (D.4), $\overline{w}^{\star\star\star}=\overline{w}^{\circ}+\widetilde{w}^{\star\star\star}$ and $\overline{r}^{\star\star\star}=\overline{r}^{\circ}+\left(\widetilde{w}^{\star\star\star}-\widetilde{u}^{\star\star\star}\right)/\bar{\theta}$, which satisfies the property described in Proposition 5.1. We now show that $(\overline{w}^{\star\star\star},\overline{w}^{\star\star\star})$ can be sustained as a separating equilibrium by the retailer's posterior belief that the manufacturer is of type $\left(\bar{\theta},\bar{\lambda}\right)$ upon contract $(\overline{w}^{\star\star\star},\overline{w}^{\star\star\star})$ being offered and is otherwise of type $(\underline{\theta},\underline{\lambda})$.

- The manufacturer $(\underline{\theta},\underline{\lambda})$'s profit of deviating to $(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star})$ and hence being mistaken as of type $\left(\bar{\theta},\bar{\lambda}\right)$ is, by definition, dominated by her equilibrium profit according to the constraints of (D.4): $\Pi\left(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star}\mid 1,(\underline{\theta},\underline{\lambda})\right)\le\underline{\pi}^{\circ}$. Among all $(\underline{w},\underline{r})\ne(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star})$, under which the manufacturer is believed to be of type $(\underline{\theta},\underline{\lambda})$, the symmetric-information contract $(\underline{w}^{\circ},\underline{r}^{\circ})$ maximizes her profit: $\Pi\left(\underline{w},\underline{r}\mid 0,(\underline{\theta},\underline{\lambda})\right)<\Pi\left(\underline{w}^{\circ},\underline{r}^{\circ}\mid 0,(\underline{\theta},\underline{\lambda})\right)$. Therefore, the manufacturer $(\underline{\theta},\underline{\lambda})$ indeed has no incentive to deviate from her symmetric-information contract terms $(\underline{w}^{\circ},\underline{r}^{\circ})$.

- For manufacturer $\left(\bar{\theta},\bar{\lambda}\right)$, we need to show that she has no incentive to deviate to any $(\overline{w},\overline{r}) \neq (\overline{w}^{\star\star\star},\overline{r}^{\star\star\star})$ and hence to be mistaken as of type $(\underline{\theta},\underline{\lambda})$, namely

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)\le\Pi\left(\overline{w}^{\star\star\star},\overline{r}^{\star\star\star}\,\middle|\,1,(\bar{\theta},\bar{\lambda})\right)=&\overline{\pi}^{\circ}-\frac{\beta}{2\bar{\lambda}}\left[\bar{\lambda}\left(\widetilde{w}^{\star\star\star}\right)^2+\bar{\lambda}^c\left(\widetilde{u}^{\star\star\star}\right)^2\right]\\=&\overline{\pi}^{\circ}-\frac{\Delta\lambda\left[(\Delta\alpha)^2+2\alpha_l\Delta\alpha\right]}{8\beta}=\underline{\pi}^{\circ},\end{aligned}$$

which holds, because $\delta=\left(\underline{\theta}\underline{\lambda}^c-\bar{\theta}\bar{\lambda}^c\right)/\bar{\theta}=0$ and (D.3) imply that

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\,\middle|\,0,(\bar{\theta},\bar{\lambda})\right)=&\frac{1}{2}w\left(\alpha_l+\underline{\lambda}\Delta\alpha-\beta w\right)+\frac{1}{2}\left(\underline{\lambda}^c w-\bar{\lambda}^c\bar{\theta}r\right)\left[\Delta\alpha-\beta/\underline{\lambda}(w-\underline{\theta}r)\right]^+\\=&\frac{1}{2}w\left(\alpha_l+\underline{\lambda}\Delta\alpha-\beta w\right)+\frac{1}{2}\left(\underline{\lambda}^c w-\underline{\lambda}^c\underline{\theta}r\right)\left[\Delta\alpha-\beta/\underline{\lambda}(w-\underline{\theta}r)\right]^+\\=&\Pi\left(\overline{w},\overline{r}\mid(\underline{\theta},\underline{\lambda}),0\right)\le\underline{\pi}^{\circ}.\quad\square\end{aligned}$$

