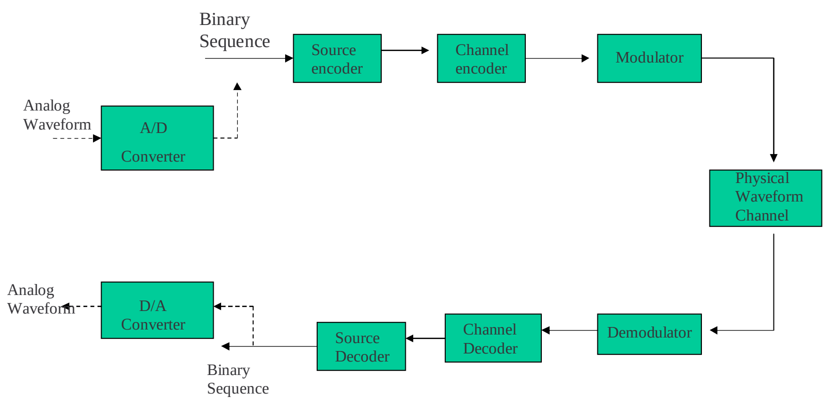
Introduction to Digital Modulation

Introduction

- In a digital communication system, the source to be transmitted is discrete both in time and amplitude
- Digital information carrying signals must be first converted to an analog waveform prior to transmission
- At the receiving end, analog signals are converted back to a digital format before presentation to the end user
- The conversion process at the transmitting end is known as modulation
- The receiving end is known as demodulation or detection

Functional Block Diagram of a Binary Digital Communication System



Digital Modulation

- Overview:
  - In digital wireless communication systems, the modulating signal may be represented as a time sequence of symbols or pulses, where each symbol has m finite states. Each symbol represents n bits of information where $n = \log_2 m$ bits/symbol.
- Advantages of Digital over Analog:
  - Greater noise immunity (due to its finite process)
  - Robustness to channel impairments
  - Easier multiplexing of various forms of information like voice, data, video

Digital Modulation

- Security – by using coding techniques to avoid jamming
- Accommodation of digital error control codes which detect and/or correct transmission errors
- Equalization to improve the performance of over all communication link
- Supports complex signal conditioning and processing methods

Digital Modulation

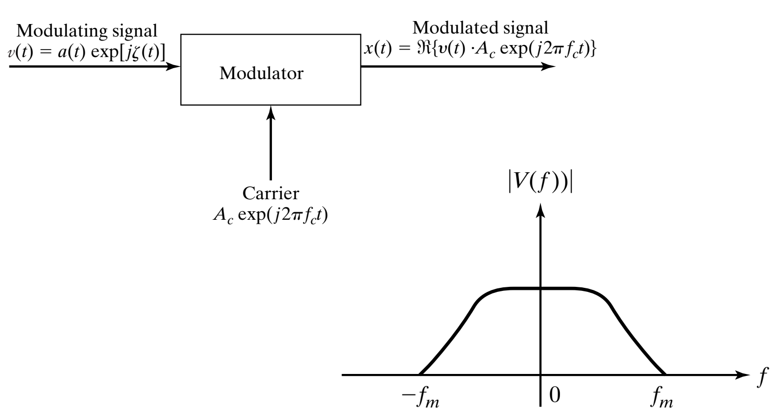
- Factors that influence Digital Modulation:
  - Low BER at low received SIR
  - performs well in multi-path and fading
  - High spectral efficiency
  - The performance of a modulation scheme is often measured in terms of its power efficiency and bandwidth efficiency
  - The power efficiency is the ability of a modulation technique to preserve the fidelity (acceptable BER) of the digital message at low power levels
  - (Good BER performance at a low SIR under conditions of co-channel interference, fading, and time dispersion)

Digital Modulation

- The source information is normally represented as a baseband (low-pass) signal
- Because of signal attenuation, it is necessary to move the baseband signal spectrum to reside at a much higher frequency band centered at f_c, called the carrier frequency, in the radio spectrum
- At the receiver end, the demodulation process removes the carrier frequency to recover the baseband information signal
- Choose different carrier frequencies for different signals
  - Modulation/demodulation process facilitates channel assignment and reduces interference from other transmissions.

Generic Bandpass Transmission

- A_c is a constant denoting the amplitude carrier



Modulation Catagories

- The modulated signal, x(t), is given by

$$\begin{aligned} x(t) &= \Re\{v(t) A_c \exp(j2\pi f_c t)\} \\ &= A_c a(t) \cos(2\pi f_c t + \zeta(t)) \\ &= A_c a(t) \cos\zeta(t) \cos(2\pi f_c t) - A_c a(t) \sin\zeta(t) \sin(2\pi f_c t). \end{aligned}$$

- The modulation can be classified into two categories:
  - Linear modulation: A modulation process is *linear* when both $a(t)\cos\zeta(t)$ and $a(t)\sin\zeta(t)$ terms are linearly related to the message information signal.
  - Nonlinear modulation: when the modulating signal, $v(t)$, affects the frequency of the modulated signal. The definition of nonlinear is that superposition does not apply.

Modulation Examples

- Examples of linear modulation include
  - amplitude modulation, where the modulating signal affects only the amplitude of the modulated signal (i.e., when $\zeta(t)$ is a constant $\forall$ t (for any t)), and
  - phase modulation (with a rectangular phase shaping function) where the modulating signal affects only the phase of the modulated signal (i.e., when $\zeta(t)$ is a constant over each signaling (symbol) interval and a(t) is a constant $\forall$ t).
- Example of the nonlinearly modulated signal is

$$x(t) = a \cdot A_c \cos(2\pi f_c t + \zeta(t))$$

where the angle, $\zeta(t)$ is the integral of a frequency function

Binary Digital Modulation Examples

- Binary amplitude shift keying (ASK): $\zeta(t)=0$.
  - amplitude component a(t):
  - a(t)=1 for symbol "1" and a(t)=0 for symbol "0".
  - The modulated signal is

$$x(t) = \begin{cases} A_c \cos(2\pi f_c t), & \text{symbol "1"} \\ 0, & \text{symbol "0"} \end{cases}.$$

- Binary phase shift keying (PSK): a(t)=1
  - the phase component $\zeta(t)$:
  - $\zeta(t)=0$ for symbol "1" and
  - $\zeta(t)=\pi$ for symbol "0"
  - The modulated signal is

$$x(t) = \begin{cases} A_c \cos(2\pi f_c t), & \text{symbol "1"} \\ A_c \cos(2\pi f_c t + \pi) = -A_c \cos(2\pi f_c t), & \text{symbol "0"} \end{cases}.$$

Binary Digital Modulation Examples

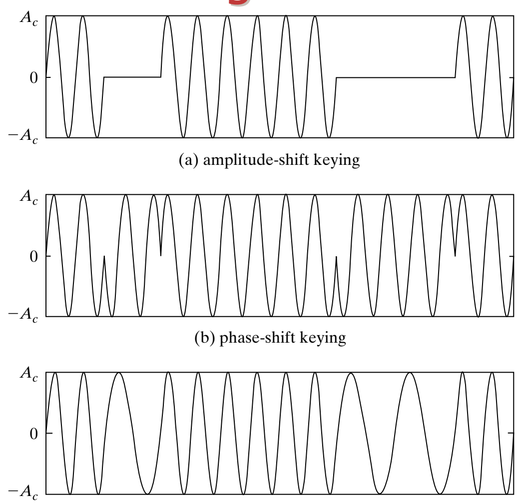
- Binary frequency shift keying (FSK): a(t)=1
  - $\zeta(t)$: $\zeta(t)=2\pi(\Delta/2)t + \phi_1$ for symbol "1"
  - $\zeta(t)=-2\pi(\Delta/2)t + \phi_2$ for symbol "0"
    - where Δ is the frequency separation between the signals for symbols "1" and "0" respectively, ϕ_1 and ϕ_2 are any constants in $[-\pi,+\pi]$.
  - The modulated signal is

$$x(t) = \begin{cases} A_c \cos[2\pi(f_c + 0.5\Delta)t + \phi_1], & \text{symbol "1"} \\ A_c \cos[2\pi(f_c - 0.5\Delta)t + \phi_2], & \text{symbol "0"} \end{cases}.$$