Appendix E: Positive Marginal Production Cost

In this appendix, we explore the situation with a positive manufacturer's marginal production cost, denoted as $c \geq 0$. We find that a positive marginal cost does not qualitatively impact our insights established in the paper. That is, under asymmetric information about either returns risk or demand potential, signaling requires the separating type (i.e., the less risky or high-demand manufacturer) to suitably distort her returns cost away from the symmetric-information level (via the induced retailer's regular and safety stocks). A positive marginal production only acts to reduce the induced symmetric-information regular and safety stocks, but it does not affect the direction of distortions *relative to the symmetric-information benchmark* (as it does not enter the retailer's quantity decision). As the only nuance, under asymmetric information about returns risk, if the marginal cost is sufficiently high (i.e., $\lambda\Delta\alpha \le \beta c < \alpha$), then it is no longer profitable for either less risky or riskier manufacturer to induce returns, i.e., to induce the retailer to carry a safety stock and return unsold inventory. As a result, the manufacturer's returns risk is not relevant to the retailer and there is no need for the manufacturer to signal her returns risk.

E.1. Returns Risk.

Given a marginal production cost $c \geq 0$, the manufacturer's expected profit function (4.1) in the paper needs to be modified as

$$\begin{aligned}\Pi\left(w,r\,\middle|\,\widehat{\theta},\theta\right):=&(w-c)s^R\left(w,r,\widehat{\theta},\lambda\right)-\frac{1}{2}\lambda^c\theta r\left[\Delta\alpha-\beta/\lambda\left(w-\widehat{\theta}r\right)\right]^+\\=&\frac{1}{2}(w-c)(\alpha-\beta w)+\frac{\lambda^c}{2}(w-\theta r-c)\left[\Delta\alpha-\beta/\lambda\left(w-\widehat{\theta}r\right)\right]^+.\end{aligned}$$

It is straightforward to verify that the symmetric-information contract is given by

$$w^{\circ}(\theta)\equiv\frac{\alpha+\beta c}{2\beta}\quad\text{and}\quad r^{\circ}(\theta)\begin{cases}\in\left[0,\frac{\alpha_l+\beta c-\lambda\Delta\alpha}{2\beta\theta}\right], & \text{if}\quad \lambda\Delta\alpha\le\beta c\le\alpha,\\ =\frac{\alpha_l}{2\beta\theta}, & \text{if}\quad \beta c\le\lambda\Delta\alpha,\end{cases}$$

where we recall $\alpha=\alpha_l+\lambda\Delta\alpha$ is the average baseline demand. Under the symmetric-information contract, the retailer's induced regular and safety stock are

$$s_r^{\circ}=\frac{(\alpha-\beta c)^+}{4}\quad\text{and}\quad s_s^{\circ}=\frac{\lambda^c}{4}\left(\Delta\alpha-\beta c/\lambda\right)^+,\quad\text{respectively;}$$

and the manufacturer's expected profit is given by

$$\pi^\circ = \frac{\left[(\alpha-\beta c)^+\right]^2}{8\beta} + \frac{\lambda^c\left[(\lambda\Delta\alpha-\beta c)^+\right]^2}{8\beta\lambda} = \frac{2}{\beta\lambda^c}\left\{\lambda^c\left(s_r^\circ\right)^2 + \lambda\left(s_s^\circ\right)^2\right\}.$$

In words, a positive marginal production cost acts to shift the symmetric-information wholesale price upward and the induced regular and safety stocks downward, relative to the case of zero marginal production cost in the paper (see Lemma 2). More specifically, regardless of the manufacturer's type θ,

- for $0 \le \beta c < \lambda\Delta\alpha < \alpha$, the retailer still orders both positive regular and safety stocks (and incurs unsold inventory of amount s_s°/λ^c in case of low baseline demand realization) and the manufacturer earns positive profit;
- for $\lambda\Delta\alpha \le \beta c < \alpha$, the retailer only orders positive regular stock but *no* safety stock (and hence no returns regardless of baseline demand realization) and the manufacturer earns positive profit (from selling regular stock);
- for $\beta c \ge \alpha$, the retailer orders no regular nor safety stocks (and hence no returns regardless of baseline demand realization) and the manufacturer earns *no* profit (i.e., exits the market). Thus, it is meaningful to only focus on the parameter range $\beta c \in [0, \alpha]$.

When returns risk θ becomes the manufacturer's private information, the riskier manufacturer still offers her symmetric-information contract and earns her symmetric-information profit π° and the less risky manufacturer distinguishes herself by offering a contract, say $(\overline{w}, \overline{r})$, which may need to be distorted away from her symmetric-information counterpart. Using the same variable transformation as in the paper, we can work with the retailer's induced quantity decision:

$$\text{regular stock} \quad s_r(\overline{w}) := \frac{1}{2}\left(\alpha - \beta\overline{w}\right), \quad \text{and}$$
$$\text{safety stock} \quad s_s(\overline{w}, \overline{r}) := \frac{\lambda^c}{2}\left[\Delta\alpha - \beta/\lambda\left(\overline{w} - \bar{\theta}\overline{r}\right)\right].$$

Notably, the retailer's quantity decision above is independent of the marginal production cost c.

Consequently, the less risky manufacturer's profit from offering contract $(\overline{w}, \overline{r})$ can be expressed as

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\theta}, \overline{\theta}\right) = \pi^\circ - \underbrace{\frac{2}{\beta\lambda^c}\left\{\lambda^c\left[s_r^\circ - s_r(\overline{w})\right]^2 + \lambda\left[s_s^\circ - s_s(\overline{w}, \overline{r})\right]^2\right\}}_{\text{less risky manufacturer's signaling cost}}, \tag{E.1}$$

and the riskier manufacturer's gain from mimicry can be expressed as

$$\begin{aligned}\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\theta}, \underline{\theta}\right) - \pi^\circ = & \lambda^c\Delta\theta \cdot \underbrace{\frac{2}{\beta(\lambda^c)^2\bar{\theta}}\left[\frac{\lambda^c\alpha_l}{2} - \lambda^c s_r(\overline{w}) + \lambda\bar{s}_s(\overline{w}, \overline{r})\right]\bar{s}_s(\overline{w}, \overline{r})}_{\text{returns cost}} \\ & \underbrace{- \frac{2}{\beta\lambda^c}\left\{\lambda^c\left[s_r^\circ - s_r(\overline{w})\right]^2 + \lambda\left[s_s^\circ - \bar{s}_s(\overline{w}, \overline{r})\right]^2\right\}}_{\text{signaling cost}}.\end{aligned} \tag{E.2}$$

We note that (E.1) and (E.2) are exactly the same as (4.6) and (4.7) in the paper, respectively. The only difference is that both s_r° and s_s° are lower than their counterparts in (4.6) and (4.7). We also note that any $(\overline{w}, \overline{r})$ such that $\bar{s}_s(\overline{w}, \overline{r}) = 0$ can always make the riskier manufacturer's gain from mimicry in (E.2) non-positive. Thus, separation is always feasible.

- If $s_s^\circ = 0$ (i.e., $\lambda\Delta\alpha \le \beta c < \alpha$), cheap separation is achievable (i.e., the symmetric-information contract is automatically separating and the less risky manufacturer's signaling cost is zero), because $s_r(\overline{w}) = s_r^\circ$ and $\bar{s}_s(\overline{w}, \overline{r}) = s_s^\circ = 0$ makes (E.2) equal to zero.
- Otherwise (i.e., $\beta c < \lambda\Delta\alpha$ and $s_s^\circ > 0$), as in the returns risk case in the paper (see Section 4), the efficient separation must entail *upward* distortion of the regular stock $s_r(\overline{w})$ and *downward* distortion of the safety stock $\bar{s}_s(\overline{w}, \overline{r})$, which translate to *downward* distortion of both the wholesale and returns prices.

E.2. Demand Potential.

Given a marginal production cost $c \ge 0$, the manufacturer's expected profit function (5.1) in the paper needs to be modified as

$$\begin{aligned}\Pi\left(w, r \,\middle|\, \widehat{\lambda}, \lambda\right) :=& (w-c)s^\star\left(w, r, 1, \widehat{\lambda}\right) - \frac{1}{2}\lambda^c r\left[\Delta\alpha - \beta/\widehat{\lambda}\,(w-r)\right]^+ \\ =& \frac{1}{2}(w-c)(\widehat{\alpha} - \beta w) + \frac{1}{2}\left[\widehat{\lambda}^c(w-c) - \lambda^c r\right]\left[\Delta\alpha - \beta/\widehat{\lambda}\,(w-r)\right]^+,\end{aligned}$$

where $\widehat{\alpha} := \alpha_l + \widehat{\lambda}\Delta\alpha$.