  - The instantaneous frequency of the modulated signal x(t) is
    - $f(t)=f_c+0.5\Delta$ for symbol "1" and $f(t)=f_c-0.5\Delta$ for symbol "0".
  - The instantaneous phase of x(t) is given by

$$\phi(t) = \int_{-\infty}^{t} 2\pi f(\tau)\, d\tau.$$

Modulated Signal Waveforms



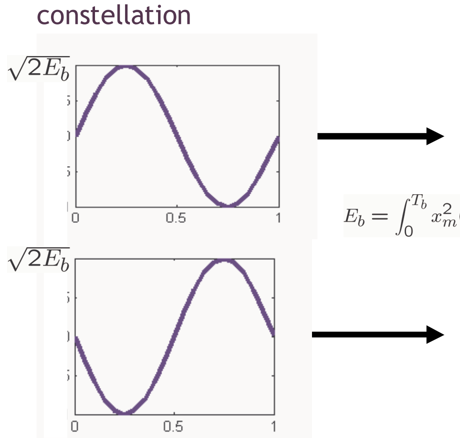
(a) amplitude-shift keying

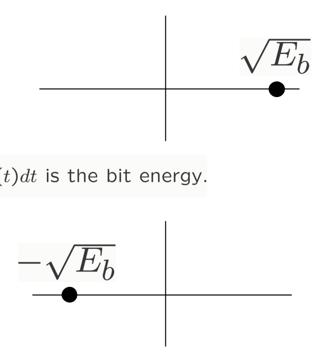
(b) phase-shift keying

(c) frequency-shift keying

Constellation Representation

- Typical signal waveforms for BPSK transmission and constellation



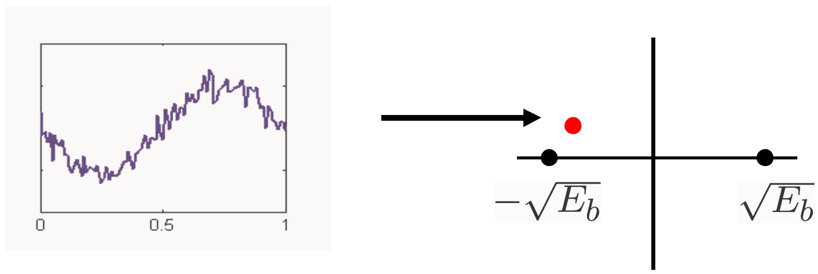


$E_b = \int_0^{T_b} x_m^2(t)dt$ is the bit energy.

Decision Regions

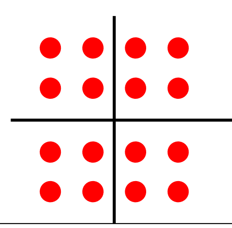
- Waveforms in noise
- Noise adds uncertainty to the location of the signal state

Boundary of two decision regions



Signal Space and Decision Regions

- Time domain → Signal space domain representation
- Vector-Space Representation of M-ary Signals
- The digital source generates digital symbols for transmission at a rate of R_s symbols per second.
- The symbols are taken from an alphabet of size $M=2^l$
- Each symbol can be represented by l binary digits.
- The transmission rate: $R_b=lR_s$ bits per second (bps), where R_b is the bit rate.



Higher size constellation

Common Digital Modulation Techniques

- M-ary Phase Shift Keying (MPSK):
- During the signaling interval, T_s, one of the waveforms is selected

$$x_m(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi(m-1)}{M}\right), \quad 0 \le t \le T_s, \quad m = 1, 2, \cdots, M,$$

where E_s is the symbol energy given by

$$E_s = \int_{-\infty}^{\infty} x_m^2(t)\, dt$$

- As each waveform represents l binary digits, we have $E_s = lE_b$ where E_b is the bit energy.