Accordingly, the asymmetric-information contract is given by

$$w^\circ(\lambda) \equiv \frac{\alpha + \beta c}{2\beta} \quad \text{and} \quad r^\circ(\lambda)\begin{cases}\in \left[0, \frac{\alpha_l + \beta c - \lambda\Delta\alpha}{2\beta}\right], & \text{if} \quad \beta c \ge \lambda\Delta\alpha, \\ = \frac{\alpha_l}{2\beta}, & \text{if} \quad \beta c \le \lambda\Delta\alpha,\end{cases}$$

where $\alpha := \alpha_l + \lambda\Delta\alpha$. The corresponding retailer's regular and safety stocks under symmetric information are given by

$$s_r^\circ(\lambda) = \frac{(\alpha - \beta c)^+}{4} \quad \text{and} \quad s_s^\circ(\lambda) = \frac{\lambda^c}{4}\left(\Delta\alpha - \beta c/\lambda\right)^+, \quad \text{respectively;}$$

and the manufacturer's profit is given by

$$\Pi^\circ(\lambda) = \frac{\left[(\alpha - \beta c)^+\right]^2}{8\beta} + \frac{\lambda^c\left[(\lambda\Delta\alpha - \beta c)^+\right]^2}{8\beta\lambda} = \frac{2}{\beta\lambda^c}\left\{\lambda^c\left[s_r^\circ(\lambda)\right]^2 + \lambda\left[s_s^\circ(\lambda)\right]^2\right\}.$$

Again, the effect of a positive marginal production cost is only to shift the symmetric-information wholesale price upward and the induced regular and safety stocks downward, relative to the case of zero marginal production cost in the paper (see Lemma 3). In particular, when $\beta c \ge \lambda\Delta\alpha$, the retailer's safety stock becomes zero (and hence no returns regardless of baseline demand realization).

For subsequent notational convenience, we denote

$$\begin{aligned}\bar{s}_r^\circ =& s_r^\circ(\bar{\lambda}), \quad \bar{s}_s^\circ = s_s^\circ(\bar{\lambda}), \quad \overline{\pi}^\circ = \Pi^\circ\left(\bar{\lambda}\right); \quad \text{and} \\ \underline{s}_r^\circ =& s_r^\circ(\underline{\lambda}), \quad \underline{s}_s^\circ = s_s^\circ(\underline{\lambda}), \quad \underline{\pi}^\circ = \Pi^\circ\left(\underline{\lambda}\right).\end{aligned}$$

When demand potential λ becomes the manufacturer's private information, the low-demand manufacturer still offers her symmetric-information contract and earns her symmetric-information profit $\underline{\pi}^\circ$ and the high-demand manufacturer distinguishes herself by offering a contract, say $(\overline{w}, \overline{r})$, which may need to be distorted away from her symmetric-information counterpart. Using the same change of variables as in the paper, we will work with the retailer's induced quantity decision:

$$\text{regular stock} \quad \bar{s}_r(\overline{w}) := \frac{1}{2}\left(\bar{\alpha} - \beta\overline{w}\right), \quad \text{and}$$

$$\text{safety stock} \quad \bar{s}_s(\overline{w},\overline{r}) := \frac{\bar{\lambda}^c}{2}\left[\Delta\alpha - \beta/\bar{\lambda}\,(\overline{w}-\overline{r})\right].$$

Again, we note that the retailer's quantity decision above is independent of the marginal production cost c.

Subsequently, the high-demand manufacturer's profit from offering contract $(\overline{w},\overline{r})$ can be expressed as

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\bar{\lambda}\right) = \overline{\pi}^\circ - \underbrace{\frac{2}{\beta\bar{\lambda}^c}\left\{\bar{\lambda}^c\left[\bar{s}_r^\circ - \bar{s}_r(\overline{w})\right]^2 + \bar{\lambda}\left[\bar{s}_s^\circ - \bar{s}_s(\overline{w},\overline{r})\right]^2\right\}}_{\text{high-demand manufacturer's signaling cost}}, \tag{E.3}$$

and the low-demand manufacturer's gain from mimicry can be similarly expressed as

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\underline{\lambda}\right) - \underline{\pi}^\circ = &\overline{\pi}^\circ - \underline{\pi}^\circ - \underbrace{\frac{2}{\beta\bar{\lambda}^c}\left\{\bar{\lambda}^c\left[\bar{s}_r^\circ - \bar{s}_r(\overline{w})\right]^2 + \bar{\lambda}\left[\bar{s}_s^\circ - \bar{s}_s(\overline{w},\overline{r})\right]^2\right\}}_{\text{signaling cost}} \\ &- \Delta\lambda \cdot \underbrace{\frac{2}{\beta(\bar{\lambda}^c)^2}\left[\frac{\bar{\lambda}^c\alpha_l}{2} - \bar{\lambda}^c\bar{s}_r(\overline{w}) + \bar{\lambda}\bar{s}_s(\overline{w},\overline{r})\right]\bar{s}_s(\overline{w},\overline{r})}_{\text{returns cost under contract } (\overline{w},\overline{r})}.\end{aligned} \tag{E.4}$$