Differential PSK (DPSK)

- DPSK is noncoherent form of phase shift keying which avoids the need for a coherent reference signal at the receiver.
- Input binary sequence is first differentially encoded and then modulation using a BPSK modulator.
- Differentially encoded sequence $\{d_k\}$ is generated from the input binary sequence $\{m_k\}$ by complementing the modulo-sum of m_k and d_{k-1}
  - Effect is to leave the symbol d_k unchanged from the previous symbol if m_k is 1, and toggle d_k if m_k is 0.

$$d_k = \overline{m_k \bigoplus d_{k-1}}$$

- Decoding

$$m_k = \overline{d_k} \bigoplus d_{k-1}$$

- Demodulation

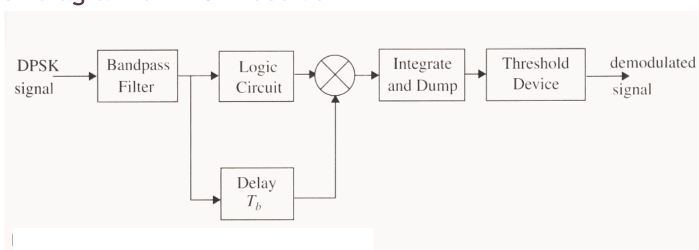
$$x_k = \overline{r_k} r_{k-1}$$

DPSK

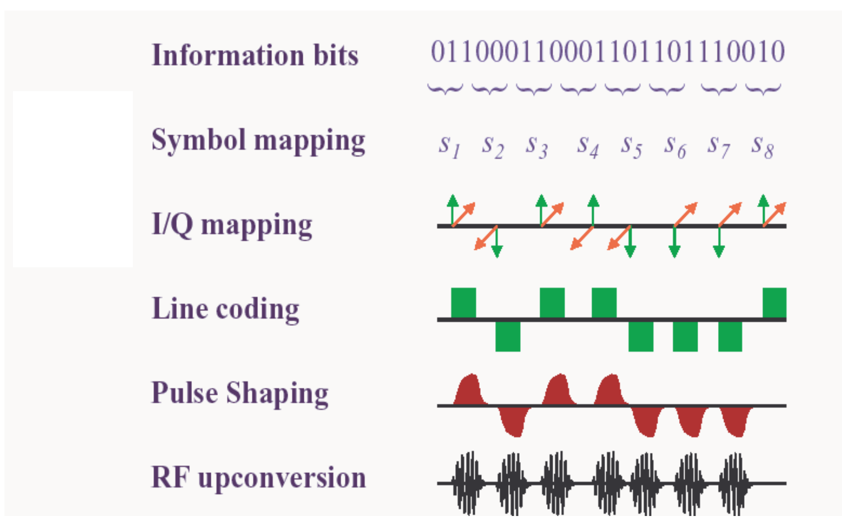
- Illustration of the differential encoding process

$\{m_k\}$		1	0	0	1	0	1	1	0
$\{d_{k-1}\}$		1	1	0	1	1	0	0	0
$\{d_k\}$	1	1	0	1	1	0	0	0	1

- Block diagram of DPSK receiver



Digital Modulation Stages



Pulse Shaping → Bandwidth Limitation

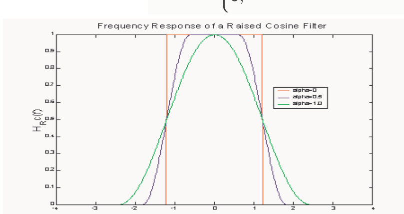
- Why pulse shaping?
  - ISI can be minimized by increasing the channel bandwidth
  - Mobile communication systems operate with minimal BW
  - Hence pulse shaping techniques are used to reduce ISI and spectral BW
- Nyquist criterion for ISI Cancellation:
  - Effects of ISI could be completely nullified if, at every sampling instant, the response due to all symbols except the current symbol is made equal to zero

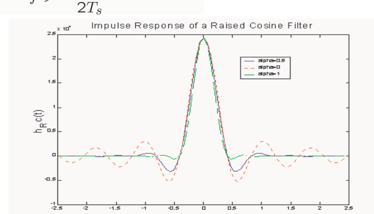
$$p(nT_s) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Raised Cosine Filter

- Raised Cosine Roll-off filter:
  - It satisfies the Nyquist criterion
  - The spectral efficiency offered by raised cosine filter only occurs if exact pulse shape is preserved at the carrier
- The transfer function of raised cosine filter

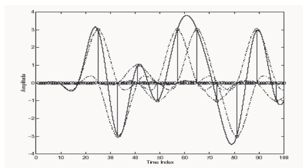
$$|H(f)| = \begin{cases} 1, & 0 \le f \le \frac{1-\alpha}{2T_s} \\ \sqrt{\frac{1}{2}\left\{1 - \sin\left[\frac{\pi(2fT_s - 1)}{2\alpha}\right]\right\}}, & \frac{(1-\alpha)}{2T_s} \le f \le \frac{(1+\alpha)}{2T_s}, \\ 0, & f > \frac{(1+\alpha)}{2T_s} \end{cases}$$

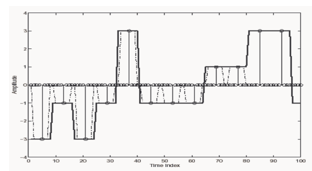


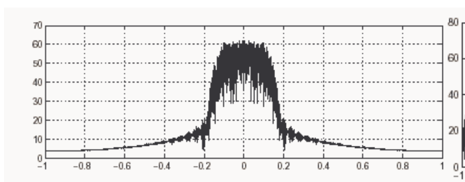


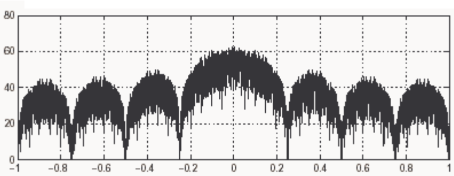
Signal Spectrum after Pulse Shaping

- After pulse-shaping before pulse-shaping



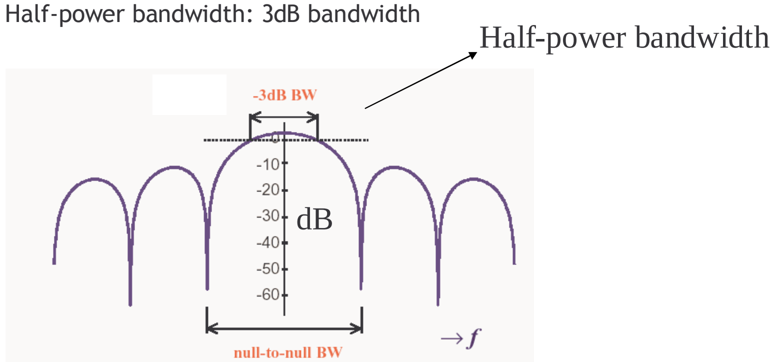






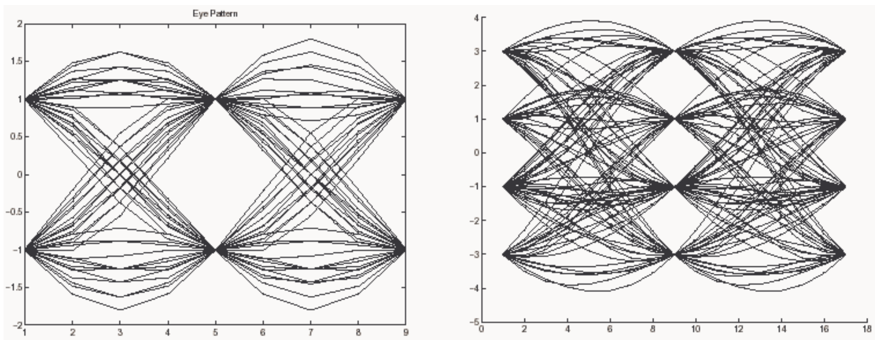
Bandwidth Definitions

- Measures of Bandwidth (BW):
  - 99% BW→ freq. range where 99% of power is
  - Absolute BW : Range of frequencies over a non-zero spectrum
  - Null-to-Null BW : Width of the main spectral lobe
  - Half-power bandwidth: 3dB bandwidth