We note that (E.3) and (E.4) are exactly the same as (5.5) and (5.6) in the paper, respectively. The only difference is that both s_r° and s_s° are lower than their counterparts in (5.5) and (5.6). We claim that the low-demand manufacturer's gain from mimicry must be non-positive for some $\bar{s}_r(\overline{w}) \in [0, \bar{s}_r^\circ]$ and $\bar{s}_s(\overline{w},\overline{r}) \in [\bar{s}_s^\circ, \frac{\bar{\lambda}^c}{2}\Delta\alpha]$. Namely, separation is always feasible. Indeed, substituting $\bar{s}_r(\overline{w}) = 0$ and $\bar{s}_s(\overline{w},\overline{r}) = \frac{\bar{\lambda}^c}{2}\Delta\alpha$ into (E.4) yields

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\underline{\lambda}\right) - \underline{\pi}^\circ = &\overline{\pi}^\circ - \underline{\pi}^\circ - \frac{2}{\beta\bar{\lambda}^c}\left\{\bar{\lambda}^c(\bar{s}_r^\circ)^2 + \bar{\lambda}\left[\bar{s}_s^\circ - \frac{\bar{\lambda}^c}{2}\Delta\alpha\right]^2 + \frac{\bar{\lambda}^c}{4}\Delta\lambda\Delta\alpha\bar{\alpha}\right\} \\ = &-\underline{\pi}^\circ - \frac{2}{\beta\bar{\lambda}^c}\left\{\bar{\lambda}\left[\bar{s}_s^\circ - \frac{\bar{\lambda}^c}{2}\Delta\alpha\right]^2 - \bar{\lambda}\left[\bar{s}_s^\circ\right]^2 + \frac{\bar{\lambda}^c}{4}\Delta\lambda\Delta\alpha\bar{\alpha}\right\} \le 0,\end{aligned}$$

where we note that (i) if $\bar{s}_s^\circ = 0$, then $\left[\bar{s}_s^\circ - \frac{\bar{\lambda}^c}{2}\Delta\alpha\right]^2 - [\bar{s}_s^\circ]^2 \ge 0$, and (ii) if $\bar{s}_s^\circ > 0$, then

$$\left[\bar{s}_s^\circ - \frac{\bar{\lambda}^c}{2}\Delta\alpha\right]^2 - [\bar{s}_s^\circ]^2 = \left[\frac{\lambda^c}{4}(\Delta\alpha + \beta c/\lambda)\right]^2 - \left[\frac{\lambda^c}{4}(\Delta\alpha - \beta c/\lambda)\right]^2 \ge 0.$$

Therefore, as in the demand potential case in the paper (see Section 5), the efficient separation must entail *downward* distortion of the regular stock $\bar{s}_r(\overline{w})$ and *upward* distortion of the safety stock $\bar{s}_s(\overline{w},\overline{r})$, which translate to *upward* distortion of both the wholesale and returns prices. In particular, as verified below, cheap separation is not achievable.

- If $\bar{s}_s^\circ > 0$ and $\underline{s}_s^\circ > 0$ (i.e., $\beta c < \underline{\lambda}\Delta\alpha < \bar{\lambda}\Delta\alpha$), substituting $\bar{s}_r(\overline{w}) = \bar{s}_r^\circ$ and $\bar{s}_s(\overline{w},\overline{r}) = \bar{s}_s^\circ$ into (E.4) yields positive mimicry gain for the low-demand manufacturer:

$$\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\underline{\lambda}\right) - \underline{\pi}^\circ = \frac{\Delta\lambda}{8\beta}\left[(\Delta\alpha)^2 - \frac{(\beta c)^2}{\bar{\lambda}\underline{\lambda}} + \alpha_l\Delta\alpha + \frac{\beta c}{\bar{\lambda}}\alpha_l\right] > 0.$$

- If $\bar{s}_s^\circ > 0$, $\underline{s}_s^\circ = 0$ and $\underline{s}_r^\circ > 0$ (i.e., $\underline{\lambda}\Delta\alpha \le \beta c < \min\{\underline{\alpha}, \bar{\lambda}\Delta\alpha\}$), substituting $\bar{s}_r(\overline{w}) = \bar{s}_r^\circ$ and $\bar{s}_s(\overline{w},\overline{r}) = \bar{s}_s^\circ$ into (E.4) yields positive mimicry gain for the low-demand manufacturer:

$$\begin{aligned}\Pi\left(\overline{w},\overline{r}\,\middle|\,\bar{\lambda},\underline{\lambda}\right) - \underline{\pi}^\circ = &\frac{1}{8\beta}\left[(\bar{\alpha}-\beta c)^2 + (\bar{\lambda}^c/\bar{\lambda})(\bar{\lambda}\Delta\alpha - \beta c)^2 - (\underline{\alpha}-\beta c)^2 - (\Delta\lambda/\bar{\lambda})\alpha_l(\bar{\lambda}\Delta\alpha - \beta c)\right] \\ = &\frac{1}{8\beta}\left[(1/\bar{\lambda})(\bar{\lambda}\Delta\alpha - \beta c)^2 - (\underline{\lambda}\Delta\alpha - \beta c)^2 + (2 - \Delta\lambda/\bar{\lambda})\alpha_l(\bar{\lambda}\Delta\alpha - \beta c)\right. \\ &\left.+ 2\alpha_l(\beta c - \underline{\lambda}\Delta\alpha)\right] > 0.\end{aligned}$$

- If $\bar{s}_s^\circ > 0$, $\underline{s}_s^\circ = 0$ and $\underline{s}_r^\circ = 0$ (i.e., $\underline{\lambda}\Delta\alpha < \underline{\alpha} \le \beta c < \bar{\lambda}\Delta\alpha$ and hence $\alpha_l < \Delta\lambda\Delta\alpha$), substituting $\bar{s}_r(\overline{w}) = \bar{s}_r^\circ$ and $\bar{s}_s(\overline{w}, \overline{r}) = \bar{s}_s^\circ$ into (E.4) yields positive mimicry gain for the low-demand manufacturer:

$$\begin{aligned}\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) - \underline{\pi}^\circ =& \frac{1}{8\beta}\left[(\bar{\alpha} - \beta c)^2 + (\bar{\lambda}^c/\bar{\lambda})(\bar{\lambda}\Delta\alpha - \beta c)^2 - (\Delta\lambda/\bar{\lambda})\alpha_l(\bar{\lambda}\Delta\alpha - \beta c)\right] \\ =& \frac{1}{8\beta}\left[\alpha_l^2 + (1/\bar{\lambda})(\bar{\lambda}\Delta\alpha - \beta c)^2 + (2 - \Delta\lambda/\bar{\lambda})\alpha_l(\bar{\lambda}\Delta\alpha - \beta c)\right] > 0.\end{aligned}$$

- If $\bar{s}_s^\circ = 0$, $\underline{s}_s^\circ = 0$ and $\bar{s}_r^\circ > \underline{s}_r^\circ > 0$ (i.e., $\underline{\lambda}\Delta\alpha < \bar{\lambda}\Delta\alpha \le \beta c < \underline{\alpha}$ and hence $\alpha_l > \Delta\lambda\Delta\alpha$), substituting $\bar{s}_r(\overline{w}) = \bar{s}_r^\circ$ and $\bar{s}_s(\overline{w}, \overline{r}) = \bar{s}_s^\circ$ into (E.4) yields positive mimicry gain for the low-demand manufacturer:

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) - \underline{\pi}^\circ = \frac{1}{8\beta}\left[(\bar{\alpha} - \beta c)^2 - (\underline{\alpha} - \beta c)^2\right] > 0.$$

- If $\bar{s}_r^\circ > 0$, $\bar{s}_s^\circ = 0$, $\underline{s}_s^\circ = 0$ and $\underline{s}_r^\circ = 0$ (i.e., $\underline{\lambda}\Delta\alpha < \max\{\bar{\lambda}\Delta\alpha, \underline{\alpha}\} \le \beta c < \bar{\alpha}$), substituting $\bar{s}_r(\overline{w}) = \bar{s}_r^\circ$ and $\bar{s}_s(\overline{w}, \overline{r}) = \bar{s}_s^\circ$ into (E.4) yields positive mimicry gain for the low-demand manufacturer:

$$\Pi\left(\overline{w}, \overline{r} \,\middle|\, \overline{\lambda}, \underline{\lambda}\right) - \underline{\pi}^\circ = \frac{1}{8\beta}(\bar{\alpha} - \beta c)^2 > 0.$$