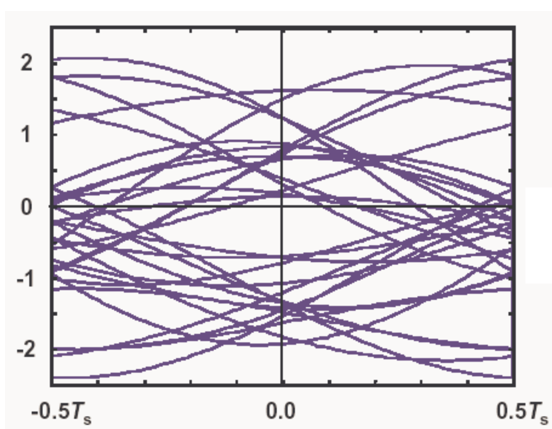
Eye Diagram

- Below figures show an eye diagram:
  - Examples of 2-level and 4-level eye patterns
- The wider the “eye” opens, the better the signal quality is.
- Empirical measure of the quality of the received signal.



Intersymbol Interference

- Intersymbol interference channel



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Probability of Transmission Error

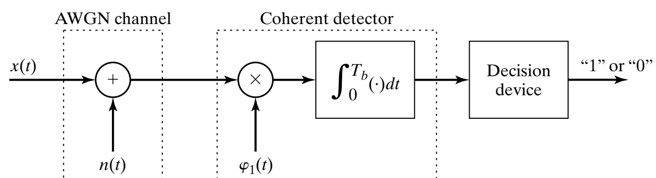
- Coherent Reception in an AWGN Channel
- BPSK: The transmitted signal

$$x(t) = \begin{cases} \sqrt{E_b}\varphi_1(t), & \text{for symbol "1"} \\ -\sqrt{E_b}\varphi_1(t), & \text{for symbol "0"} \end{cases},$$

- In an AWGN channel, the received signal is

$$r(t) = x(t) + n(t)$$

  - where n(t) represents the white Gaussian noise process with zero mean and two-sided psd $N_0/2$.
- Coherent reception of BPSK in an AWGN channel



Probability of Bit Error

- The conditional probability at the output of correlator:

$$f_{r_1}(x) = \frac{1}{\sqrt{2\pi}\sigma_N}\exp\left[-\frac{(x-\mu_i)^2}{2\sigma_N^2}\right], \quad i = 0, 1; -\infty < x < \infty.$$

- The decision rule is as follows:

$$\begin{cases} \text{if } r \geq 0, \text{ symbol "1" was sent;} \\ \text{if } r < 0, \text{ symbol "0" was sent.} \end{cases}$$

- With equally likely symbols "1" and "0", the probability of symbol (bit) error,

$$\begin{aligned} P_b &= P(r \geq 0|\text{symbol "0" was sent})P(\text{symbol "0" was sent}) \\ &\quad + P(r < 0|\text{symbol "1" was sent})P(\text{symbol "1" was sent}) \\ &= \frac{1}{2}\left\{\int_0^{\infty}\frac{1}{\sqrt{2\pi}\sigma_N}\exp\left[-\frac{(x-\mu_0)^2}{2\sigma_N^2}\right]dx + \int_{-\infty}^{0}\frac{1}{\sqrt{2\pi}\sigma_N}\exp\left[-\frac{(x-\mu_1)^2}{2\sigma_N^2}\right]dx\right\} \\ &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad \text{where } Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^{\infty}\exp(-z^2/2)dz \end{aligned}$$